Fermionic Dark Matter: From models to collider searches

A thesis submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy

in the
Grupo de Fenomenología de Interacciones Fundamentales (GFIF)
Instituto de Física

Author: Amalia Betancur Rodríguez

Advisor: Dr. Óscar Alberto Zapata
Co-Advisor: Dr. Diego Alejandro Restrepo

February 7, 2019
Declaration of Authorship

I, Amalia Betancur Rodríguez, declare that this thesis titled, “Fermionic Dark Matter: From models to collider searches” and the work presented in it are my own. This thesis is based on the following works:


In this thesis, we investigate from diverse point of views, the dark matter problem. First, we study the doublet-triplet fermion model, a simple extension of the Standard Model with an extra $Z_2$ symmetry. In this extension, it is possible to have a dark matter candidate at the electroweak scale that evades current strong direct detection constraints. We also include a double-triplet scalar in order to generate neutrino masses at loop-level and to relax the tension on the fermion sector from the current Higgs diphoton decay measurement. In the second part, we again consider the doublet-triplet fermion model but this time under a non-standard cosmology and multi-component dark sectors scenarios. We study restrictions on the model from collider searches, direct detection, and indirect detection experiments. In the third part of this work, we study the case of dark matter production at the LHC as the end product of a short cascade event and we study how to constrain it. We use the Matrix Element Method in order to show that even with very little information, it is possible to obtain the value of the most relevant parameters of the event.
Acknowledgements

I would like to thank everyone that has been part of this long journey, especially to my family: my husband, sons, and parents who have always been there for me. I want to give many thanks to my advisor for teaching me so much with patience and understanding and mostly for putting up with me. I am very grateful to all the students at GFIF, especially the ones I got to share an office with. Without their help and support, I would have never completed this work. Last but not least, I would also like to thank the professors at GFIF, including my co-advisor, I learned a lot from all of them.
# Contents

Declaration of Authorship iii

Abstract v

Acknowledgements vii

Contents ix

1 Introduction 1

2 Theoretical Background 5
  2.1 The Standard Model ........................................ 5
    2.1.1 The SM Lagrangian .................................. 6
    The gauge Lagrangian ........................................ 7
    The Higgs Lagrangian ....................................... 7
    The Fermion and Yukawa Lagrangian ......................... 10
    Gauge Boson interactions after SSB: ......................... 12
    Quantum Chromodynamics (QCD): ............................. 13
  2.2 The Standard Model successes and shortcomings: ........ 14
    2.2.1 The DM problem: ..................................... 14
    WIMP Relic density ........................................ 17
    DM detection ............................................... 18
    2.2.2 DM and neutrinos ..................................... 25
  2.3 Summary .................................................. 28

3 The Doublet Triplet DM with neutrino masses 29
  3.1 The Model .................................................. 30
    3.1.1 Scalar sector ......................................... 31
    3.1.2 Fermion sector ....................................... 32
  3.2 DM phenomenology .......................................... 33
    3.2.1 Fermion DM ........................................... 34
    3.2.2 Scalar DM ............................................ 40
  3.3 Neutrino masses ............................................ 41
  3.4 Summary .................................................. 44
List of Figures

2.1 Behavior of the tree-level potential for the case $\mu^2 > 0$ in the top figure and $\mu^2 < 0$ in the lower figure. ........................................ 8

2.2 An example of the discrepancy between the observed rotational curve followed by stars within the galaxy (shown with yellow and blue dots with error bars) and the expected rotational curve according to gravitational laws for the M33 galaxy, from Wikipedia, Public Domain. ... 15

2.3 Upper limits on SI WIMP-nucleon scattering cross section provided at 90% C.L published by the LUX collaboration in [71] ................... 20

2.4 A slice of the CMS experiment showing the different detection forms [85]. ................................................................. 22

2.5 Feynman diagrams for a two body semi-invisible decay and double sided semi-invisible decay where the variables inside the parenthesis represents the particles four-momentum. .................. 25

2.6 Feynman diagrams as shown in [104] for tree-level realizations of the $d = 5$ Weinberg operator, where the mass is generated via the exchange of a singlet fermion (type I), a triplet scalar (type II) and a triplet fermion (type III) .................................................. 26

2.7 Feynman diagram for the radiative seesaw showing that neutrino mass generation after SSB. The loop is mediated by the right handed neutrino ($N_k$), and the neutral CP-even and CP-odd scalars ($H^0$ and $A^0$) [28]. ................................................................. 27

3.1 Possible fermion DM annihilation channels in the early universe. The annihilation occurs via the exchange of one of the heavier $Z_2$-odd fermions into either neutral or charged weak gauge bosons as shown. The left panel shows annihilation into charged weak gauge bosons. The right panel shows DM annihilation into $Z$ bosons via the exchange of a neutral particle. In both cases, the top figure shows the $t$-channel while the bottom one shows the $u$-channel. .......................... 34

3.2 The triplet fermion mass as a function of the dark matter mass for the symmetric case ($y_1 = y_2 = y$) with $\chi_1^0$ being the DM particle. The color code denotes the allowed values of $y$ and the dashed line corresponds to the points satisfying $M_\Sigma = -M_\Psi$. ................................. 35
3.3 Diagrams representing the main contribution to the spin-independent direct detection. The field \( \chi \) represents either a charged \( Z_2 \)-odd fermion or a heavier neutral one, in which case the mediating gauge boson is charged or neutral, respectively.  

3.4 Parameter space of the electroweak DM region accounting for the observed DM relic abundance and consistent with direct and indirect searches of DM. In the top row the color bar corresponds to the allowed values of \( M_\Sigma \) while in the bottom row it corresponds to the allowed values of \( \delta_m \), as defined in the text. The region in the left (right) panels satisfies \( M_\Sigma < -m_{\chi_1^0} \) (\( M_\Sigma > -m_{\chi_1^0} \)), which corresponds to the region below (above) the dashed line in Fig. 3.2. Solid, dashed and dotted-dashed lines represent the maximum value for \( m_{\chi_1^0} \) consistent with the Higgs diphoton decay rate reported in Refs. [119], [120] and [121], respectively (see text for details).  

3.5 Feynman diagrams of the Higgs diphoton decay induced by the \( Z_2 \) odd charged particles. The left diagram (A) shows the contribution from the charged fermions. The two diagrams on the right (B) show the contribution from the charged scalars.  

3.6 The combination \( \lambda_3 + \lambda_3' \) as a function the \( m_{\chi_1^0} \) for the set of points that present \( R_{\gamma\gamma} \) within the experimental limits. The color bar shows the relation between the charged scalar \( \kappa_1 \) and the dark matter mass. The left figure corresponds to the region where \( M_\Sigma \) is always negative and the right figure to the region where \( M_\Sigma \) is mostly positive. All points satisfy, additionally, perturbativity, vacuum stability, relic abundance, direct detection, and electroweak precision constraints.  

3.7 Feynman diagrams leading to one-loop neutrino masses. For the type a topology there are two Feynman diagrams, one with charged particles running in the loop and one with neutral particles, only the neutral one is shown.  

4.1 Scan on the parameter space of the model against \( R_{\gamma\gamma} \) for the region where \( M_\Sigma < -m_{\chi_1^0} \) (left panels) and the region where \( M_\Sigma > -m_{\chi_1^0} \) (right panels). The solid and dashed horizontal lines represent the lowest bound at a 2\( \sigma \) deviation from the central value reported by the ATLAS and CMS collaboration respectively, while the color indicates the value of \( M_\Sigma \).  

4.2 Same as Fig. 4.1 but with the color indicating the value of the Yukawa coupling.  

4.3 Impact of the cosine of the mixing angle \( \theta \) on the Higgs diphoton decay rate for the allowed values of the DM mass. The conventions are the same as those of Fig. 4.1.
4.4 DM mass versus lightest chargino mass for the regions where \( M_\Sigma < -m_{\chi_1^0} \) (left panel) and \( M_\Sigma > -m_{\chi_1^0} \) (right panel). The region below the blue dashed line is excluded from CMS electroweak production while the regions bounded by the blue (green) solid line represents the exclusion by the ATLAS (CMS) collaboration using compressed spectra. Points below the solid (dashed) black contour are excluded by the \( R_{\gamma\gamma} \) results reported by the ATLAS (CMS) collaboration.

4.5 Spin-independent cross section for the regions \( M_\Sigma < -m_{\chi_1^0} \) (left) and \( M_\Sigma > -m_{\chi_1^0} \) (right). The blue curve represents the upper limit imposed by XENON1T \[152\] while the green curve shows the projected sensitivity of DARWIN \[153\]. Points in the region \( M_\Sigma < -m_{\chi_1^0} \) that are above the red curve are excluded by CMS electroweak production while points in the region \( M_\Sigma > -m_{\chi_1^0} \) that are below the black solid curve are excluded by ATLAS search on compressed spectra. The color bands indicate the value of the Yukawa coupling (shown in the top) and the value of the triplet mass (shown in the bottom). The black dashed line represents the limit when the \( R_{\gamma\gamma} \) restriction from CMS is considered. All points satisfy the ATLAS \( R_{\gamma\gamma} \) restriction.

4.6 ID restrictions and prospects coming from the observation of dSphs of the Fermi satellite applied to the regions \( M_\Sigma < -m_{\chi_1^0} \) (left panel) and \( M_\Sigma > -m_{\chi_1^0} \) (right panel). The blue and green curve show current limits from the \( W^+W^- \) channel for 6 years of observation and 15 dSphs, and the projected sensitivity for 45 dSphs and 15 years of observation, respectively. Points below the black dashed line are excluded when the CMS experiment \( R_{\gamma\gamma} \) restriction is considered, while points left to the red curve on the right panel are excluded from ATLAS compressed spectra searches, as shown in Sec 4.2.3.

4.7 \( \epsilon_{\chi_1^0} \) vs. \( m_{\chi_1^0} \) for \( M_\Sigma < -m_{\chi_1^0} \) (left panel) and \( M_\Sigma > -m_{\chi_1^0} \) (right panel). All points satisfy collider bounds presented in Sec. 4.2.

4.8 Direct detection results for \( M_\Sigma < -m_{\chi_1^0} \) (left panel) and \( M_\Sigma > -m_{\chi_1^0} \) (right panel), the conventions are the same as in Fig. 4.5.

5.1 The event topologies considered in this chapter: (A) single and (B) pair production of a \( W \)-like resonance decaying leptonically.

5.2 Unit-normalized lepton \( P_{T\ell} \) distributions for single \( W \) production in the limit of \( \Gamma_W = 0 \). The invisible particle mass \( M_{\nu} = 500 \text{ GeV} \), and the mass of the parent particle is varied as shown.

5.3 Unit-normalized distributions of the lepton transverse momentum \( P_T \) (left) and longitudinal momentum \( P_z \) (right), for different values of \( M_W \) and \( \Gamma_W = 0 \) obtained using analytical expressions and, in the case of \( P_z \), pdfs from LHAPDF \[198\]. The mass \( M_{\nu} \) of the invisible particle has been fixed from the measurement (5.11).
5.4 $\chi^2/d.o.f.$ fit to $M_\nu$ from $P_{T_{\ell z}}$ templates in $W^+$ production (left) and $W^-$ production (right). The dashed vertical line represents the minimum.

5.5 The same as Fig. 5.2, but for fixed $M_W = 1000$ GeV, and several values of the width $\Gamma_W$ as shown.

5.6 Results from a $\chi^2$ fit to the one-dimensional $P_{T_{\ell z}}$ distribution for a data sample of 10,000 events. The fitted parameters are $M_\nu$ and $\Gamma_W/M_W$, with $M_W$ computed from (5.11). The study point ($\times$) has $M_W = 1000$ GeV, $M_\nu = 500$ GeV, and $\Gamma_W = 50$ GeV. The dashed line marks the flat direction (5.14). The color bar indicates the $\chi^2/d.o.f.$ (we used 100 bins).

5.7 One-dimensional scan in $M_\nu$ for a fixed width of 5% and $M_W$ given by eq. (5.12) (left) and a one-dimensional scan in $\Gamma_W$ with $M_W$ and $M_\nu$ fixed (right), for the $W^+$ sample, where the likelihood has been calculated using 10,000 events.

5.8 A simultaneous measurement of the mass $M_W$ and the width, $\Gamma_W$, of the heavy resonance with the MEM method for $W^+$ (left) and $W^-$ (right) production. The $\times$ (+) marks the input values (the result from the fit). The dashed line represents the relation (5.14). Contours represent the negative log likelihood from 1,000 events.

5.9 Left: A fit to the chirality of the quark and lepton couplings to the heavy resonance. The input study point has $M_W = 1000$ GeV, $\Gamma_W = 50$ GeV, $\varphi_{\ell} = 0$, and $\varphi_q = 0$. Contours represent the negative log likelihood from 1,000 events. Right: Lepton $P_z$ distributions for different chiralities, obtained from analytic expressions and LHAPDF pdfs [198] for $M_W = 1000$ GeV and $\Gamma_W = 0$.

5.10 Simultaneous measurement of all four parameters: the daughter particle mass $M_\nu$ (blue), the parent particle mass parameter $\mu$ defined in (5.11) (orange), the parent particle width parameter $\gamma$ defined in (5.17) (green), and the relative chirality $\varphi_{\text{rel}}$ defined in (5.16) (black). We show normalized distributions of the measured values for each parameter over 100 samples of 1000 events each, using MEM. The input values for our study point were $M_\nu = 500$ GeV, $M_W = 1000$ GeV, $\Gamma_W = 50$ GeV and $\cos^2 \varphi_{\text{rel}} = 0.5$, which translates into $2\mu = 750$ GeV and $\gamma = 62.5$ GeV.

5.11 The same as Fig. 5.5, but for the case of pair production, where we study the $M_{T2}$ distribution instead of $P_T$. In the lower panel we show the bin-by-bin ratio of the number of events for different widths, normalized to the case of $\Gamma_W = 50$ GeV.
5.12 The same as Fig. 5.3, but for the case of pair production. We showcase several variables whose distributions are sensitive to the overall mass scale: the dilepton invariant mass $m_{\ell\ell}$ (upper left), the transverse lepton momentum $P_{\ell T}$ (upper right), the larger of the two longitudinal lepton momenta (in absolute value), $\max(|P_{\ell z}|, |P_{\bar{\ell} z}|)$ (lower left), and the other longitudinal lepton momentum with its sign chosen as $\text{sgn}(P_{\ell z} P_{\bar{\ell} z}) \min(|P_{\ell z}|, |P_{\bar{\ell} z}|)$ (lower right).

5.13 The same as the left panel in Fig. 5.8, but for pair-production, i.e. the second diagram in Fig. 5.1. Contours represent the negative log likelihood from 500 events. The $\times$ (+) marks the input values (the result from the fit).

5.14 A fit to the chirality of the lepton couplings in the case of pair production as in the second diagram of Fig. 5.1 using 600 events. In the left panel, the couplings were chosen to be purely chiral, $\varphi_\ell = 0$, while in the right panel they were vectorlike, $\varphi_\ell = 45^\circ$.

A.1 Topologies that lead to the annihilation of DM into two photons. The external straight lines represent the DM particles, whereas the internal ones represent a charged $Z_2$ odd fermion (shown with a cyan solid line), gauge boson or Goldstone boson (shown with a black solid line). The external wavy lines represent the photons (the gamma-rays).
List of Tables

2.1 Bosonic content of the SM. ................................. 6
2.2 Fermionic content of the SM. ............................... 6

5.1 The extent to which various observables depend on the input parameters (5.6). ✓ (∼, ×) indicates strong (weak, no or almost no) dependence. 71
A mi familia.
Chapter 1

Introduction

The Large Hadron Collider (LHC) has closed an important chapter in particle physics with its finding of the Higgs boson by the CMS and ATLAS collaborations. The particle content of the Standard Model (SM) is now complete, however, some questions remain unanswered:

- What is the origin of the neutrino masses?
- What is the particle nature of the Dark Matter (DM)?
- What is the origin of the matter anti-matter asymmetry?

The preferred DM candidate is a new particle [1, 2], for instance, a Weakly Interacting Massive Particle (WIMP). The preference on particle DM relies on the fact that with it is possible to explain the DM behavior at very different scales in the universe, from stellar formation to BBN and CMB [1, 2]. Many beyond the Standard Model (BSM) theories have a natural DM as a WIMP, such as the neutralino in Supersymmetry (SUSY), the lightest Kaluza-Klein particles in models with Universal Extra Dimension, Little Higgs models, etc [3]. Another approach is, rather than having a whole new theory, with a slew of particles, symmetries and/or dimensions, just extend the SM with a few fields and symmetries. These types of models are referred to as Simple Extensions of the SM. In this category, the DM may be a scalar, fermion or vector boson. In the scalar sector, some well studied models are the singlet scalar DM model [4–6], the inert doublet model (IDM) [7, 8] and inert triplet model (ITM) [9–11]. On the other hand, for fermion models, we have the singlet fermion [12–14], the singlet-doublet fermion [15–18] and the doublet-triplet fermion [19]. These models share also the fact that the DM may communicate with the SM through the Higgs portal, hence becoming even more relevant after the discovery of the Higgs boson [20].

In the doublet-triplet fermion dark matter (DTFDM) model [19] a vectorlike doublet with $Y = -1$ and a Majorana triplet are added to the SM, both odd under a $Z_2$ symmetry that stabilizes the DM. The model includes invariant and renormalizable terms that mix the new fields, and so the particle spectrum contains two charged fermions and three Majorana fermions, with the lightest Majorana fermion being the DM candidate. The viable DM regions are the ones featuring masses around the
electroweak scale and above 1 TeV [19–21]. When the DM particle is mainly doublet (triplet), the correct relic abundance is explained for DM masses around $\sim 1 (2.8)$ TeV and when the mixing is arbitrary. But when the dark sector is invariant under an $SU(2)_R$ symmetry, the abundance can be correctly explained for low masses, $\lesssim 100$ GeV. Such global symmetry is known to be broken the SM but its breaking is related to the $\rho$ parameter. Now, in order to obtain the correct relic density for the later region, there must be a large splitting between the new charged fermions and the DM (which suppresses the annihilation channels), this has a direct impact on the Higgs diphoton decay, and as a result, the model is severely constrained by the $h \to \gamma\gamma$ measurement of the ATLAS and CMS collaboration [22, 23]. However, this conclusion can be modified if extra scalar charged particles are added in such a way that the Higgs diphoton decay is altered.

Taking into account the Higgs diphoton decay rate suppression and considering that the model fails to account for other evidence of physics beyond the SM, such as neutrino masses, we consider also an extension that includes a scalar sector, also odd under the $Z_2$ symmetry. In that case, neutrino masses are still exactly zero at tree-level but are non-vanishing at the one-loop level. The radiative seesaw [24], a thoroughly studied model, is an extension of the SM with an Inert Doublet and three generations of a right handed neutrino, all odd under the discrete symmetry. The model has a viable DM candidate and Majorana neutrino masses are generated in one particular realization of the Weinberg operator. Other works such as, [25] have studied models with this feature (DM and radiative neutrino masses in the same topology) while generating Majorana or Dirac masses. Another work [26], considered models with viable DM candidates and generating radiative Majorana neutrino masses in the topologies classified in [27]. For the case of the doublet-triplet fermion model, if we add both a doublet and a triplet scalars the resulting model, which corresponds to the T-1-2-F model with $\alpha = -1$ in Ref. [26], allows the generation of radiative neutrino masses through four different topologies, namely, T-3, T1-I, T1-II, and T1-III [27]. This model presents the complete set of irreducible topologies leading to realizations of the Weinberg operator at one loop [27, 28], with the interesting feature that all of the $Z_2$-odd fields have an active role in the neutrino mass generation. The model is discussed in Ch. 3, where we show that it is possible to satisfy constraints coming from $h \to \gamma\gamma$ and direct DM detection. Additionally, we calculate the neutrino mass-matrix and show that it is compatible with mixing angle data for sensible values of the relevant Yukawa couplings.

Another possibility in the fermionic sector of the model is to relax the relic density constraint, in such case the charged fermions may be closer to the DM mass which leads to a less restrictive suppression on the diphoton decay rate without the need for a scalar sector to counteract the suppression. There are good reasons to consider this approach, since, the standard WIMP relic density calculation makes assumptions of what happens to the universe (and the WIMP) previous to Big Bang Nucleosynthesis (BBN) that as of today we have no way to probe. If, for instance,
the entropy of matter-radiation is not conserved previous to BBN, or WIMP decoupling does not occur during the radiation dominated era or there is an underlying mechanism for non-thermal WIMP production, it has been shown that the relic density can be significantly different than the one from the standard calculation without necessarily affecting other DM observables [29]. Moreover, there are models that naturally predict a non-standard cosmology prior to BBN such as gravitino, moduli, thermal inflation, etc [30]. Works such as [30–34] have shown that non-standard cosmology scenarios may have a big impact in the relic density, thus, generating abundances that are enhanced or suppressed compared to the standard case. Moreover, in a more recent work a scalar Higgs portal model was studied under different non-standard scenarios [35]. Taking this into account, we set out to investigate experimental constraints on the fermionic sector of the model (DTFDM) with DM at the electroweak scale assuming that the relic abundance arises from a non-standard cosmology scenario prior to BBN in Ch. 4. In this regard, we look at current constraints (and in some cases prospects) for collider, direct detection (DD) and indirect detection (ID) experiments. We find that some portions of the parameter space are already ruled out, while in the upcoming years most of the allowed values for the free parameters will be probed.

Since the discovery of the Higgs boson, the LHC physics programme includes large efforts to do precision measurements of Higgs physics and to find and constrain BSM physics. Both of these require difficult analysis due to the complexity of the problems. Very often this comes in the form of multivariate analysis such as boosted decision trees or neural networks [36]. Another type of such methods is the Matrix Element Method (MEM) which consists on using the knowledge of the expected physics that gives rise to events in a collider in order to calculate a likelihood that the events were the results of different ansatz of the unknown quantities (masses, couplings, widths, etc.). The Feynman amplitude of the assumed physics setup is used to compute the likelihood. The ansatz that yields the highest likelihood is thus the correct value of the unknown quantities. The MEM was developed at Tevatron for the measurement of the top quark mass, where it proved to be successful [37–40]. More recently, it has been used to find the spin and parity of the Higgs boson [41] and properties of its decay in the four lepton channel [42]. The method could be used as well to constrain BSM physics discovered at the LHC. In the case of DM it could prove to be very useful, since, in many models with a DM candidate, the lightest neutral particle is expected to be accompanied by other particles, thus the DM could be produced as the end product of a heavier BSM resonance, none of this could be reconstructed by the detectors. For this reason, in Ch. 5, we use the MEM to show that in such scenario, it is possible to find the masses, couplings and width of the new particles assuming that the production arises from two simple topologies which are the most challenging cases. The method is computationally involved but we show that with the publicly available package MadWeight [43], it is feasible to find the likelihood for the different parameters to be constrained and that
those ansatz which give the highest likelihood are in fact the correct ones.
Chapter 2

Theoretical Background

In this chapter, we give an overview of the SM of particle physics and study the reasons for the need for an extension of the model, such as the neutrino mass and DM problem. For the DM problem, we focus on weakly interacting massive particles WIMPs and overview how the relic density is achieved as well as its various detection mechanism. For neutrinos, we review the evidence for their mass and we focus on how this could be connected to the DM problem through scotogenic models.

2.1 The Standard Model

The SM of particle physics is the best-known description of the subatomic world. The model is based on the idea that nature follows certain symmetries, in particular, the model is based on the local gauge symmetries $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. In the context of these symmetries three forces of nature are described: the strong, weak and electromagnetic forces.

The electromagnetic force is perceived in a wide variety of daily life phenomena from lightning to the proper functioning of the computer used to write this thesis. The strong and weak forces are more subtle. The strong force is responsible for holding the protons and neutrons together in the nucleus, and the weak force is responsible for the decay of a neutron into a proton, a positron and a neutrino. For the symmetries to be local, mediators of the forces have to exist, these are the 8 gluons, the weak $W^\pm$ and $Z$ bosons, and the photon. But forces act upon matter and so the SM includes matter fields which are all fermions and are divided into quarks and leptons, each of them organized into three families or flavors. Only the quark sectors carry charges under the $SU(3)_C$ symmetry group, in fact, they are organized in triplets whereas the leptons do not carry color, they are singlets. Under the $SU(2)_L$ all fermions transform differently according to their chirality, left-handed fields transform as doublets while right-handed fields are singlets. Additionally, all matter fields carry hypercharge $Y$ which accounts for the strength of the $U(1)_Y$ interaction.

On the other hand, the symmetries of the model, or gauge invariance, impose that all vector bosons or mediators must be massless, however, this comes in stark
Chapter 2. Theoretical Background

<table>
<thead>
<tr>
<th>Boson</th>
<th>Spin</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 gluons</td>
<td>+1</td>
<td>Strong</td>
</tr>
<tr>
<td>$W^\pm, Z$</td>
<td>+1</td>
<td>Weak</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>+1</td>
<td>Electromagnetic</td>
</tr>
<tr>
<td>$H$</td>
<td>0</td>
<td>Mass origin</td>
</tr>
</tbody>
</table>

**Table 2.1:** Bosonic content of the SM.

<table>
<thead>
<tr>
<th>Fermion</th>
<th>$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarks</td>
<td>(3, 2, $\frac{1}{6}$)</td>
<td>$u_L$</td>
<td>$c_L$</td>
<td>$t_L$</td>
</tr>
<tr>
<td></td>
<td>(3, 1, $\frac{2}{3}$)</td>
<td>$d_L$</td>
<td>$s_L$</td>
<td>$b_L$</td>
</tr>
<tr>
<td></td>
<td>(3, 1, $-\frac{1}{3}$)</td>
<td>$u_R$</td>
<td>$e_R$</td>
<td>$t_R$</td>
</tr>
<tr>
<td></td>
<td>(3, 1, $-\frac{1}{3}$)</td>
<td>$d_R$</td>
<td>$s_R$</td>
<td>$b_R$</td>
</tr>
<tr>
<td>Leptons</td>
<td>(1, 2, $-\frac{1}{2}$)</td>
<td>$\nu_{\tau L}$</td>
<td>$\nu_{e L}$</td>
<td>$\nu_{\tau L}$</td>
</tr>
<tr>
<td></td>
<td>(1, 1, $-1$)</td>
<td>$e_L$</td>
<td>$\nu_{\mu L}$</td>
<td>$\tau_R$</td>
</tr>
<tr>
<td></td>
<td>(1, 1, $-1$)</td>
<td>$e_R$</td>
<td>$\mu_L$</td>
<td>$\mu_R$</td>
</tr>
</tbody>
</table>

**Table 2.2:** Fermionic content of the SM.

The discrepancy between the range of the weak interaction and gauge invariance is solved by adding one piece to the puzzle, the Higgs boson. This is a scalar field that, by acquiring a vacuum expectation value (v.e.v) breaks spontaneously the $SU(2)_L \otimes U(1)_Y$ and the remaining symmetry is the $U(1)_{EM}$. The spontaneous symmetry breaking (SSB) explains also why the fermions of the SM have mass, all of them except for the elusive neutrino.

The particle content of the SM is presented in Tables 2.1 and 2.2. In Table 2.2 the roman numbers I, II and III stand for the family or flavors of the fields.

### 2.1.1 The SM Lagrangian

The relevant information about particles and its interactions is encoded in the Lagrangian, in the case of the SM this is:

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{fermion} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}.$$  \hspace{1cm} (2.1)

In the following, each of part of the Lagrangian will be explained.
The gauge Lagrangian

Interactions among gauge bosons are given as follows:

\[ \mathcal{L}_{\text{gauge}} = -\frac{1}{4} G^{a\mu\nu} G_{a\mu\nu} - \frac{1}{4} W^{a\mu\nu} W_{a\mu\nu} - \frac{1}{4} B^\mu B_\mu. \]  

(2.2)

where \( G^{a\mu\nu} \) is the field strength associated to the \( SU(3)_C \) group and is given by

\[ G^{a\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^c_\mu G^b_\nu, \]  

(2.3)

where the index \( a \) runs from 1 to 8, from the 8 generators of the group, and \( f^{abc} \) the structure constants that satisfy

\[ [T^a, T^b] = i f^{abc} T^c, \]  

(2.4)

and \( T^a \) are the generators of the group and \( T^a = \frac{\lambda^a}{2} \) with \( \lambda^a \) the Gell-Mann matrices. The field associated to the \( SU(2)_L \) group is the \( W^{a\mu} \), with its field strength given by:

\[ W^{a\mu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_L \epsilon^{abc} W^b_\mu W^c_\nu. \]  

(2.5)

In this case \( a \) runs from 1 to 3 for the three generators of the group which satisfy \( T^a = \frac{\sigma_a}{2} \) where \( \sigma_a \) are the Pauli matrices. The group structure constant for \( SU(2) \) is \( \epsilon^{abc} \) the completely antisymmetric tensor.

The \( U(1)_Y \) group, unlike the others, is abelian, the associated field is \( B^\mu \) and the field strength is:

\[ B^\mu = \partial_\mu B_\nu - \partial_\nu B_\mu. \]  

(2.6)

The Higgs Lagrangian

As stated before, the Higgs field is the one responsible for the masses of the weak gauge bosons and the fermion fields through the SSB. To understand how the mechanism works, we first define the Higgs field as:

\[ H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}, \]  

(2.7)

where the field contains four degrees of freedom (d.o.f) two charged and two neutral ones. The Higgs Lagrangian is:
Chapter 2. Theoretical Background

\[ \langle h \rangle (\text{GeV}) \]

\[ V(h) (\text{GeV}^4) \]

\[ 0.0 \quad 0.5 \quad 1.0 \quad 1.5 \quad 2.0 \quad 2.5 \]

\[ \mu^2 > 0 \]

\[ \mu^2 < 0 \]

**Figure 2.1:** Behavior of the tree-level potential for the case $\mu^2 > 0$ in the top figure and $\mu^2 < 0$ in the lower figure.

\[ \mathcal{L}_{\text{Higgs}} = (D_{\mu} H)^\dagger (D^\mu H) - \mu^2 H^\dagger H - \frac{\lambda_1}{2} (H^\dagger H)^2. \quad (2.8) \]

The first term which will be discussed later is the kinetic part of the Lagrangian, the second and third term are part of the scalar potential, which may have different behaviors according to the sign of $\mu$. As shown in Fig. 2.1 if $\mu^2 > 0$, the potential has a minimum at the origin, that is the potential is symmetric. If, on the other hand, $\mu^2 < 0$ the potential has a minimum away from the origin at $\langle H \rangle = v = \sqrt{-\frac{\mu^2}{\lambda}}$ (and the origin becomes a local maximum). This is a sign of SSB, which is fundamental for the masses of the gauge bosons. To understand this, first, we parametrize the Higgs in the unitary gauge, where the Goldstone bosons are not present, as follows:

\[ H = \begin{pmatrix} 0 \\ \frac{|h + v|}{\sqrt{2}} \end{pmatrix}. \quad (2.9) \]

On the other hand, for the kinetic terms in Eq. (2.8), the covariant derivative is needed to ensure that the theory is invariant under local gauge transformations. This derivative is defined as:
\[ D_\mu = \partial_\mu - i \frac{g_L}{2} r_\mu W^a_\mu - i \frac{g_Y}{2} B_\mu - i \frac{g_s}{2} \lambda_\mu G^a_\mu. \] (2.10)

In this case the electric charge (the Gell-Man-Nishijima equation) becomes

\[ Q = T_3 + Y \]

where \( T_3 \) is the diagonal generator.

In case of the Higgs, the fourth term on the right is not relevant since it doesn’t interact through the strong force. On the other hand, the second and third term on the right may be written as

\[
\begin{pmatrix}
-i \left( \frac{g_L}{2} W^3_\mu + g_Y B_\mu \right) \\
-i \left( \frac{g_L}{\sqrt{2}} W^-_\mu \right)
\end{pmatrix}
\begin{pmatrix}
-i \left( \frac{g_L}{2} W^3_\mu + g_Y B_\mu \right) \\
-i \left( \frac{g_L}{\sqrt{2}} W^-_\mu \right)
\end{pmatrix}^T,
\] (2.11)

hence the interaction \((D^\mu H)^T (D_\mu H)\) contains the terms:

\[
\frac{1}{4} g_L W^+ W^- v^2 + \frac{1}{8} g_L v^2 (W^3)^2 + \frac{1}{4} g_L g_Y v^2 B W^3 + \frac{1}{4} g_L g_Y v^2 W^3 B - \frac{1}{2} g_Y^2 v^2 B^2.
\]

all these terms are mass-terms. In the case of the \( W^\pm \) fields, the mass can be read directly. The neutral fields, on the other hand, mix, and so their mass-matrix must be diagonalized. Taking into account that \( Y = \frac{1}{2} \) for the Higgs, we find that the mass-matrix is:

\[
\begin{pmatrix}
-\frac{g_L^2}{2} & -g_L g_Y \\
eg L g_Y & g_L^2
\end{pmatrix}.
\] (2.12)

In order to diagonalize the mass-matrix, the neutral fields must be rotated, which leads to the new mass-eigenstates:

\[ Z_\mu = \cos \theta_w W^3_\mu + \sin \theta_w B_\mu, \quad A_\mu = \sin \theta_w W^3_\mu - \cos \theta_w B_\mu, \]

where \( \theta_w \) is the Weinberg angle which satisfies \( \cos \theta_w = \frac{1}{2} v g_L \) and \( A_\mu \) is a massless field associated with the Electromagnetic interaction, in other words, it is the photon field. Due to the structure of Eq. (2.12) one field is massless which is precisely the photon. The masses of the other fields are:

\[ M_W = \frac{g_Y v}{2}, \quad M_Z = \frac{M_W}{\cos \theta_w}. \]

The beauty of SSB is that the Goldstone bosons that are no longer present have been absorbed by the longitudinal components of the gauge bosons, hence acquiring mass, and thus fixing the vector boson mass problem.
It is important to add that Eq. (2.8) gives us much more than bosons mass. First, it allows a mass for the Higgs field which is \( m_H = \sqrt{2\lambda v} \). Second, it also tells us that the Higgs field interacts with the gauge bosons through three particle vertex (two vectors and one scalar) and through a four particle vertex (two vectors, two scalars) and with itself through a three and four particle vertex.

### The Fermion and Yukawa Lagrangian

In the case of the fermion sector, the Lagrangian includes the kinetic terms and the Yukawa interaction which are:

\[
L_{\text{fermion}} + L_{\text{Yuk}} = \sum_{\Psi=\ell_R, Q_{uR}, d_R} i\bar{\Psi} \gamma^\mu D_\mu \Psi - Y_{Lij} \bar{L}_i H l_R^j - Y_{ij} \bar{Q}_i H u_R^j - Y_{dij} \bar{Q}_i \bar{H} d_R^j, \quad (2.13)
\]

where \( \bar{H} = i\sigma_2 H^* \). Here, the indices \( i, j \) indicate family indices, that is, they go from 1-3. The \( L(l_R) \) is any of the lepton doublets (singlets) shown in Table 2.2. In a similar way \( Q(u_R, d_R) \) is any of the Quark doublets (singlets) shown in the same table. In this case \( u_R(d_R) \) represents a singlet quark with the highest (lowest) hypercharge of each family.

#### Quark Masses

Let us first consider the Yukawa interaction for quarks, the third and fourth terms of Eq. (2.13). After expanding the fields, the mass terms may be written as:

\[
L_{\text{Yuk}} \supset -d_i \bar{L}_i^j M_{d}^j d_i - u_i \bar{L}_i^j M_{u}^j u_i, \quad (2.14)
\]

where \( \bar{d}_i \) and \( \bar{u}_i \) are three component vectors, each component corresponds to a different family. And the mass-matrices \( M_{d} \) and \( M_{u} \) are such that:

\[
(M_{d})_{ij} = Y_{dij} \frac{v}{\sqrt{2}}, \quad (M_{u})_{ij} = Y_{ij} \frac{v}{\sqrt{2}}. \quad (2.15)
\]

We can choose a basis in which one of the matrices, for instance \( M_{u} \) is diagonal, this is done by re-defining the fields \( \bar{u}_R^i \) [44]. However, this process can’t be done simultaneously for the down-type quark, and so its mass-matrix needs to be diagonalized in order to find the mass eigenstates, this is done via \( M_{d}' = H_d U_d = S_d^M M_d S_d U_d \) with \( H_d = \sqrt{M_d' M_d'^T} \). The resulting matrix \( M_d \) is diagonal, hermitian and positive-definite, hence, the mass-terms in the Lagrangian become:

\[
L_{\text{Yuk}} \supset -\bar{d} M_d d - \bar{u} M_u u, \quad (2.16)
\]
where \( d_L = S_dd_L' \) and \( d_R = S_dU_dd_R' \) are the mass eigenstates. Though it is good news that it is possible to diagonalize the mass-matrix \( M_d' \) we now have gauge interactions that are not necessarily diagonal, this will be seen shortly [45].

**Lepton Masses:** To study the Yukawa interactions of leptons we must remember that there are no right-handed neutrinos in the SM and that the left-handed neutrinos are massless so, unlike the case of the quarks, for leptons, there is only one mass-term given by

\[
\mathcal{L}_Y \supset -\overline{L}_i M_L e^i_R,
\]

and again, by choosing an appropriate basis, the matrix \( M_L \) is diagonal.

**Fermion-gauge boson interactions** The information on fermion-gauge interactions is encoded in the kinetic term. We can consider separately interactions related to charged gauge bosons, e.g. charged currents, and neutral gauge bosons, e.g. neutral and electromagnetic currents.

**Electromagnetic currents:** Electromagnetic interactions are of the form:

\[
eQ\overline{f}\gamma^\mu f,
\]

where \( Q \) is the charge of the fermion and \( e = g_L \sin \theta_W = g_Y \cos \theta_W \). As expected, neutral fermions do not couple to the photon.

**Neutral currents:** In the case of the \( Z \) gauge boson, the interactions include three terms, which stems from the fact that the \( Z \) boson is a mix of \( B_\mu \) and \( W^3_\mu \), and from the fact that the \( W^3_\mu \) interaction is proportional to the third component of the weak isospin \( \tau^3 \), which for right handed fields is zero, in this way, the fermion interactions with \( Z \) are of the form:

\[
\frac{g_L}{c_W} f_L^\mu \gamma^\mu (\tau^3 - 2Qs_W^2) f_L - \frac{Qs_W^2}{c_W} f_R^\mu \gamma^\mu f_R,
\]

where \( f_L \) is any of the fermion doublets (left-handed) and \( f_R \) is any of the fermions singlets (right-handed) and \( Q \) is the electric charge. Notice that since the \( Z \) interactions does not involve a mix of left-handed and right-handed fields, they are diagonal, even for the case of the \( d '_L \) and \( d '_R \), and since they are flavour diagonal, there are no flavour changing neutral currents (FCNC) in the SM. The interactions in Eq.(2.19) are sometimes expressed in a different way. Taking into account that a field \( f_{L,R} = P_{L,R} f \) where \( P_{L,R} = \frac{1 \pm \gamma^3}{2} \) are the usual chirality projection operators, the interaction with the \( Z \) boson is:
\[\frac{e}{2c_W s_W} f \gamma^\mu (g_V - g_A) f Z_{\mu}, \quad (2.20)\]

where \(g_V = \tau^3 - 2 Q_{s_W}^2\) and \(g_A = \tau^3\).

**Charged currents in the quark sector:** For quarks, charged current interactions which involve a \(W^\pm\) include terms of the following form \(\bar{u}'_L d'_L = \bar{u}_L V d_L\) where \(V\) is the Cabibo-Kobayashi-Maskawa (CKM) matrix (defined below), hence, the interaction is:

\[\frac{g}{2\sqrt{2}} W^+_\mu \bar{u}_i \gamma^\mu (1 - \gamma^5) V_{ij} d_j, \quad (2.21)\]

where \(V_{ij}\) are elements of the CKM matrix, which is responsible of mixing quark flavours. The CKM matrix is usually parametrized in terms of angles as following \[46\]

\[V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}, \]

where \(\delta_{ij}\) is the only complex parameter and thus the only source of CP violation. Also, \(c_{ij}\) and \(s_{ij}\) stand for \(\cos(\theta_{ij})\) and \(\sin(\theta_{ij})\) which may be defined as that \(c_{ij}, s_{ij} \geq 0\). 

**Charged currents in the lepton sector:** In the lepton sector, as was shown earlier, interactions are diagonal due to the absence of neutrino masses, that is:

\[\frac{g}{2\sqrt{2}} W^+_\mu \bar{\nu}_i \gamma^\mu (1 - \gamma^5) \nu_{ij}, \quad (2.22)\]

where \(\nu_{i}\) (\(e_{i}\)) refers to any of the neutral (charged) fermions, but without flavour mixing.

**Gauge Boson interactions after SSB:**

As we saw earlier, the SSB rotates the gauge eigenstates \(B_\mu\) and \(W^3_\mu\) into the mass eigenstates \(Z_\mu\) and \(A_\mu\). Now, we can study how these mass eigenstates interact among themselves. Using Eqs. (2.2),(2.5), and (2.6) we find that the electroweak gauge bosons have interactions among themselves which include either three or four of them. The triple gauge interaction is:
2.1. The Standard Model

\[ L_3 = -i \, e \cot \theta_W \left( W^{\mu \nu} W^\dagger_\mu Z_\nu - W^\dagger_\mu W^{\mu \nu} Z_\nu - W^\dagger_\mu W_\mu Z^{\mu \nu} \right) \] (2.23)

\[ + i \, e \left( W^{\mu \nu} W^\dagger_\mu A_\nu - W^\dagger_\mu W^{\mu \nu} A_\nu - W^\dagger_\mu W_\nu A^{\mu \nu} \right), \] (2.24)

where \( F^{\mu \nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \) is the electromagnetic field strength tensor.

On the other hand, the quartic gauge interaction is:

\[ L_4 = -\frac{e^2}{2 s^2 \theta_W} \left( (W^\dagger_\mu W^\mu)^2 - W^\dagger_\mu W^{\mu \nu} W^\nu \right) \]

\[ - e^2 \cot^2 \theta_W \left( W^\dagger_\mu W^{\mu \nu} Z_\nu - W^\dagger_\mu Z^{\mu \nu} W_\nu \right) \]

\[ + e^2 \cot \theta_W \left( 2 W^\dagger_\mu W^{\mu \nu} A_\nu - W^\dagger_\mu Z^{\mu \nu} W_\nu A_\nu - W^\dagger_\mu A^{\mu \nu} W_\nu \right) \]

\[ - e^2 \left( W^\dagger_\mu W^{\mu \nu} A_\nu A^\mu - W^\dagger_\mu A^{\mu \nu} W_\nu \right). \] (2.25)

From (2.23) and (2.25) we find that these interactions always involve at least two charged bosons.

Quantum Chromodynamics (QCD):

Baryons such as the proton or neutron are bound states of three quarks, whereas mesons are bound states of a quark and anti-quark pair. To explain the observations of these non-fundamental particles, it is necessary to introduce a new quantum number, color, associated with the already mentioned SU\((3)_C\) symmetry. Since the symmetry is respected by nature, all observed states are color singlets, this is known as the confinement hypothesis, and explains why quarks are not observed as free-particles [45]. When a quark is created, for instance due to a high energy collision, it hadronizes, thus forming again a color singlet. The QCD lagrangian is:

\[ L_{QCD} = -\frac{1}{4} (\partial^\mu G^\mu_a - \partial^\nu G^\nu_a) (\partial_\mu G^\mu_a - \partial_\nu G^\nu_a) + \sum_f \bar{q}_f \left( i \gamma^\mu \partial_\mu - m_f \right) q^a_f \]

\[ - g_s G^\mu_a \sum_f \bar{q}_f (i \lambda_a \gamma^\mu) \gamma_\lambda \frac{\lambda_a}{2} q^\beta_f \]

\[ + g_s^2 \frac{f_{abc}}{4} (\partial^\mu G^\mu_a - \partial^\nu G^\nu_a) G^\nu_b G^\mu_c - \frac{g_s^2}{4} f_{abc} f_{ade} G^\mu_b G^\nu_c G^\rho_d G^\sigma_e, \] (2.26)

where the indices \( a, b, c, d, e = 1, 2, ....8 \) and the indices \( a \beta = 1, 2, 3 \). The first and second term are kinetic terms for the gluons and quarks respectively. The third terms shows the interactions of two quarks with a gluon and the two last terms show the interactions among three and four gluons, similar to that of Eq (2.23) and Eq. (2.25).
2.2 The Standard Model successes and shortcomings:

The SM is considered one of the most successful theories of nature. Some of its achievements include (but are not limited to) the prediction of the existence the $W^\pm$, $Z$ bosons and gluons. The prediction of the masses of the weak gauge bosons which are in stark agreement with current measurements and, perhaps the greatest success, the prediction of the Higgs boson which has been observed at the LHC by the ATLAS [47] and CMS [48] experiments. However, we now know that the SM cannot be the final theory. Some of the most convincing experimental evidence are:

- The observation that the universe is made mostly of matter and not anti-matter, also known as the matter anti-matter asymmetry problem. The evidence for this comes from several sources such as Big Bang Nucleosynthesis (BBN), the CMB measurements [49] and the lack of observational evidence that there are regions in the universe where matter and anti-matter are annihilating. The measurement of this asymmetry comes from the measurement of the ratio of the baryon number density to photon number density $\eta$, this number is (at 95 % C.L.) $5.8 \times 10^{-10} < \eta < 6.6 \times 10^{-10}$ [50], in a symmetric universe this number should be about nine orders of magnitude smaller [51].

- The observation of neutrino oscillations which means that neutrinos change flavor as they travel. The evidence for this comes from solar, atmospheric, reactors and neutrinos produced at colliders in experiments such as KamLAND Super-Kamiokande, K2K, MINOS, T2K, Double Chooz, Daya Bay, RENO and Opera collaboration [52]. The oscillation probability is related to the mass, which implies that neutrinos must have mass that although tiny it is non-zero with $(\sum_j m_j \leq (0.3 - 1.3)\text{eV})$ [53].

- The disagreement between the amount of matter needed to explain the observed gravitational interactions an the number of baryons in the universe. This discrepancy, also known as the DM problem, in its current form states that about 80% of the matter content of the universe [54, 55]is an unknown type of matter. More on this problem below.

2.2.1 The DM problem:

Although there has been an astonishing advance in the understanding of the fundamental building blocks of our universe, the 20th century has brought the outstanding discovery that most of the matter content of the universe is unknown to us, this is what is called DM. The following historical part is mostly based on the discussion of Ref. [56]. The first evidence of the existence of DM comes from 1933 when the astronomer Fritz Zwicky studied the Coma cluster, which was one of the most massive galaxy cluster known at that time. Zwicky inferred the amount of normal matter by observing the starlight. Additionally, he calculated the total matter of the cluster by observing the motion of individual galaxies. He came to the conclusion that
there was a large discrepancy between the two measurements, and so he inferred that most of the matter was not seen [56, 57]. In 1960 Vera Rubin and Kent Ford found out that the rotational velocity of stars within 60 galaxies had an unexpected behavior. As one looks away from the center of a galaxy, the rotational velocity of starts is supposed to fall because the gravitational pull is weaker. However, Rubin and Ford found out that the velocity remained almost constant instead. An example of this discrepancy is shown in Fig. 2.2. Again, the conclusion is that the stars are experiencing a larger gravitational force due to unseen matter, the DM.

The CMB is the earliest photograph of our universe. When the universe was hot enough so that no neutral elements could exist, the photons interacted with baryons and the universe could be modeled as a photon-baryon fluid. At this time, the fluid oscillated because, due to its mass, it tended to collapse in regions where there was some overdensity, but due to the charge of the protons the fluid heats up and a pressure is generated which works against the collapse, this becomes a cycle. But once the universe cools to allow the formation of neutral elements, the gas becomes transparent to photons allowing them to travel freely yet keeping the information of their place in the cycle right before the last scattering. This is seen today as small variations in temperature. The variations are thus related to the gravitational potential wells that became the seeds of structure formation. The problem is that they are too small (about $30 \pm 5 \mu K$), and so, there is not enough time to form the structures seen today. This implies that a new form of matter must be present, one that is not affected by electromagnetic interactions, the DM [57]. The best and latest measurement of the amount of DM comes from the PLANCK satellite measurement of the fluctuations and the results are $\Omega_bh^2 = 0.02237 \pm 0.00015$ and $\Omega_ch^2 = 0.1200 \pm 0.0012$ [58] where
\( \Omega_b h^2 \) is the total baryonic matter density and \( \Omega_c h^2 \) is the total DM density.

There is another earlier indirect evidence that supports the existence of particle DM, this is Big Bang Nucleosynthesis (BBN) which refers to the formation of light elements in the early universe. BBN took place at around \( t \sim 0.1 - 10^4 \) sec [59]. During this time, the universe had been cooling very rapidly and now it was possible for baryons to form and then collide, thus forming elements such as Helium and Lithium as well as their isotopes. The relevance of this for particle DM goes as follows: It is possible to predict from the evolution of the universe and the known nuclear cross sections how much of each of these elements should have been produced. These predictions depend on the density of baryons, and so observation of their abundance gives an indirect measurement of this density which is (at 95 \% C.L.) \( 0.021 \leq \Omega_b h^2 \leq 0.024 \) [60]. On the other hand, the CMB gives a measurement of the total matter content of the universe. As it turns out, there is a large disagreement between the baryon and total matter content, thus, another form of matter must be present in large quantities [59, 61]. Clearly, our universe is mostly filled with DM.

One additional and recent evidence is the observation that in the collisions of galaxy clusters such as the bullet cluster, the gas from both sides of the collision heats up and slows down. This is observed using X-ray telescopes. However, using weak gravitational lensing it is found that most of the matter kept moving unperturbed, this has put constraints on the self-interaction properties of DM [62]. These are only some of the strongest evidence of DM, but, it is worth mentioning that there are more observations and measurements that hint its existence.

What do we know about this elusive type of matter?

- It is non-baryonic. As stated above, this is supported by the fact that the amount of baryons we have measured, agrees with the formation of light elements, also known as BBN. Moreover, additional baryons would change the CMB that we see today [63].

- It is stable, at least in cosmological times. If DM could decay with short lifetimes, we would not see the amount that we see today. Also, there are stringent constraints on its width decay coming from cosmic rays, photons and neutrinos, which limit its lifetime to be at least 9 orders of magnitude larger than the age on the universe [64].

- It is neutral or has a very small electric charge. We know this because for one it has not been seen through telescopes. On the other hand, if it had electric charge it would couple to the early photon-baryon fluid in the early universe. This, in turn, would affect the CMB spectrum [63].

- It is cold or non-relativistic at the time of decoupling, this is important for structure formation. What it is found is that small scale structures formed before large scale structures. However, if DM was relativistic at time of decoupling, it would have prevented small scale structures from forming early on,
2.2. The Standard Model successes and shortcomings:

hence creating a disagreement with the current knowledge of the structures in the universe [55].

Although neutrinos were the first particle DM candidates, it is now known that they can’t be since they were relativistic when they decoupled, thus violating the last requirement. All the evidence mentioned above for DM implies that we only know its effects from gravitational interactions but we want to further understand its nature. There are theories of how DM works, for instance, a popular one is that is made of one (or several) particles that we have not detected yet. Among the most widely studied particles are the WIMPs and the Axion. There are also theories that include primordial black holes and modified theories of gravity. This work focuses mostly on WIMPS.

WIMP Relic density

The WIMP DM candidate has been, perhaps, the most studied candidate up to this date. The reason behind the seeming obsession that particle physicists have with it is due to the so-called "WIMP miracle", which works in the following way: WIMPs are in thermal equilibrium in the early universe (as many other SM particles were), then they decouple from the thermal bath, and due to the nature of their interaction their cross section must be \( \sim 3 \times 10^{-26} \text{cm}^3/\text{s} \), this cross section leads to the right amount of DM seen today.

Now, to understand how the process works, we first consider a DM particle \( \chi \) which annihilates through processes such as \( \chi \bar{\chi} \leftrightarrow Y \bar{Y} \) where \( Y \) is a SM particle and \( m_\chi > m_Y \). In the early universe, due to the high temperatures (higher than \( m_\chi \)), the process proceeded both in the left and the right direction (hence the arrow in two directions). However, as the universe cools and the temperature drops below \( m_\chi \) the only favoured process is the annihilation of DM particles, and no creation occurs. On the other hand, the universe is also expanding, and so the number of DM particles gets diluted which makes it harder for DM particles to find each other and annihilate. At this point the co-moving number density of WIMPs becomes fixed, a process known as freeze-out. To study this, the Boltzmann equation must be solved [65]:

\[
\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma_{\chi\bar{\chi}}v \rangle \left( n_\chi^2 - n_{\chi,eq}^2 \right),
\]

(2.27)

where \( n_\chi(n_{\chi,eq}) \) is the number density (equilibrium number density) of DM particles, \( H \) is the Hubble constant and \( \langle \sigma_{\chi\bar{\chi}}v \rangle \) is the thermally averaged annihilation cross section of the DM particles. The term \( 3Hn_\chi \) accounts for the dilution due to the expansion of the universe and the left side accounts for the interaction with other
Chapter 2. Theoretical Background

particles [66]. An approximate solution to this equation yields the relic density as

\[ \Omega_\chi h^2 \simeq \frac{s_0}{\rho_c h^2} \left( \frac{45}{\pi^2 g_*} \right)^{1/2} \frac{1}{x_f M_{pl}} \frac{1}{\langle \sigma v \rangle} \]  

(2.28)

where \( s_0 \) is the entropy today, \( \rho_c \) is the critical density, \( g_* \) are the effective degrees of freedom, \( M_{pl} \) is the reduced Planck mass, \( x_f = m_\chi / T_f \) with \( T_f \) the temperature at freeze-out [67].

Though Eq. (2.27) is the most standard form of the Boltzmann equation for relic density calculation, there are many more forms of it, all model-dependent. For instance, K. Griest and D. Seckel showed in their famous work [68] that there are three exceptions to the usual relic density calculation, which are:

- **Resonances.**
- **Thresholds.**
- **Co-annihilations.**

Resonances happen when the DM annihilates via an intermediate particle \( B \) and \( m_\chi \sim m_B / 2 \) at this point the cross section spikes due to the pole of the propagator and the relic abundance gets considerably reduced. Thresholds occur when the DM mass is smaller than the mass of the particles it is annihilating to. In principle, this is kinematically forbidden, but the authors found that if the mass difference lies between 5% and 10%, the annihilation still proceeds, thus affecting the relic abundance. Co-annihilations occur when the mass of the DM particles is nearly degenerate with other particles, in this case, it is not only the annihilation of the DM but also the annihilation of the other particles that set the relic abundance.

**DM detection**

Since the particle nature of DM is yet to be understood, physicists have put forward large efforts to uncover it. The most common ways are [57]:

- **Direct detection**
- **Indirect detection**
- **Production at colliders**

Each of the methods will be discussed in this thesis.

**Direct detection** Direct detection is based on the idea that DM particles are continuously passing through the Earth, and they might eventually scatter off a nucleus, which will make the nucleus recoil, and consequently the recoil energy could be measured. To do this, the differential recoil spectrum is calculated as [69]:
2.2. The Standard Model successes and shortcomings:

\[
\frac{dR}{dE_R}(E_R, t) = \frac{\rho_0}{m_x m_A} \int v f(v, t) \frac{d\sigma}{dE_R}(E_R, \nu) \, d^3v,
\]

(2.29)

where \(m_A\) is the nucleus mass, \(\rho_0\) and \(f(v, t)\) are astrophysical parameters that depend on the local DM density, the velocity \(v\) is defined in the rest frame of the detector, \(E_R\) is the recoil energy and \(\frac{d\sigma}{dE_R}\) is the differential cross section which depends both in particle physics and nuclear physics. The recoil energy can be approximated as:

\[
E_R = \frac{\mu_N^2 v^2 (1 - \cos \theta^*)}{m_N},
\]

(2.30)

with \(\theta^*\) the scattering angle in the center of mass frame of the detector and \(\mu_N\) the nucleon reduced mass. The event rate per kilogram per day is calculated by integrating over all possible velocities, that is [29]:

\[
R = \int_{E_T}^{\infty} \frac{\rho_0}{m_N m_x} dE_R \int_{v_{\text{min}}}^{\infty} v f(v, t) \frac{d\sigma}{dE_R}(E_R, \nu) dv,
\]

(2.31)

The most general form for the differential cross section is:

\[
\frac{d\sigma}{dE_R} = \frac{m_A^2}{2\mu_A^2 v^2} \left( \sigma_{SI}^2 F_{SI}^2(E_R) + \sigma_{SD}^2 F_{SD}^2(E_R) \right),
\]

(2.32)

where \(\mu_A = \frac{m_A m_x}{m_A + m_x}\), where \(F(E_R)\) is the nuclear form factor which determines the spectrum of the recoil nucleus [70], while SI and SD refers to the spin independent and spin dependent contribution respectively. In the case of SI, the interaction arises from scalar-scalar and vector-vector couplings and the cross section is given by:

\[
\sigma_{SI} = \sigma_P \frac{\mu_A^2}{\mu_P^2} [Z f^p + (A + Z) f^n],
\]

(2.33)

the superscript \(p\) and \(n\) refer to the proton and neutron respectively and \(f^{(n)}\) represent the contributions of the quarks to the mass of the proton (neutron) [29]. Usually \(f^p \sim f^n\) which renders the SI cross section to be proportional to \(A^2\) [69]. This dependence generates and enhancement in the cross section compared to that of SD, for this reason I will focus on the SI interaction. For fermionic DM, one can formulate an effective Lagrangian, which yields a scalar coupling:

\[
\mathcal{L} \supset S_F \overline{\chi} \chi f,
\]

(2.34)
where $f$ refers to a SM fermion, as usual $\chi$ is the DM and $S_F$ parametrizes the strength of the effective interaction. In the case of Higgs portal, this interaction arises from two terms:

$$L_S = y_\chi H\bar{\chi} + \sum_q H\bar{q}q,$$

(2.35)

With this information, the transition amplitude for the scattering may be calculated, which in the case of a Majorana particle is:

$$|M_F|^2 = 4 \times 64 \frac{y_\chi^2 m_\chi^2}{m_H^4} \left( \sum_q \frac{y_q f_q}{m_q} \right)^2,$$

(2.36)

where $y_\chi$ is replaced by $-\frac{1}{4} \frac{\lambda_{hf}}{\Lambda^2}$ where $\Lambda$ is the effective scale of the model. The transition amplitude then yields the SI cross section:

$$\sigma^{SI} = (4) \times \frac{\lambda_{hf}^2}{16\pi\Lambda^2} \frac{M_N^4 m_\chi^2}{m_H^4 (M_N + m_\chi)^2} \left( \sum_q f_q \right)^2,$$

(2.37)

It is important to note that the model dependent part is mostly included in $m_\chi$ and $y_\chi$ (or $\frac{\lambda_{hf}}{\Lambda}$). Hence, by varying the mass and the couplings, it is possible to calculate the cross section, and then with $\frac{d\sigma}{dE}$, the event rate is calculated. The experiments, on the other hand, measure the event rate and since no evidence on WIMP-nucleus interaction has been found, the information has so far been used to put constrains on the DM interaction cross section with the nucleus (such as the one shown in Fig. 2.3).
2.2. The Standard Model successes and shortcomings:

**Indirect Detection** DM halo models predict large densities in the center of galaxies, this means that in such dense regions, DM particles might find each other and thus annihilate producing SM particles such as gamma rays, neutrinos, positrons, etc. For gamma rays, this might happen through direct annihilation or by loop-mediated annihilation. As a result, we might observe, for instance, an excess of gamma rays or gamma-ray lines in the center of galaxies. Gamma rays are specially appealing for DM searches because they travel through the galaxy unperturbed and so they can give an indication of the location of its source. To calculate the expected flux of photons we rely on the astrophysics as well as the particle physics nature of DM. If the DM is a Majorana particle, the flux is given by \[72, 73\]:

\[
\Phi_\gamma(n, E) = \langle \sigma v \rangle \int_{\text{line of sight}} \rho(n, l)^2 d\psi,
\]

where \(l\) is the path length along the line of sight in the direction \(n\), \(dN_\gamma/dE_\gamma\) is the gamma ray spectrum that is produced in an annihilation and \(\rho(n, l)\) is the DM density along the line of sight. For \(\rho\) a density profile is used, the most widely used is the Navarro-Frenk-White profile \[74\] with:

\[
\rho(r) = \rho_0 \left( \frac{r/s}{1 + r/s} \right)^{-\gamma - 3},
\]

This profile fits well N-body simulations of galaxies \[75\]. Another widely used DM density profile is the Einasto profile which is \[76\]:

\[
\rho(r) = \rho_0 \exp \left( -\frac{2}{a} \left[ \left( \frac{r}{s} \right)^a - 1 \right] \right),
\]

where \(\rho_0\) is the local DM density, \(s\) is the scale radius, \(\gamma\) (not to be confused with the photon) is usually taken to be 1 due to computer simulations and \(a=0.17\). This profile is preferred for low mass haloes, such as those of dwarf galaxies \[75\]. The terms that are related to particle physics in the flux are \(\langle \sigma v \rangle\) and \(dN_\gamma/dE_\gamma\). The thermally averaged cross section times velocity might be the same as the one calculated for DM relic density, although, if there is an s-wave suppression, the \(\langle \sigma v \rangle\) will depend on the actual velocity of DM which is much lower than the one at freeze out.

Some experiments look for an excess of gamma rays coming from the center of our galaxy or from dwarf spheroidal galaxies. The excess might show up as a continuous broad bump or a line-like feature. The continuous spectra results when DM annihilates to quarks, Higgs, leptons and gauge bosons, such particles subsequently decay into pions that later on produce photons \[77\]. On the other hand, when DM annihilates to a neutral SM particle (including a photon) and a photon, the result is
a mono-chromatic gamma-ray with energy $E_\gamma = m_\chi (1 - m_\chi^2 / 4m_{SM}^2)$. No astrophysical source is known to produce this type of line-like spectra [78], thus, this is called the "smoking gun" of DM annihilation. The gamma-ray photons in the form of a continuous, as well as line-like excess, are searched by the FERMI-LAT and H.E.S.S collaborations. The Large Area Telescope (hence the LAT) on board the FERMI satellite is an imaging high-energy gamma-ray telescope that searches for photons in the range of 20 MeV to 300 GeV [79]. The High Energy Spectroscopic System (H.E.S.S) is a system of imaging atmospheric Cherenkov telescopes that investigates cosmic gamma-rays that range from GeV to TeV [80]. The latest results published from both collaborations in works such as [81–84] provide the most stringent constraints in the $\langle \sigma v \rangle$ of DM annihilation to mono-energetic photons and to $W^+ W^-$ which will later be used in this thesis.

**DM at the LHC** The LHC is the largest and most ambitious experiment built by mankind. It consists of a ring with a perimeter of 27 km where protons are accelerated near the speed of light. At certain points in the ring two opposite beams of protons meet and so they collide at a high energy (currently 13 TeV). There are four main experiments at the LHC that study such collisions, ATLAS, CMS, LHCb and ALICE. Since there is enough energy from the collision, a myriad of particles is produced. Some of them are directly detected by the experiments within the LHC, while others are not, and so their presence must be inferred. As of today, the largest achievement of the LHC is the discovery of the Higgs boson, the last particle predicted by the SM to be detected.

Due to the high energies of the accelerator, new beyond the SM (BSM) particles may be produced and a great effort is being put forth in order to either find them or put constraints on the parameter space of specific BSM models. To understand
2.2. The Standard Model successes and shortcomings:

how this is done, one first must understand how particles are detected, for illustration purposes the CMS experiment will be used as an example. A slice of the transverse plane of the CMS experiment is shown in Fig. 2.4. The left most point of the figure is where the collision occurs, after that, particles fly off. The first sensor they encounter in their path is the silicon tracker, where the position of charged particles is measured, hence the path can be reconstructed. Due to the superconducting magnetic field of 3.8T (shown as a cross), the path of these charged particles will be spiral and its curvature gives information about their momentum. The next layer is the electromagnetic calorimeter, in this part, electrons and photons will be stopped and this will give information of their initial energy. The following layer is the hadron calorimeter where particles that interact with the strong force will deposit most of its energy. The final layers are part of the muon chambers, only muons and weakly interacting particles such as neutrinos will fly through this layer. Since muons are charged their path will bend due to the superconducting magnet, and their path will be inferred. Neutrinos, on the other hand, leave no signature so additional methods must be used in order to deduce their presence and characteristics such as momentum. For this, the simplest method is the use of the transverse momentum. Since protons travel along the beam pipe (usually considered the $z$-axis) initial momentum in the transverse plane ($x−y$ plane) is zero, due to momentum conservation the same must be true for the final momentum.

As for DM, if a WIMP particle has some interaction with SM particles and its mass lies at or below the TeV scale, it might be produced at the LHC, hence, many searches are focused on either finding it or constraining the parameter space of popular DM models. Since WIMP DM is only weakly interacting (like neutrinos), it will not be seen and indirect measurements must be made. Additionally, it is important to note that in many WIMP models the stability of DM is guaranteed by a new symmetry for instance in Supersymmetry this is the $R$-parity and in many BSM models is the $Z_n$ symmetry with $n = 2$ being probably the most popular one. This implies that DM particles may be produced along with other new particles, very often in pairs, if that is the case they might be produced back to back and so transverse momentum conservation will not yield any information. A caveat to this scenario is when the two WIMPs are accompanied by another particle, in that case, there will be additional momentum information, which could be for instance coming from initial state radiation (ISR). Another common BSM possibility is that the DM comes from the decay of a heavier unstable new particle which yields a visible particle and missing energy, hence called two-body semi-visible. The left Feynman diagram of Fig. 2.5 shows such process with $p\ p \rightarrow A \rightarrow B\ell$ where the slashed notation in this case indicates invisible (e.g. the WIMP). From energy-momentum conservation, it is clear that only certain values of the parent mass are consistent with the information provided in the event by the transverse momentum of the visible particle. In this case, it is convenient to define a transverse mass $M_T$ which is defined as [86]
\[ M^2_T = m^2_B + m^2_{\ell} + 2(E_{TB}E_{T\ell} - \vec{p}_{TB} \cdot \vec{p}_{T\ell}), \]  
(2.41)

where

\[ E^2_{T_i} = m^2_i + \vec{p}^{\perp 2}_{T_i}. \]  
(2.42)

By construction \( M_T \leq M_A \) which means that a histogram of many of these events will exhibit an endpoint in \( M_T \) and that endpoint will be the boundary between the allowed mass of the parent and the disallowed mass for a particular ansatz of the mass of the daughter particle. Moreover, it has been shown by [87] that this variable exhibits a kink, e.g is continuous but not differentiable, at the true daughter mass, thus both the parent and daughter mass may be found.

A similar situation is depicted on the right side of Fig. 2.5 where two BSM particles, the parent particles \( A \) are produced and each of them decay to a visible particle \( B \) and an invisible particle \( C \). Notice that even though the diagram has two equal sides with the same particles, their momentums need not be the same (hence the different labels for each equal particle). In such case, the appropriate kinematic variable is the Cambridge \( M^2_{T2} \) which is defined as [88]:

\[ M^2_{T2} = \min_{\vec{p}_{Tc1} + \vec{p}_{Tc2}} \left\{ \max \left[ M^2_T(\vec{p}_{TB1}, \vec{p}_{TC1}), M^2_T(\vec{p}_{TB2}, \vec{p}_{TC2}) \right] \right\}, \]  
(2.43)

where the minimization is done over all possible invisible transverse momenta of \( C_1 \) and \( C_2 \) such that \( \vec{p}_{Tc1} + \vec{p}_{Tc2} = \vec{p}_{T inv} \). There are several useful features with this variable, one, it defines a bound on the parent mass because, by construction \( M_A \geq M_{T2} \) and two, it exhibits a kink at the truth daughter mass as in the case of \( M_T \). Hence, by means of \( M_T \) and \( M_{T2} \) it is in principle possible to find both the parent and daughter (DM) mass for the two aforementioned topologies. However, it has been shown that the kinks are hard to find unless there is hard ISR, which requires a loss of statistics. Moreover, if new particles are spotted at the LHC, the mass will not be the only relevant parameter to be measured. Experimentalists will probably be interested in other properties such as the width of the parent particle, chirality of the couplings, spin, etc. For this reason, kinematic variables such as \( M_T \) and \( M_{T2} \) fall short and more complicated calculations will be necessary such as the MEM.

The MEM is a multivariable analysis which requires a previous knowledge (or at least an informed guess) of the interaction that leads to the event. With all the available information, it is then possible to calculate the theoretical differential cross section for an ansatz of the unknown quantities such as mass, couplings, etc. This is then used to calculate the likelihood \( L \) that the process was produced as a function of the different hypothesis used for the unknown quantities. A minimization of
2.2. The Standard Model successes and shortcomings:

- In $L$ returns the best fit values of the unknown quantities. The MEM has been successfully used at Tevatron for the measurement of the top quark mass [89] and more recently used by both ATLAS and CMS collaboration for measurement of the properties of the Higgs Boson such as spin and parity [90, 91]. This method will be presented more in-depth in chapter 5.

2.2.2 DM and neutrinos

Neutrinos are the second most abundant SM particles in the universe after photons, yet they are puzzling and there is still a lot to be learned about them. Their presence was first inferred in the early part of the 20th century due to the continuous electron spectrum in $\beta$-decays. This could be explained only through the production of a new particle in the experiment. Later on, Fermi applied this idea to formulate his beta decay theory [92]. Some years later, Reines and Corwen were able to directly detect neutrinos through inverse beta decay, this leads to the discovery of the antineutrino [93]. In the 1960s the Lederman- Schwartz-Steinberger experiment lead to the discovery of the muon neutrino, which showed that they come in flavors just as the other SM fermions [94].

On the other hand, experiments such as the Homestake experiment started showing the solar neutrino flux was one-third of its expected value [95], the deficiency has been confirmed by other experiments such as the SNO [96] and Super-Kamiokande experiment [97]. Additionally, the Super-Kamiokande experiment also gave an indication that there was a shortage of muon neutrinos coming from the atmosphere [98]. A solution to the discrepancies between the expected and measured flux was put forward by Pontecorvo in 1957-1958. He suggested that in the path to the detector, neutrinos were oscillating in a form analogous to the one in the $K^0 - \bar{K}^0$ system, however, it was only an oscillation between active and sterile neutrinos [99, 100]. However, it was almost a decade later when Maki, Nakagawa, Sakata [101] and Pontecorvo himself (with a subsequent paper), laid the basis of neutrino oscillations which proved to be the answer to the deficits [102].
Chapter 2. Theoretical Background

The probability that a neutrino of flavour \( a \) transitions to a neutrino of flavour \( b \) is given by [103]:

\[
P(\nu_a \rightarrow \nu_b, t) = \left| \sum_{i=1}^{3+n_s} U_{bi} e^{-2i\Delta_{pi} t} U_{ai}^* \right|^2,
\]

(2.44)

where \( U_{ij} \) are the elements of a unitary \((3 + n_s) \times (3 + n_s)\) matrix that rotates from interaction eigenstates to mass eigenstates and

\[
\Delta_{pi} = \frac{\Delta m^2_{pi} L}{4E}, \quad \Delta m^2_{pi} = m_i^2 - m_p^2,
\]

(2.45)

where \( L \) is the distance between the source of neutrinos and the detector and \( p \) is an arbitrary index.

As can be seen from Eq. (2.45) in order for neutrinos to oscillate, they must have mass, which as mentioned before cannot be accounted for in the context of the SM because with the particle content it is not possible to build a gauge invariant term of neutrinos with the Higgs field that after SSB yields a mass-term. This is, as of now, the strongest experimental evidence that the model must be extended.

In order to build an extension, one must take into account that neutrinos are at least six orders of magnitude lighter than other SM fermions, and so the mechanism must generate tiny masses. On the other hand, the question of the nature of neutrinos remains, are they Dirac or Majorana particles? If they are Dirac particles their mass term has the form \( \bar{\psi} \psi \) but that requires a right-handed neutrino which has not been observed. If they are Majorana particles the mass term has the form \( \psi^T C^{-1} \psi \).

Note that the second term breaks lepton number by two units, so a Majorana mass term is evidence of the breaking of a global symmetry \( U(1) \).

\[\]
2.2. The Standard Model successes and shortcomings:

Now, since the neutrino mass is so small, it could be generated by physics beyond the electroweak scale [27]. In 1979 Weinberg found that at the lowest non-renormalizable order, the only operator that allowed such term is [27, 105]:

$$\mathcal{L}^{d=5} = \frac{\lambda}{\Lambda} L \tilde{H}^T \tilde{H} L^T, \quad (2.46)$$

where $\Lambda$ is the suppression scale of the new physics that gives rise to the operator, and $\lambda$ is a model-dependent coefficient, while $H$ and $L$ are the SM Higgs and lepton doublets respectively.

At tree-level this mass-term is realized via the exchange of a singlet fermion (the right handed neutrino), a triplet scalar or a triplet fermion in the type I, II and III Seesaw mechanism respectively as shown in Fig. 2.6. However, in order to generate the small mass term with couplings of $O(1)$, the suppression scale must be very large and so most likely out of reach of current experiments [27].

Another interesting possibility is that the neutrino mass is generated radiatively, in that case, due to the loop suppression, the scale of the new physics could be the electroweak scale. In this category, some well-studied models include the Zee model [106] where a charged scalar singlet and another scalar doublet are introduced. The problem with this model is that through the interaction of the new doublet with the leptons it is possible to generate flavor changing neutral currents which are severely constrained. However, a restricted version, called the Zee-Wolfenstein model where the leptons do not couple to the second doublet, is ruled out from neutrino physics.

Another well-studied model is the radiative seesaw [28] where the SM is extended with another scalar doublet that does not acquire a v.e.v (the Inert Doublet) and three generations of a right handed neutrino, all particles are odd under a $Z_2$ symmetry. This model has sparked particular interest in the literature because the new neutral particles participate in the neutrino mass generation while the lightest neutral one (either fermion or scalar) is stable and so is a DM candidate. Thus, the model connects two of the most important problems in particles physics and one gets a solution of the type "two for the price of one". The radiative seesaw is not the only one to have these interesting aspects, works such as [26, 107–109] have studied the phenomenology of these types of model with varying particle contents. Ch. 3 is dedicated to the study of one such model.

![Figure 2.7: Feynman diagram for the radiative seesaw showing that neutrino mass generation after SSB. The loop is mediated by the right handed neutrino ($N_R$), and the neutral CP-even and CP-odd scalars ($H^0$ and $A^0$) [28].]
2.3 Summary

In this Chapter, a short overview of the SM was presented. We then moved to BSM physics, in particular we review the DM problem where the candidate is a WIMP. We showed how its relic density is calculated through the solution of the Boltzmann equation. Furthermore, we show how experiments search for WIMPs. In the case of direct detection, experiments look for the evidence of a DM interaction with a heavy nucleus, such as Xenon. In the case of indirect detection, experiments look for evidence of DM annihilation in regions of high DM density such as the center of the Milky Way. We also review the case of DM that could be produced at the LHC and how it could be constrained.

On the other hand, we review the neutrino mass problem and how it could be solved with a radiative seesaw which connects the solution also to the solution of the DM problem.
Chapter 3

The Doublet Triplet DM with neutrino masses

As was reviewed in Ch. 2, DM comprises most of the matter-content of the universe, yet, there is not a DM candidate within the SM. Moreover, in order to keep the symmetries of the SM, it is not possible to have massive neutrinos, which goes against the observation of neutrino mixing. It is possible that the physics that gives rise to the solution of one problem is also implicated in the solution of the other. It was in that direction in which the Scotogenic or Radiative Seesaw model was proposed by E. Ma in [24]. In the model, the $Z_2$ symmetry that stabilizes the DM, also forbids neutrino masses a tree-level, yet, a Majorana mass is generated at the one-loop level in the topology classified as T-3 in [27] which is a one-loop realization of the $d = 5$ Weinberg operator. The symmetry is a global discrete symmetry where fields either transform as odd (charged) or even (uncharged). On the other hand, the Scotogenic model is a simple extension of the SM with an extra $Z_2$-odd doublet and several copies of a right-handed neutrino also charged under the symmetry. The DM candidate could be either the lightest right-handed neutrino or one of the neutral components of the doublet, the CP-even or the CP-odd. Several such Scotogenic models that connect the WIMP DM candidates and one-loop Majorana masses have also been proposed. For instance in [26] it was found that by considering fields (fermion and scalars) transforming as singlets, doublets and triplets under the $SU(2)$ symmetry, and with all new fields odd under the $Z_2$ symmetry, it was possible to construct 35 such models. In this chapter, we consider one such model, classified as T-1-2-F with $a = -1$ in [26], where a vectorlike doublet with $Y = -1$ and a Majorana triplet are added to the SM. Additionally, in the scalar sector, a $Y = 1$ scalar doublet and a real triplet are also added. All new fields are odd under a $Z_2$ symmetry, while the SM is even.
3.1 The Model

We extend the SM by adding an SU(2)\textsubscript{L} vectorlike doublet with \( Y = -1 \) and a Majorana triplet, both odd under a \( Z_2 \) symmetry. These fields can be expressed as

\[
\psi_L = \left( \begin{array}{c} \psi_1^L \\ \psi_2^L \\ \end{array} \right), \quad \psi_R = \left( \begin{array}{c} \psi_1^R \\ \psi_2^R \\ \end{array} \right), \quad \Sigma_L = \left( \begin{array}{cc} \Sigma_0^L / \sqrt{2} & \Sigma_1^L / \sqrt{2} \\ \Sigma_1^L / \sqrt{2} & -\Sigma_0^L / \sqrt{2} \end{array} \right).
\]

(3.1)

On the other hand, the scalar sector is enlarged by a \( Z_2 \)-odd doublet with \( Y = 1 \) and a \( Z_2 \)-odd real triplet,

\[
H_2 = \left( \begin{array}{c} H^0 \\ \frac{H^0 + i \rho_0}{\sqrt{2}} \end{array} \right), \quad \Delta = \frac{1}{2} \left( \begin{array}{cc} \Delta_0 & \sqrt{2} \Delta^+ \\ -\sqrt{2} \Delta^- & -\Delta_0 \end{array} \right).
\]

(3.2)

It follows that the most general \( Z_2 \)-invariant Lagrangian of the model is

\[
\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_F + \mathcal{L}_S + \mathcal{L}_1,
\]

(3.3)

where \( \mathcal{L}_{\text{SM}} \) is the SM Lagrangian, which comprises the scalar potential of the Higgs doublet \( H_1 \) as shown in Eq. 2.8. Besides, \( \mathcal{L}_F \) refers to the the kinetic and mass terms of the \( Z_2 \)-odd fermion fields,

\[
\mathcal{L}_F = \bar{\psi} i \gamma^\mu D_\mu \psi - M_\psi (\bar{\psi}_R \psi_L + \text{h.c.}) + \text{Tr}[\bar{\Sigma}_L i \gamma^\mu D_\mu \Sigma_L] - \frac{1}{2} \text{Tr}(\bar{\Sigma}_L M_\Sigma \Sigma_L + \text{h.c.}),
\]

(3.4)

whereas \( \mathcal{L}_S \) contains the kinetic, mass and self-interaction terms of the the \( Z_2 \)-odd scalar fields,

\[
\mathcal{L}_S = |D_\mu H_2|^2 - \mu_2^2 |H_2|^2 - \frac{\lambda_2}{2} |H_2|^4 + \frac{1}{2} |D_\mu \Delta|^2 - \mu_\Delta^2 |\Delta|^2 - \frac{\lambda_\Delta}{2} \text{Tr}[\Delta|^2].
\]

(3.5)

Lastly, \( \mathcal{L}_1 \) contains the different interaction terms between the \( Z_2 \)-odd fermion particles and the SM particles:

\[
\mathcal{L}_1 = \left[ -y_1 H_1^2 \bar{\Sigma}_L H_1 - \bar{\zeta}_1 L_{L_i}^\dagger \bar{\psi} R_i e^{\Sigma_L} H_2 - \rho_i \bar{\psi}_L H_2 e^{R_i} - f_i L_{L_i} \bar{\Delta} \psi_R + \text{h.c.} \right] - \mathcal{V}_1.
\]

(3.6)

Here \( L_{L_i} \) and \( e_{R_i} \) represent the SM lepton SU(2)\textsubscript{L} doublets and singlets, respectively, \( y_1, y_2, \zeta_i, \rho_i \) and \( f_i \) are Yukawa couplings controlling the new interactions \( (i = 1, 2, 3) \). By field redefinitions we make \( M_D \) and \( y_{1,2} \) to be positive whereas \( M_\Sigma \) is assumed to be real [19]. The last term in Eq. (3.6) accounts for the interaction potential,

\[
\mathcal{V}_1 = \lambda_3 |H_1|^2 |H_2|^2 + \lambda_4 |H_1^2 H_2|^2 + \frac{\lambda_5}{2} \left[ (H_1^2 H_2)^2 + \text{h.c.} \right] + \lambda_3 |H_1|^2 \text{Tr}[\Delta^2] + \lambda_6 |H_2|^2 \text{Tr}[\Delta^2] + \mu \left[ H_1^2 \Delta H_2 + \text{h.c.} \right],
\]

(3.7)
3.1. The Model

where $\lambda_5$ and $\mu$ have been taken to be real. This allows us to reduce the parameter space and avoid effects of CP violation which require a further analysis and is beyond our goal.

3.1.1 Scalar sector

In order to preserve the $Z_2$ symmetry once electroweak symmetry breaking occurs, a zero vacuum expectation value for $H_2$ and $\Delta$ is assumed, along with $\mu_1^2 > 0$, $\mu_2^2 > 0$ and $\mu_\Delta^2 > 0$. This entails that the trilinear $\mu$ term in the scalar potential is the only term responsible for the mixing among the CP-even neutral components as well as the charged components of the doublet and triplet fields. By parametrizing the Higgs doublet as $H_1 = (0, (h + v)/\sqrt{2})^T$, with $h$ being the SM Higgs boson and $v = 246$ GeV, we have that the CP-even neutral and charged mass matrices in the basis $(H^0, \Delta^0)$ and $(H^\pm, \Delta^\pm)$, respectively, read

$$M_{S^0} = \begin{pmatrix} \mu_1^2 + \lambda_1 v^2/2 & \frac{1}{2} \mu v \\ \frac{1}{2} \mu v & \mu_\Delta^2 + \frac{1}{2} \lambda'_3 v^2 \end{pmatrix}, \quad M_{S^±} = \begin{pmatrix} \mu_2^2 + \frac{1}{2} \lambda_3 v^2 & -\frac{1}{2} \mu v \\ -\frac{1}{2} \mu v & \mu_\Delta^2 + \frac{1}{2} \lambda'_3 v^2 \end{pmatrix},$$

(3.8)

where $\lambda_L = (\lambda_3 + \lambda_4 + \lambda_5)/2$ controls the trilinear interaction between the SM Higgs and $H^0$. The neutral physical states are

$$\begin{pmatrix} H^0 \\ \Delta^0 \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}, \quad \sin(2\alpha) = \frac{\mu v}{m_{\eta_2}^2 - m_{\eta_1}^2}, \quad (3.9)$$

where $m_{\eta_i}^2$ are the physical masses. For the charged sector the mass eigenstates $\kappa_i$ (with masses $m_\kappa$) are defined as

$$\begin{pmatrix} H^+ \\ \Delta^+ \end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \kappa_1 \\ \kappa_2 \end{pmatrix}, \quad \sin(2\beta) = \frac{-\mu v}{m_{\kappa_2}^2 - m_{\kappa_1}^2}. \quad (3.10)$$

Note that the angles $\alpha$ and $\beta$ are related due to the $\mu$ dependence. Finally, $A^0$ remains as the only CP-odd state in the spectrum with a mass given by $m_{A^0}^2 = \mu_2^2 + (\lambda_3 + \lambda_4 - \lambda_5)v^2/2$.

The set of $Z_2$-odd scalar masses allows expressing some of the parameters in the scalar potential in terms of them in such a way that the set of 10 free parameters of this model can be chosen to be

$$m_{A^0, m_\kappa, m_{\eta_1}, m_{\eta_2}, \lambda_2, \lambda_\Delta, \lambda_3, \lambda'_3, \lambda_6, \mu},$$

(3.11)
with the quartic couplings subject to the following vacuum stability and perturbativity conditions

\[\lambda_3 + \sqrt{\lambda_1 \lambda_2} > 0; \lambda_3 + \lambda_4 - |\lambda_5| + \sqrt{\lambda_1 \lambda_2} > 0; \lambda_3' + \sqrt{\lambda_1 \lambda_\Delta} > 0; \lambda_{H_\Delta} + \sqrt{\lambda_2 \lambda_\Delta} > 0; \lambda_3, \lambda_3', \lambda_6 < 4\pi; \lambda_2, \lambda_\Delta < \frac{4\pi}{3}. \quad (3.12)\]

Thus we can expect that for an appropriate choice of scalar couplings, \(\eta_1, \eta_2\) or \(A^0\) can be the lightest particle in the \(Z_2\)-odd scalar spectrum.

### 3.1.2 Fermion sector

Since \(H_1\) is the only scalar having a non zero vev, the \(y_{1,2}\) terms in Eq. (3.6) are the only ones that generate a mixing, in this case between \(\psi\) and \(\Sigma_L\). The \(\zeta_i, \rho_i\) and \(f_i\) terms represent pure interaction terms: they may induce both new co-annihilation and lepton flavor violation (LFV) processes and two of them (\(\zeta_i\) and \(f_i\)) enter in the neutrino mass generation. Consequently, the \(Z_2\)-odd fermion spectrum of the doublet-triplet DM model (DTDM) model is the one of the DTFDM model [19]. Thus, the neutral (in the basis \(\Xi_0 = (\Sigma_0, \psi_0, \psi_0^c)^T\)) and the charged (in the basis \(\Xi^- = (\Sigma_+^c, \psi_-, \psi_0^c)^T\) and \(\Xi_- = (\Sigma_-, \psi_-)\)) fermion mass matrices are given by

\[
M_{\Xi_0} = \begin{pmatrix}
M_\Sigma & \frac{1}{\sqrt{2}} y v \cos \beta & \frac{1}{\sqrt{2}} y v \sin \beta \\
\frac{1}{\sqrt{2}} y v \cos \beta & 0 & M_\psi \\
\frac{1}{\sqrt{2}} y v \sin \beta & M_\psi & 0
\end{pmatrix}, \quad M_{\Xi^-} = \begin{pmatrix}
M_\Sigma & y v \cos \beta \\
y v \sin \beta & M_\psi
\end{pmatrix},
\]

with \(y = \sqrt{(y_1^2 + y_2^2)/2}\) and \(\tan \beta = y_2/y_1^1\). It follows that the \(Z_2\)-odd particle spectrum of this model includes two charged fermion particles \(\chi_{1,2}^\pm\) with masses

\[
m_{\chi_{1,2}^\pm} = \frac{1}{2} \left[ M_\psi + M_\Sigma \mp \sqrt{(M_\psi - M_\Sigma)^2 + 2y^2v^2} \right] \text{ implying that } m_{\chi_2^\pm} > m_{\chi_1^\pm}, \text{ and three neutral Majorana states, namely } \chi_1^0, \chi_2^0 \text{ and } \chi_3^0 (\text{no mass ordering is implied}). \text{ Their masses can be computed from the characteristic equation,}
\]

\[
(M_\Sigma - m_{\chi_1^0})(m_{\chi_2^0}^2 - M_\psi^2) + \frac{1}{2}y^2v^2(M_\psi \sin 2\beta + m_{\chi_1^0}) = 0. \quad (3.14)
\]

Clearly, the \(Z_2\)-odd physical states are a mixture of the triplet and two doublets, with non zero couplings to the \(Z\) and Higgs bosons. On the contrary, in the symmetric case \(y_1 = y_2\) (\(\tan \beta = 1\)) one of the neutral states is an equal mixture of the doublet fermions without a triplet component and does not get a mass from the electroweak symmetry breaking. This means that the neutral spectrum has one pure doublet state with a mass given by the vector-like mass. This can be easily understood after

\[^1\text{Note that these mass matrices are reminiscent of the very well-known neutralino and chargino mass matrices in the minimal supersymmetric standard model [110]}.\]
3.2. DM phenomenology

considering the similarity transformation $M'_{\Xi_0} = O^\dagger M_{\Xi_0} O$, with

$$
M'_{\Xi_0} = \begin{pmatrix}
M_{\Sigma} & y_v & 0 \\
y_v & M_{\psi} & 0 \\
0 & 0 & -M_{\psi}
\end{pmatrix}, \quad \text{and} \quad O = \begin{pmatrix}
1 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{pmatrix}.
$$

Thus we have that $\chi_0^1 = \frac{1}{\sqrt{2}}(\psi_0^0 + \psi_0^c)$ with $m_{\chi_0^1} = -|M_{\psi}|$, and the charged eigenstates degenerate with the other two neutral states, i.e. $m_{\chi_0^2} = m_{\chi_0^+}$ and $m_{\chi_0^3} = m_{\chi_0^{\pm}}^2$. In addition to this, the fermion sector also presents other interesting features, in the symmetric case, namely $y_1 = \pm y_2$, it presents a custodial symmetry which ensures that the contribution of the new fermion fields to the $T$ and $U$ parameters are zero, and the diagonal coupling of the mass eigenstates to the $Z$ boson vanishes [19, 111].

Moreover, when $y_1 = y_2$ the Lagrangian is also invariant under a global $SU(2)_R$ symmetry. Second, when the condition $M_{\Sigma} < (y^2 v^2 - 4M_{\psi}^2) / (4M_{\psi})$ is fulfilled (and all the $Z_2$-odd scalars are heavier than $\chi_0^0$), the resulting DM candidate is pure doublet ($|m_{\chi_0^0}| < |m_{\chi_0^1}|, |m_{\chi_0^3}|$) with a vanishing diagonal coupling to the Higgs boson $g_{\chi_1 \chi_1} h$ at tree level. These features have a profound impact on DM phenomenology analysis since the direct detection through the $Z$-boson exchange would be zero at tree level and the same for the Higgs boson exchange when $\chi_0^1$ is the DM candidate (see below).

### 3.2 DM phenomenology

If the lightest $Z_2$-odd particle is electrically neutral, either a fermion or a scalar, it will play the role of the DM particle. Consequently, two main scenarios emerge according to whether the new co-annihilation processes are affecting or not the DM annihilation. For the scenario where the new co-annihilation processes are suppressed the resulting DM phenomenology will be very much that of doublet-triplet fermion (doublet-triplet scalar) DM model if the DM particle is a fermion (scalar), with the $Z_2$-odd scalars ($Z_2$-odd fermions) not playing any role in the DM annihilation and detection processes. Since the new co-annihilation channels to be effective demand large Yukawa couplings ($\zeta_i, f_i, \rho_i \gg 1$) or/and that the $Z_2$-odd scalar and fermion sectors to be mass degenerate, in what follows we will only consider the scenario where the new co-annihilation processes are inefficient. Moreover, this scenario is in turn preferred by the bounds coming from lepton flavor processes which, in general, favor small Yukawa couplings.
Chapter 3. The Doublet Triplet DM with neutrino masses

3.2.1 Fermion DM

When the DM particle is a fermion, the model can account for the observed DM relic abundance in two distinct regions [19–21]: one where the DM candidate lies at the electroweak scale and one above the TeV scale. The former region results when the DM particle is \( \chi_1^0 \), and \( y_1 = y_2 = y \), since its diagonal couplings to the \( Z \) and Higgs bosons, vanish at tree level, the DM does not annihilate through the \( s \)-channel. Fig. 3.1 shows the possible annihilation channels, that is, \( t \)- and \( u \)-channels with final state gauge bosons \( W^+W^- \) and \( ZZ \) via the exchange of one of the \( Z_2 \)-odd heavier fermion eigenstates. The annihilation channels are thus suppressed, and relic abundance is satisfied for low DM masses, around 100 GeV but only with large Yukawa coupling, \( y \gtrsim 1 \). To understand this last requirement, we may take a look at the expression for \( m_{\chi_2^\pm} \), if \( y \) is small, one of the heavier masses will be nearly degenerate with the DM, thus making the annihilations very efficient. This possibility may be considered in a way similar to that of the doublet fermion DM which requires masses around the TeV scale [16, 112, 113], which is far from the much more appealing electroweak region. On the other hand, if \( y \sim 1 \), the splitting between the neutral eigenstates is large, annihilation is less efficient and relic abundance is saturated at

\[ M_{\phi} + M_{\Sigma} = 0. \]  

(3.16)

Note that the charged states (and therefore \( \chi_2^0 \) and \( \chi_3^0 \)) are degenerate when

That is, all the fermion spectrum but \( \chi_1^0 \) is degenerate with a mass given by \( M_{\phi} \left[(1 + y^2v^2)/(2M_{\phi})\right]^{1/2} \).
masses around 100 GeV. In Figure 3.2 we display the resulting parameter space consistent with the DM relic abundance measured by Planck $\Omega_{DM} h^2 = 0.1199 \pm 0.0027$ [54] after scanning over the free parameters of the model$^3$. The maximum value for $y$ is set as a perturbativity condition. Below the minimum value for $M_\psi \sim 80$ GeV DM annihilations are no longer effective$^4$ while beyond $\sim 210$ GeV they are too effective. Note also that for any value of the pair $y$-$M_\psi$ that saturates relic density, there are two different allowed triplet mass regions, one where $M_\Sigma$ is always negative ($M_\Sigma < -M_\psi$) and one where it can be either positive or negative but larger than $-M_\psi$. In regards to direct detection for this scenario, the dispersion with nuclei is not possible at tree level because spin-independent interactions rely on $Z$ and/or $h$ mediation and neither of them is present at leading order. However, spin-independent interactions are allowed through loops mediated by the heavier $Z_2$-odd fermions and gauge bosons and through box diagrams involving gauge bosons and twist-2 operators. The latter contributions have been found to be two orders of magnitude smaller than the ones leading to an effective Higgs coupling and hence not considered here. In regards to the former type of diagrams, e.g. the vertex corrections shown in Fig. 3.3, the loop suppression could, in principle, make the cross section out of reach of the current experiment’s sensitivity. However, due to the large Yukawa couplings required by the relic abundance constraint, the LUX experiment [118] does place constraints on a portion of the parameter space, as it was shown in Ref. [20].

$^3$The model was implemented in SARAH-4.4.2 [114] which generates an output to SPheno [115, 116] to obtain the physical spectrum, which was then exported to MicrOMEGAS [117] to calculate the relic abundance.

$^4$To be consistent with collider bounds we have demanded that the lightest charged fermion is heavier than 93 GeV [20].
Chapter 3. The Doublet Triplet DM with neutrino masses

Since there are two allowed triplet mass regions, direct detection places different constraints in these two regions. To analyze them, we computed the effective $h - \chi^0_1 - \chi^0_1$ coupling using the expressions given in Ref. [20], we then calculated the spin-independent cross section, and imposed the most recent bound reported by the LUX experiment [71]. Figure 3.4 shows the allowed parameter space in the $y - M_{\Sigma}$ plane with the color bar representing the triplet mass $M_{\Sigma}$. In the region where $M_{\Sigma} \lesssim -100$ GeV (left panel), for the lowest $M_{\psi}$, $M_{\Sigma}$ can be as low as $\sim -1900$ GeV, however as $M_{\psi}$ increases, $M_{\Sigma}$ becomes heavily restricted very rapidly, to the point that, for $M_{\psi}$ around 85 GeV $M_{\Sigma}$ cannot be less than $\sim -500$ GeV. Upon further increasing $M_{\psi}$, direct detection becomes even more restrictive, leaving only a narrow strip in the $y$ vs. $M_{\psi}$ plane and with the largest $M_{\psi}$ being less than 130 GeV, at that point $M_{\psi} \sim -M_{\Sigma}$. On the other hand, in the region where $M_{\Sigma} \gtrsim -100$ GeV (right panel), at low values of $M_{\psi}$, direct detection restricts $M_{\Sigma}$ to be larger than $\sim 120$ GeV. As the dark matter mass increases, negative triplet masses are favored, in fact, the largest DM mass considered requires $M_{\Sigma} \sim -120$ GeV. Notice that in this region all values of the Yukawa coupling considered, are still viable, in contrast to the former region where $y < 1.8$. The direct detection bounds also imply that the splitting between the DM mass and the next heavier fermion are also constrained by LUX [71]. Defining $\delta_m$ as $(|m_{\chi^0_2}| - |m_{\chi^0_1}|)/|m_{\chi^0_1}|$ we find that, in the region where $M_{\Sigma}$ is mostly positive, small $\delta_m$ are only allowed for low DM mass and low $y$, for instance, for $\delta_m < 1$ the DM mass must be less than 110 GeV and $y$ less than 1.5 GeV. As the DM mass increases so must $\delta_m$, with the largest allowed value being $\sim 1.6$. A similar situation arises in the region where $M_{\Sigma}$ is always negative, for $\delta_m < 1$ the dark matter mass must be less than 90 GeV, larger DM mass require greater $\delta_m$, with the largest allowed value being 1.8.

For the other DM allowed region, the DM particle, either $\chi^0_2$ or $\chi^0_3$, is a mixture of
3.2. DM phenomenology

**Figure 3.4:** Parameter space of the electroweak DM region accounting for the observed DM relic abundance and consistent with direct and indirect searches of DM. In the top row the color bar corresponds to the allowed values of $M_{\Sigma}$ while in the bottom row it corresponds to the allowed values of $\delta m$, as defined in the text. The region in the left (right) panels satisfies $M_{\Sigma} < -m_{\chi_1^0}$ ($M_{\Sigma} > -m_{\chi_1^0}$), which corresponds to the region below (above) the dashed line in Fig. 3.2. Solid, dashed and dotted-dashed lines represent the maximum value for $m_{\chi_1^0}$ consistent with the Higgs diphoton decay rate reported in Refs. [119], [120] and [121], respectively (see text for details).

A mass splitting is only induced by loop corrections, thus small, hence, coannihilations are very efficient and relic abundance is only satisfied for heavy DM around the TeV scale [20]. If the DM is mostly doublet, the resulting model is similar to the pure doublet fermion DM and the lowest mass to saturate the relic abundance is $\sim$ 1 TeV [16, 112, 113], whereas, for mostly triplet, the lowest mass is $\sim$ 2.8 TeV [9, 107]. Since in this case, DM particle has a non-vanishing diagonal coupling to the Higgs boson (which in turn is controlled by $y$), new DM annihilation channels and DM scattering processes with nuclei are expected. This means that the DM mass may go beyond a few TeV’s.

Despite being an appealing and promising scenario, the symmetric case in the low mass region ($\sim$ 100 GeV) is severely constrained by the LHC measurement of the Higgs diphoton decay rate [120, 121]$^5$ since the $Z_2$-odd charged states induce the

---

$^5$From the LHC Run 1, the reported measurements are $1.17 \pm 0.27$ by ATLAS [122] and $1.14^{+0.26}_{-0.23}$ by CMS [123].
Chapter 3. The Doublet Triplet DM with neutrino masses

38

\[ h \kappa^i \kappa^j \gamma \gamma h \kappa^i \kappa^j \gamma \gamma h \]

(a)

(b)

\[ F \text{IGURE 3.5: Feynman diagrams of the Higgs diphoton decay induced by the } Z_2 \text{ odd charged particles. The left diagram (A) shows the contribution from the charged fermions. The two diagrams on the right (B) show the contribution from the charged scalars.} \]

Higgs decay to photons at one-loop\(^6\). In fact, in the DTFDM model, almost all the parameter space of this scenario is excluded due to the large suppression on \( R_{\gamma\gamma} \) induced by the two \( Z_2 \)-odd charged fermions (such a suppression arises because the fermion contribution is always positive, that is, opposite in sign to SM contribution, and sizable due to the large values of \( y \)).

Thus, in principle, the same should occur in the DTDM model. However, in the DTDM model, there are extra charged scalar fields, \( \kappa_{1,2} \), that also mediate the Higgs decay to two photons at one loop and may help to increase \( R_{\gamma\gamma} \).

To investigate the impact of the new \( Z_2 \)-odd charged scalar we consider the limit where \( \mu \ll v \), which is favoured by electroweak precision observables. The Feynman diagrams contribution from the new charged particles are shown in Fig. 3.5, this corresponds to a decay ratio that reads [19–21]:

\[
R = \left| 1 + \frac{1}{A_{SM}} \left[ \frac{\lambda_3 A_5 (\tau_{\chi_1})}{4m_{\chi_1}^2} + \frac{\lambda_3' A_5 (\tau_{\chi_2})}{4m_{\chi_2}^2} + \frac{y^2 v^2}{m_{\chi_1}^2 - m_{\chi_2}^2} \left( \frac{A_F (\tau_{\chi_1}^\pm)}{m_{\chi_1}^\pm} - \frac{A_F (\tau_{\chi_2}^\pm)}{m_{\chi_2}^\pm} \right) \right] \right|^2,
\]

(3.17)

where \( A_{SM} = -6.5 \) is the SM contribution from charged fermions such as the top quark and charged gauge bosons, with the loop functions \( A_F (\tau) = 2\tau^{-2}[\tau + (\tau - 1) \arcsin^2 \sqrt{\tau}] \) and \( A_5 (\tau) = -\tau^{-2}(\tau - f(\tau)) \) for \( \tau \leq 1 \), and \( \tau_X = m_X^2 / (4m_X^2) \). It follows that there are two possibilities in which the scalar contribution may modify the ratio to yield a result that is in agreement with experimental measurements:

- A sufficiently negative \( \lambda_3, \lambda_3' \) that may counteract the positive contribution from the \( Z_2 \)-odd fermions. However, this possibility is subject to the requirement of vacuum stability Eq. (3.12), which demands \( \lambda_3, \lambda_3' \gtrsim -1 \).

- A sufficiently positive \( \lambda_3, \lambda_3' \) could also yield the correct \( R_{\gamma\gamma} \) by generating a

\(^6\)The DM region above TeV scale does not suffer from such difficulties since the \( Z_2 \)-odd charged fermion lie above the TeV scale and therefore their effect on Higgs diphoton decay rate is negligible.
3.2. DM phenomenology

Figure 3.6: The combination $\lambda_3 + \lambda_3'$ as a function the $m_{\chi_0^1}$ for the set of points that present $R_{\gamma\gamma}$ within the experimental limits. The color bar shows the relation between the charged scalar $\kappa_1$ and the dark matter mass. The left figure corresponds to the region where $M_\Sigma$ is always negative and the right figure to the region where $M_\Sigma$ is mostly positive. All points satisfy, additionally, perturbativity, vacuum stability, relic abundance, direct detection, and electroweak precision constraints.

Negative new physics contribution of order 2, which in turn renders the observed ratio. We checked this possibility but found out that the restriction that $\mu_1^2 > 0$, $\mu_2^2 > 0$ and $\mu_3^2 > 0$ imposes limits on the allowed values of positive $\lambda_3$ and $\lambda_3'$, hence, the correct ratio is not achieved in this way.

To explore the impact on $R_{\gamma\gamma}$ we performed a random scan, taking as input points those that satisfy relic abundance and direct detection constraints. The relevant parameters are varied as follows:

$$0 < \lambda_2, \lambda_\Delta < \frac{4\pi}{3}; \; \lambda_3, \lambda_3' < 4\pi; \; 1.2 < m_{\kappa_1, \kappa_2} / m_{\chi_0^1} < 3.0; \; (3.18)$$

$$1.2 < m_{\eta_1, \eta_2} / m_{\chi_0^1} < 3.0; \; m_{A^0} = m_{\eta_1}; \; \mu \ll v.$$

The last two conditions, together with $m_{\kappa_1} - m_{\eta_1} \lesssim 85$ GeV, which we checked, ensure that the new scalars do not give sizable contributions to the electroweak precision observables, namely the $S$ and $T$ parameters. The constraint on the mass-ratio between the scalar and the DM is imposed to ensure that there are no coannihilation effects, hence, the expected DM phenomenology is not spoiled. The scan results are shown in Figure 3.6, with all points satisfying the constraints mentioned above along with a Higgs diphoton decay rate within experimental limits. The region where $M_\Sigma$ is mostly positive is shown on the right panel. It is evident from the plot that only $\lambda_3 + \lambda_3' < 0$ in order to achieve the correct ratio with the largest $m_{\chi_0^1}$ at 125 (GeV) and the ratio between the mass of $\kappa_1$ and the DM mass less than 2. The left panel shows the results for $M_\Sigma < -m_{\chi_0^1}$. In this case, there is a narrow region where $\lambda_3 + \lambda_3' < 0$ and $m_{\kappa_1} / m_{\chi_0^1} > 2$ coming from the fact that in that portion of the parameter space fermion suppression is not as strong and so the scalar enhancement is not essential.
Again, in this region, the DM mass must be less than 125 (GeV). It is worth mentioning that the same results would be obtained for \( \kappa_1 \to \kappa_2 \) in other words, both charged scalar have the same impact on the decay rate.

A final comment is in order on collider bounds which may constrain up to some extent the DM symmetric scenario at the electroweak scale. The main production processes associated with \( Z_2 \)-odd fermions at the LHC are the same as those of the DTFDM model \([19, 20]\). Since the \( Z_2 \)-odd scalars are coupled to \( Z_2 \)-odd fermions through the term controlled by \( \zeta_i, \rho_i \) and \( f_i \), which are assumed small (\( \ll 1 \)) in order to be compatible with LFV bounds, it turns out that \( \chi^\pm_1 \) can only decay to \( \chi^0_1 \) via a virtual \( W^\pm \). Thus, the characteristic signal to be looked for is events with final states involving leptons and missing transverse momentum. Moreover, for the range of the DM mass considered, the most sensitive channel is the one with three final state leptons plus missing transverse momentum, again as in the DTFDM model \([19, 20]\).

The DM production goes as follows:

\[
q\bar{q}' \to W^{\pm} \to \chi^{\pm}_{1,2,3} : \begin{cases} 
\chi^{\pm}_1 \to \chi^0_1 W^{\pm} \to \chi^0_1 \ell^\pm \nu_\ell, \\
\chi^0_{2,3} \to \chi^0_1 Z^* \to \chi^0_1 \ell^+ \ell^-.
\end{cases} \tag{3.19}
\]

This signal has been explored by the CMS Collaboration at \( \sqrt{s} = 8 \) TeV \([124]\) and \( \sqrt{s} = 13 \) TeV \([125]\), and by the ATLAS Collaboration \([126]\), for the case of supersymmetric charginos and neutralinos, in the limit of \( \chi^\pm_1, \chi^0_2 \) being wino-like with decoupled Higgsinos and squarks. When the bounds on the production cross-section are translated into this model lesser constraints are obtained \([20]\), due in part to the non wino-like character of \( \chi^\pm_1, \chi^0_2 \), which leads to the conclusion that the current LHC results does not constrain the symmetric scenario at the electroweak scale. However, the analysis made in Ref. \([20]\) regarding the future reach of the LHC, shows that this scenario can be completely explored with a luminosity of 300 fb\(^{-1}\).

### 3.2.2 Scalar DM

With respect to the DM phenomenology of this model, for the case of a heavy \( A^0 \) \((m_{A^0} \gtrsim m_{\eta_{1,2}})\), we identify three DM scenarios depending on the size of the entries of the neutral mass matrix: one where the DM is mostly doublet \((|M_{S^0}|_{11} \ll |M_{S^0}|_{22} \) and \( \mu \ll v \)), one with triplet DM \((|M_{S^0}|_{11} \gg |M_{S^0}|_{22} \) and \( \mu \ll v \)) and a scenario in between \((|M_{S^0}|_{11} \sim |M_{S^0}|_{22} \sim \mu v \)). For the first scenario, the DM particle is \( \eta_1 \sim H^0 \) and the viable DM mass regions are those of the IDM, namely, around the Higgs funnel region and above 500 GeV \([8, 127–132]\). In the latter region, the so-called high mass regime, the lowest DM mass that reproduces the observed relic DM density is obtained when the interactions of \( H^0 \) with the Higgs are negligible \((\lambda_{3,4,5} \ll 1)\), and so the main channels contributing to DM relic abundance are annihilation of the new charged (neutral) scalars via the \( t \)- and \( u \)-channels through the exchange of a neutral (charged) \( Z_2 \)-odd scalar, thus producing gauge bosons. Additionally, the scalar annihilations mediated by a gauge boson in the \( s \)-channel are also present.
3.3 Neutrino masses

As $\lambda_L$, the parameter that mediates the trilinear interaction of $H^0$ with the Higgs, increases, new annihilation channels become available and so in order to saturate the relic abundance the DM mass must be increased. A similar situation arises for the mostly triplet scenario (where $\eta_2 \sim \Delta^0$ is the DM particle), in the pure gauge limit, the channels that contribute to the DM relic abundance are similar to those of the mostly doublet case, except for the fact that gauge interactions are now stronger which makes annihilations more efficient, as a result, the lowest mass that saturates the relic density is $\sim 1.8$ TeV [9–11, 133, 134].

Regarding the mixed scenario, in the pure gauge limit, the DM particle lies between the above-mentioned cases. If the couplings $\lambda_L$ and $\lambda_3'$ are not zero, the DM mass must be higher to compensate for more annihilation channels available. Furthermore, in this scenario, if the DM particle is mainly triplet and a degenerate mass spectrum is considered, the constraint on the DM mass would loosen up to 1.1 TeV due to a net increase in the effective degrees of freedom which lowers the effective cross section. On the contrary, when the mass degeneracy within the triplet multiplet is lifted due to the mixing with the doublet (through the $\mu$ term), as well as the doublet components being heavier, it is harder to saturate the relic density constraint due to loss of the effective degrees of freedom entering in the thermally averaged cross section. It is worth noting that this mixed scenario may be significantly constrained from electroweak precision measurements.

For the case of a CP-odd DM particle, $A^0$, we have the same two viable DM mass regions of the IDM, but with the condition of having $\mu \ll v$ in order to not modify the expectations in the High mass regime since it requires the coannihilation of $A^0$ with the other two doublet components.

A last comment regarding both fermion and scalar DM is in order. If the model is considered in the regions of high mass such that $m_{DM} \gg M_{W,Z}$, the DM interaction with the $W$ and $Z$ boson might become long range. This leads to an enhancement in the annihilation cross section [11], known as the Sommerfeld enhancement, thus requiring heavier DM to yield the correct relic density. This situation is known to exist for doublets and even more for triplet extensions of the SM. However, for the case at hand, the enhancement is not present since the DM mass is of the order of $M_W$ and $M_Z$ due to the custodial symmetry.

3.3 Neutrino masses

As it was mentioned above, neither of the doublet-triplet mixed scenarios give account of neutrino masses. This occurs because the new $Z_2$-odd fields do not couple to the lepton doublet via renormalizable and gauge invariant terms. In other words, in these models the lepton number ($L$) is conserved. Nevertheless, from the combination of these models $L$ violating terms are automatically present which lead at the end to radiative neutrino masses at one-loop level. Hereby the interplay of the
doublet-triplet scalar and fermion DM models leads automatically to a framework with massive neutrinos.

In the doublet-triplet scalar (fermion) DM model the trivial lepton number assignment $L(H_2) = L(\Delta) = 0 \ (L(\Sigma) = L(\psi) = 0)$ guaranteed the $L$ conservation. However, when both models are combined several new $L$-violating terms appear. In particular, by keeping the same $L$ assignment the lepton number is violated in one unit by each of the Yukawa terms $\zeta_i, f_i$ and $\rho_i^7$. This in turn means that it is possible to generate neutrino masses at one-loop level since the seesaw mechanism at tree level is not operative due to the vanishing vev of $H_2$ and $\Delta$. Depending on which set of Yukawa couplings are used to build the one-loop neutrino mass diagram we have four different topologies (displayed in Fig. 3.7) that lead to the three finite realizations of the $d = 5$ Weinberg operator [27, 28]. Specifically, $\zeta_i$ generate $c)$ and $d)$ diagrams whereas $f_i$ generate $b)$ diagram, and both set of couplings enter in diagram $a)$. In both $a)$ and $b)$ diagrams the mixing fermion term $y_{1L}$ is mandatory and for the $b)$ diagram the mixing scalar term $\mu$ is also required. As electroweak eigenstates, the fermion triplet enters in each diagram, the scalar doublet is required in three of them, the same occurs to the triplet scalar while the doublet fermion enters in only two diagrams.

\[ \begin{array}{c}
\text{(a)} \\
\text{(b)} \\
\text{(c)} \\
\text{(d)}
\end{array} \]

**Figure 3.7**: Feynman diagrams leading to one-loop neutrino masses.
For the type $a)$ topology there are two Feynman diagrams, one with charged particles running in the loop and one with neutral particles, only the neutral one is shown.

\[ \text{In contrast, if we assign lepton numbers for the fermions such that } L(\psi) = 1 = -L(\Sigma) = 1 \text{ and all the new scalar at zero, then lepton number is violated in two units through the } y_{1L} \text{ term and also in two units by the Majorana-triplet mass term.} \]
The general expression for the Majorana mass matrix from the all one-loop contributions displayed in Fig. 3.7 can be written as

\[
M_\nu = \Lambda_\xi \tilde{\xi}_i \xi_j + \Lambda_f \xi_i \phi_j + \Lambda_f \xi_i \xi_j + f_i \phi_j,
\]

(3.20)

where

\[
\begin{align*}
\Lambda_\xi & = \frac{1}{32\pi^2} \sum_{k=1}^{3} m_{\nu_k}^0 (U_{ik})^2 \left[ c_\alpha^2 F_1 (m_{\nu_1}^2, m_{\nu_2}^2) + s_\alpha^2 F_1 (m_{\nu_2}^2, m_{\nu_3}^2) - F_1 (m_{\nu_1}^2, m_{\nu_3}^2) \right], \\
\Lambda_f & = \frac{1}{16\pi^2} \sum_{k=1}^{3} m_{\nu_k}^0 (U_{ik})^2 \left[ s_\alpha^2 F_2 (m_{\nu_1}^2, m_{\nu_2}^2) + c_\alpha^2 F_2 (m_{\nu_2}^2, m_{\nu_3}^2) \right], \\
\Lambda_{\xi f} & = \frac{1}{32\pi^2} \sum_{k=1}^{3} m_{\nu_k}^0 \left[ s_\alpha c_\alpha U_{1k} U_{3k} \left[ F_1 (m_{\nu_1}^2, m_{\nu_2}^2) - F_1 (m_{\nu_1}^2, m_{\nu_3}^2) \right] + s_\beta c_\beta \sum_{k=1}^{3} m_{\nu_k}^0 V_{1k}^L V_{2k}^R \left[ F_1 (m_{\nu_1}^2, m_{\nu_2}^2) - F_1 (m_{\nu_1}^2, m_{\nu_3}^2) \right] \right], \\
\end{align*}
\]

(3.21)

with \( U, V^L \) and \( V^R \) being the rotation matrices for the neutral, charged-left and charged-right fermions, respectively, and we have used \( s_\theta \) and \( c_\theta \) as a short-hand notation for the sine and the cosine of a given scalar mixing angle \( \theta \). The loop functions read

\[
F_1 (m_{\tau_1}^2, m_{\tau_2}^2) = \frac{m_{\tau_1}^2}{m_{\tau_1}^2 - m_{\tau_2}^2} \ln \frac{m_{\tau_1}^2}{m_{\tau_2}^2}, \quad F_2 (m_{\tau_1}^2, m_{\tau_2}^2) = \frac{m_{\tau_1}^2 \ln m_{\tau_2}^2 - m_{\tau_1}^2 \ln m_{\tau_2}^2}{m_{\tau_1}^2 - m_{\tau_2}^2}.
\]

(3.22)

Since \( M_\nu \) has a null determinant there is only one Majorana phase and the neutrino spectrum has one massless neutrino and the two non-zero neutrinos masses are set by the solar and atmospheric mass scales, e.g. for normal hierarchy \( m_{\nu_1} = 0, m_{\nu_2} = \sqrt{\Delta m_{\text{sol}}^2} \) and \( m_{\nu_3} = \sqrt{\Delta m_{\text{atm}}^2} \). Furthermore, it is possible to parametrize five of the six Yukawa couplings in terms of the neutrino observables so that there is only one free parameter. Specifically, for the case of a normal hierarchy and by considering without loss of generality \( \xi_1 \) as the free parameter, the most general Yukawa-couplings compatible with the neutrino oscillation data are given by

\[
f_i = \frac{1}{\Lambda_f} \left[ \pm \sqrt{\Lambda_f \xi_{i1} - \Lambda_{\xi f} \xi_{i1}^2} - \Lambda_{\xi f} \xi_i \right], \quad i = 1, 2, 3,
\]

(3.23)

\[
\xi_j = \frac{\pm 1}{\Lambda_f \Lambda_{\xi 11}} \sqrt{\lambda^2 m_{\nu_2} m_{\nu_3} \Lambda_{\xi 1}^2 \bar{A}(V_{e1}^* V_{\nu_2}^* - V_{e2}^* V_{\nu_1}^*)^2 (\Lambda_f \alpha_{11} - \Lambda_{\xi f} \alpha_{11}^2)} + \frac{\alpha_{1j} \xi_1}{\alpha_{1j}}, \quad j = 2, 3,
\]

(3.24)

where we have defined

\[
\bar{A} \equiv (\Lambda_f \Lambda_{\xi f} - \Lambda_{\xi f}^2), \quad \alpha_{ij} \equiv m_{\nu_2} \lambda^2 V_{e1}^* V_{\nu_2}^* + m_{\nu_3} V_{e2}^* V_{\nu_3}^*.
\]

(3.25)

In addition, we have used \( \tilde{M}_\nu = U_{\text{PMNS}}^T M_\nu U_{\text{PMNS}} \) and \( U_{\text{PMNS}} = V P \) [135], with \( \tilde{M}_\nu = \text{diag}(0, m_{\nu_2}, m_{\nu_3}) \), the matrix \( V \) containing the Dirac phase and the neutrino
mixing angles, and $P = \text{diag}(1, \lambda, 1)$ giving account of the Majorana neutrino phase. For the case of an inverted hierarchy, the parametrization would yield a similar result which we do not include. In this way, it is always possible to correctly reproduce the neutrino oscillation observables within the DTDM model.

In order to estimate the size of the Yukawa couplings $f_{1,2,3}$ and $\zeta_{2,3}$ for the electroweak DM region, we have repeated the scan over the parameter space (see Eq. (4.4)) with $10^{-3} \leq \zeta_1 \leq 1$, and assume CP conservation and a normal hierarchy. As a result, we have found that the Yukawa couplings $f_{1,2,3}$ and $\zeta_{2,3}$ can be small as $\sim 10^{-3}$. Since such couplings also control LFV processes, it follows that the corresponding rates can become rather suppressed because they generically involve the product of two squared Yukawa couplings. Consequently, in the electroweak DM region, it is also possible to be compatible with the LFV constraints [136, 137].

3.4 Summary

To conclude, we have found that, by enlarging both the fermion and scalar sectors of the SM with $SU(2)_L$ doublets and triplets and by imposing a custodial symmetry on the Yukawa couplings, e.g. $y_1 = y_2 = y$, of the new fermion fields to the Higgs doublet, we obtain a fermionic dark matter candidate at the electroweak scale that may saturate the relic density. In the custodial limit, direct detection is only possible at the loop-level through spin-independent interactions mediated by $h$. As a result, DM at the electroweak scale is still viable. On the other hand, the relic density constraint enforces large Yukawa couplings, $O(1)$ as well as large splittings between the masses of the heavier and lightest odd-sector fermion. This, in turn, results in large deviations on the Higgs diphoton decay rate due to the newly charged fermions, which is in tension with experimental results. However, we found that the scalars might lift the suppression when the sum of the couplings $\lambda_3 + \lambda'_3 < 0$, while we still ensure perturbativity, vacuum stability and that the Lagrangian mass term of all the scalar fields is positive. Once the effect of the scalars is included, we found that DM masses between 80 to $\sim 125$ GeV was consistent with the aforementioned restrictions. Furthermore, due to the $Z_2$ symmetry, neutrino masses are zero at tree level. Nonetheless, the model generates Majorana neutrino masses at the one-loop level in all possible topologies of the $d = 5$ Weinberg operator and at the same time satisfy restrictions from neutrino physics with Yukawa couplings as low as $10^{-3}$. 
Chapter 4

Phenomenology of doublet-triplet fermionic DM in non-standard cosmology and multicomponent dark sectors

In Ch. 3 we saw that the interplay between the doublet-triplet fermion and scalars could explain DM at the electroweak scale and account for the fact that neutrinos are massive. Moreover, the scalar particles could relax the strong constraints on the fermion sector due to the Higgs diphoton decay rate. In this chapter we take a different approach. Since the doublet-triplet fermion (DTF) generates and interesting WIMP DM from the phenomenological point of view (especially under the custodial symmetry), we ask if it is possible for the lightest neutral fermion to account for the DM of the universe while also evading current strong constraints? The answer to this question turns out to be mostly not. If current direct detection constraints are considered together with latest $R_{\gamma\gamma}$ results by ATLAS [22] and CMS [23] collaborations, the parameter space of the model is highly constrained. This is due to the high Yukawa couplings required for obtaining the correct relic density constraint. For this reason, it is worth considering alternative scenarios for this interesting DM candidate. Two of the several possibilities are:

- The new fermion sector arises in a non-standard cosmology.
- The WIMP is part of a multi-component dark sector.

In both cases the relic density constraint is relaxed, thus broadening the allowed parameter space of the model. For the non-standard cosmology assumption, it is worth mentioning that WIMPs decouple from the thermal plasma before BBN occurs and that we have no experimental evidence of conditions previous to BBN. On the other hand, in Ch. 2 we saw how the WIMP relic density is obtained in the standard and simplest case. Eq. 2.28 shows the dependence of the relic density with $T_f$, the freeze-out temperature, so this temperature is a key parameter in how much relic density is obtained for a given WIMP candidate. But this temperature depends
also on whether DM decouples during matter or radiation dominated era [67] in the standard calculation it is assumed that freeze-out occurs in a radiation dominated era but it could happen in a matter dominated thus changing the obtained relic density. Moreover, a late decay of a scalar field that couples to gravity could change the reheating temperature and could decay into DM particles, either one of them would affect the WIMP number density [33, 138, 139]. Thus, assumptions about what happens before and during decoupling of the WIMP that must be made, have an effect on the WIMP relic density. As a result, in a non-standard cosmology scenario, it is possible that a WIMP yields just the correct relic density, even though in the standard scenario is over or underabundant. Interestingly enough, such deviations from the standard cosmology do not affect the prospects regarding DM detection and BBN [30]. In light of this, it makes sense to study the DTF model without imposing the relic constraint, because, again, in non-standard cosmology it is possible to fulfill this requirement when the right combination of parameters is achieved.

On the other hand, although most DM models assume the candidate is just one particle, there is no reason to consider this to be the only possibility. The total relic abundance could be a result of the presence of several DM particles. This is referred in the literature as multi-component DM. In works such as [140, 141] the total relic abundance was set by the contribution of a WIMP and the Axion, another well motivated DM particle. Other works have considered the two (or more) candidates to be WIMPs. For instance in Ref. [142] the case of a two dirac fermions with an additional $U(1)_{B-L}$ was studied, while works such as [143, 144] considered the DM stability to arise from a $Z_2 \times Z_2'$ symmetry. Taking this into consideration, it also makes sense to study a model where the total relic abundance is not enforced as a constraint, but rather that is achieved by the interplay of two separate sectors.

For the reasons mentioned above, in this Chapter we consider the DTF model under the custodial symmetry but without imposing the relic density constraints. And then we study the impact of collider, direct detection, and indirect detection experiments on the parameter space of the model.

### 4.1 The Model

We now enlarge the SM with the same fermionic sector considered in Ch. 3. The new interaction Lagrangian of the DTF that is invariant under $SU(2)_L \times U(1)_Y \times Z_2$ is:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_F + \mathcal{L}_1.$$  \hfill (4.1)

Here $\mathcal{L}_F$ refers to the kinetic and mass terms of the new fermions, which are the same as those of Eq. 3.3. While $\mathcal{L}_1$ is:
\[ L_4 = -y_1 H^\dagger \Sigma L e \psi_R^c + y_2 \bar{\psi}_L^c e \Sigma_L H + \text{h.c.} \] (4.2)

From this last term, there is fermion mixing and we obtain the same particle spectrum of the fermion sector of Ch. 3. We again focus on the custodial limit scenario \( y_1 = y_2 = y \) which ensures no deviations from electroweak precision tests \[111\], and that the DM-DM coupling to the Higgs and Z boson is zero at tree-level, as a result, DD is only possible at loop level.

### 4.2 Collider Bounds

The LHC is currently running at an outstanding 13 TeV energy and has collected more than 100 fb\(^{-1}\) of data. One of its current goals is to probe BSM models either by direct production of new particles or by measuring possible deviations from the SM. In that regard, the parameter space of the DTF model may be constrained by the ATLAS and CMS experiments.

#### 4.2.1 Higgs diphoton decay

As explained in Ch. 3 the presence of the \( Z_2 \) odd fermions may generate large deviation from the results published by ATLAS \[22\] and CMS \[23\] collaborations. The new physics contribution is:

\[ R_{\gamma\gamma} = \left| 1 + \frac{1}{A_{\text{SM}}} \left[ \frac{y^2 v^2}{m_{\Sigma_2}^2 - m_{\chi^\pm_1}^2} \left( A_F(\tau_{\chi^\pm_2}) - A_F(\tau_{\chi^\pm_1}) \right) \right] \right|^2, \] (4.3)

where, \( A_{\text{SM}} = -6.5 \) and the loop factor is \( A_F(\tau) = 2\tau^{-2}[\tau + (\tau - 1) \arcsin^2 \sqrt{\tau}] \) for \( \tau \leq 1 \) where \( \tau_X = m_X^2 / (4m_{\chi}^2) \). Since we are not considering the relic density constraint, the model no longer requires large Yukawa couplings and large splitting between \( m_{\chi^\pm_1} \) and \( m_{\chi^\pm_1} \) and so the strong \( R_{\gamma\gamma} \) restriction gets relaxed. In order to obtain this restriction on the model, we performed a scan of the free parameters of as follows:

\[ 0.1 < y < 3.5, \]
\[ -2000 \text{ GeV} < m_{\Sigma} < 2000 \text{ GeV}, \]
\[ 75.0 \text{ GeV} < m_{\psi} < 500 \text{ GeV}. \] (4.4)

Additionally, we only considered models where the lightest charged fermion is heavier than 100 GeV in order to satisfy LEP limits \[145\].

The results are presented in Figs. 4.1 and 4.2 where the parameter space has been divided into two regions, \( M_\Sigma < -m_{\chi^\pm_1} \) (left panels) and \( M_\Sigma > -m_{\chi^\pm_1} \) (right panels). The scan shows that, considering the ATLAS results, for the region where
Chapter 4. Phenomenology of doublet-triplet fermionic DM in non-standard cosmology and multicomponent dark sectors

Figure 4.1: Scan on the parameter space of the model against $R_{\gamma\gamma}$ for the region where $M_\Sigma < -m_{\chi_1^0}$ (left panels) and the region where $M_\Sigma > -m_{\chi_1^0}$ (right panels). The solid and dashed horizontal lines represent the lowest bound at a 2\(\sigma\) deviation from the central value reported by the ATLAS and CMS collaboration respectively, while the color indicates the value of $M_\Sigma$.

Figure 4.2: Same as Fig. 4.1 but with the color indicating the value of the Yukawa coupling.

$M_\Sigma < -m_{\chi_1^0}$ there are no restrictions on the Yukawa coupling $y$ or on $M_\Sigma$ whereas in the region $M_\Sigma > -m_{\chi_1^0}$ the decay rate forbids $y$ values larger than 2.25 and $M_\Sigma \lesssim -60$ GeV. On the other hand, CMS results yield the severe constraint $y < 2.0$ for $M_\Sigma < -m_{\chi_1^0}$ and $y < 1.0$ for $M_\Sigma > -m_{\chi_1^0}$, whereas $M_\Sigma$ must be less than $-92$ GeV. Hence, positive triplet masses are no longer consistent with $R_{\gamma\gamma}$ results.

We also consider the impact on the fermion mixing angle from $R_{\gamma\gamma}$ (see Fig 4.3). For the region where $M_\Sigma < -m_{\chi_1^0}$ (left panel) the mixing angle must be small such that $|\cos\theta| \lesssim 0.3$ ($|\cos\theta| \lesssim 0.2$) in order to be consistent with ATLAS (CMS) measurements. Accordingly, in that region the lightest charged fermion is mostly doublet. On the other hand, the results for the region where $M_\Sigma > -m_{\chi_1^0}$ (right panel) show that the lightest charged fermion is mostly triplet, with $|\cos\theta| \gtrsim 0.94$. 

---

$M_\Sigma < -m_{\chi_1^0}$ there are no restrictions on the Yukawa coupling $y$ or on $M_\Sigma$ whereas in the region $M_\Sigma > -m_{\chi_1^0}$ the decay rate forbids $y$ values larger than 2.25 and $M_\Sigma \lesssim -60$ GeV. On the other hand, CMS results yield the severe constraint $y < 2.0$ for $M_\Sigma < -m_{\chi_1^0}$ and $y < 1.0$ for $M_\Sigma > -m_{\chi_1^0}$, whereas $M_\Sigma$ must be less than $-92$ GeV. Hence, positive triplet masses are no longer consistent with $R_{\gamma\gamma}$ results.

We also consider the impact on the fermion mixing angle from $R_{\gamma\gamma}$ (see Fig 4.3). For the region where $M_\Sigma < -m_{\chi_1^0}$ (left panel) the mixing angle must be small such that $|\cos\theta| \lesssim 0.3$ ($|\cos\theta| \lesssim 0.2$) in order to be consistent with ATLAS (CMS) measurements. Accordingly, in that region the lightest charged fermion is mostly doublet. On the other hand, the results for the region where $M_\Sigma > -m_{\chi_1^0}$ (right panel) show that the lightest charged fermion is mostly triplet, with $|\cos\theta| \gtrsim 0.94$. 

---
\(|\cos \theta| \gtrsim 0.98\) for the ATLAS (CMS) results. A comment regarding this region is in order, for \(R_{\gamma\gamma} \sim 0.2\) the mixing angle exhibits a rather complex behaviour which is seen as large changes in \(\cos \theta\) (from -0.8 to 0.6) right next to a boundary where \(\cos \theta \sim 0.8\). This stems from the fact that at this boundary the triplet mass is changing sign, thus having an impact on the mixing angle behaviour.

The results for the mixing angle in both region are important because they will have a direct impact on the production cross section of the heavier fermions at the LHC, as will be discussed below.

\[\begin{align*}
\begin{array}{ll}
\text{Figure 4.3: Impact of the cosine of the mixing angle } \theta \text{ on the Higgs diphoton decay rate for the allowed values of the DM mass. The conventions are the same as those of Fig. 4.1.}
\end{array}
\end{align*}\]

### 4.2.2 Constraints from electroweak production searches

Other LHC results that may potentially constrain the DTF model are those searching for electroweak production of neutralinos and charginos in different simplified SUSY models (with all other SUSY particles decoupled), where the relevant detection channels are those with several leptons (and missing energy) in the final state. In the DTF, \(\chi_{1,2}^\pm\) and \(\chi_{2,3}^0\) play the role of charginos and heavier neutralinos, respectively, with the same mass degeneracy that characterizes the simplified supersymmetric scenarios.

The CMS collaboration has recently published results for such searches at an energy of \(\sqrt{s} = 13\) TeV and 35.9 fb\(^{-1}\) \([146]\). For the case of \(m_{\chi_1^1} \lesssim 500\) GeV and a non-degenerate spectrum, the most sensitive channel is that with three final state leptons where at least two of them have opposite sign and same flavour. Thus, DM production proceeds via the following process

\[q\bar{q}' \rightarrow W^* \rightarrow \chi^\pm_{1,2,3} : \begin{cases} 
\chi^\pm \rightarrow \chi_{1,23}^0 W^* \rightarrow \chi_{1,23}^0 \ell^\pm \nu, \\
\chi_{2,3}^0 \rightarrow \chi_{2,3}^0 Z^* \rightarrow \chi_{1,23}^0 \ell^+ \ell^-. \end{cases}\]
where the mediators $\chi^{\pm}$ and $\chi^{0}$ are considered to be winos and thus mass degenerate, with the neutral fermion decaying 100% via Z boson. To recast the LHC constraints (and other experimental restrictions that will be discussed below) for the DTF we implemented the model in S\textsc{arah-4.12.3} package \cite{114} whose output was used with S\textsc{pheno-4.0.3} \cite{115} in order to obtain the particle spectrum and with \textsc{MadGraph5}_\textsc{aMC@NLO} to obtain the production cross sections \cite{147}.

Fig. 4.4 shows the constraints from electroweak production, where the excluded region corresponds to the points below the blue dashed line. Moreover, the points below the solid and dashed black lines yield a lower diphoton decay ratio than the one allowed by ATLAS and CMS, respectively. In the region where $M_{\Sigma} > -m_{\chi_{1}^{0}}$ (right panel) the diphoton decay ratio restricts the lightest charged and the next-to-lightest neutral fermions to be mostly doublet. A consequence of this is that the production cross section is nearly the same even for all values of the allowed Yukawa coupling and the triplet mass, which means that the boundary of the excluded region is nearly independent of $y$ and $M_{\Sigma}$. Moreover, due to the mixing angle, the production cross section resembles that of SUSY Higgsino with all scalars decoupled. The figure shows that the strongest constraints come mostly from $R_{\gamma\gamma}$, except for a small area where electroweak production cross section is more restrictive. Nonetheless, there are no additional restrictions placed on the free parameters $M_{\psi}$, $y$ and $M_{\Sigma}$.

In the region where $M_{\Sigma} < -m_{\chi_{1}^{0}}$, the diphoton decay rate restricts $\chi_{2}^{0}$ and $\chi_{1}^{\pm}$ to be mostly triplet. The production cross section is large but again independent of $y$ and $M_{\Sigma}$, and so, the region excluded by electroweak production is presented with only one contour. In this region, due to the larger production cross section, the curve

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4.png}
\caption{DM mass versus lightest chargino mass for the regions where $M_{\Sigma} < -m_{\chi_{1}^{0}}$ (left panel) and $M_{\Sigma} > -m_{\chi_{1}^{0}}$ (right panel). The region below the blue dashed line is excluded from CMS electroweak production while the regions bounded by the blue (green) solid line represents the exclusion by the ATLAS (CMS) collaboration using compressed spectra. Points below the solid (dashed) black contour are excluded by the $R_{\gamma\gamma}$ results reported by the ATLAS (CMS) collaboration.}
\end{figure}
is shifted to the left in the $m_{\chi^+_1}$ line and so $R_{\gamma\gamma}$ places the strongest constraints for the whole plain.

### 4.2.3 Constraints from compressed spectra searches

The ATLAS and CMS collaborations have also published relevant results for the DTF for the case of compressed spectra [148, 149], i.e., the next-to-lightest fermion is close in mass to the neutralino DM ($\lesssim 35$ GeV) and a mass degeneracy between the next-to-lightest neutralino and lightest chargino. In that region, the DM production proceeds via:

$$q\bar{q}' \rightarrow W^{\pm} \rightarrow \chi^{\pm}_0 \chi^{0}_{2,3}: \begin{cases} \chi^{\pm}_0 \rightarrow \chi^{0}_{1} W^{\pm}\rightarrow \chi^{0}_{1} q\bar{q}', \\ \chi^{0}_{2,3} \rightarrow \chi^{0}_{1} Z^{\pm}\rightarrow \chi^{0}_{1} \ell^{+}\ell^{-}. \end{cases} \quad (4.6)$$

The search then focuses on two leptons with opposite sign and same flavour with soft momentum and large $\not{E}$ which is present due to the two DM particles recoiling against initial state radiation. For this search, small mass splittings are required, which is motivated in order to ensure DM coannihilations. In the DTF this low mass splitting is not needed, in fact $0 \lesssim m_{\chi^+_1} - m_{\chi^0_0} \lesssim 140$ GeV, however, we may still use the constraints for small mass splittings between the next-to-lightest $\chi^\pm$ and the DM. We find that, for the region $M_{\Sigma} > -m_{\chi^0_0}$ where $\chi^+_1$ and $\chi^0_2$ are mostly triplet, and so the restrictions resemble those of the ATLAS and CMS collaboration which is shown with a solid blue and green contour respectively. In terms of the free parameters, we find that the triplet mass is now restricted to be smaller than $\sim -165$ GeV, whereas the Yukawa coupling is not constrained. For the case of $M_{\Sigma} < -m_{\chi^0_0}$, since $\chi^+_1$, $\chi^0_2$ are mostly doublet, there is a lower production cross section and so the restriction is negligible.

### 4.3 DM detection in a non-standard cosmology

In a non-standard cosmology scenario, the late decay of a heavy scalar field could either increase or decrease the DM relic abundance compared to the standard calculation. Hence, we expect the DTF model to saturate, in one way or another, the DM relic abundance. Therefore, we look into current experimental constraints coming from direct searches and indirect detection via gamma rays. Since the diphoton decay is by far more restrictive than production at colliders, in this section we impose the $R_{\gamma\gamma}$ restriction coming from ATLAS and when relevant, we present the restriction arising in this observable from the CMS experiment and from electroweak production.

Within the custodial limit scenario, the SI elastic scattering is only achieved at the loop level since both $g^{\lambda_{1}^{0}\lambda_{1}^{0}Z}$ and $g^{\lambda_{1}^{0}\lambda_{1}^{0}h}$ couplings vanish at tree-level. However, at loop-level there are, in principle, several contributions that could be relevant. As shown in Ch. 3, there is an effective non-zero $\chi^{0}_{1}\lambda_{1}^{0}h$ coupling originating from loops.
mediated by the new heavier fermions and weak gauge bosons, thus allowing for spin-independent direct detection (see Fig. 3.3). In order to compute the effective Higgs coupling, we used the results presented in [20]. Additionally, box diagrams mediated by gauge bosons and twist-2 operators of the form [150, 151]:

$$\mathcal{L}_{\text{eff}} = \frac{g_1^{(1)}}{m_{\chi_1}^2} \partial_\mu \gamma^\nu \chi_0^0 \mathcal{O}_{\mu\nu}^q + \frac{g_2^{(2)}}{m_{\chi_1}^2} \partial_\mu (i \partial_\nu) (i \partial_\nu) \chi_0^0 \mathcal{O}_{\mu\nu}^q, \quad (4.7)$$

where $\mathcal{O}_{\mu\nu}^q \equiv \frac{i}{2}(\partial_\mu \gamma_{\nu u} + \partial_\nu \gamma_{mu} - \frac{1}{2} g_{\mu\nu} \mathcal{D}) q$, contribute to the SI cross section. Additionally, box diagrams mediated by gauge bosons and twist-2 operators [150, 151] contribute to the SI cross section. In principle these two contributions should be taken into account to obtain a reliable calculation. However, it has been shown that they are sub-leading by about two orders of magnitude [19] except when the two contributions arising from the Higgs vertex corrections cancel each other out, which happens for low values of $\sigma_{SI} (\lesssim 10^{-47} \text{cm}^2)$ [151]. Moreover, the authors of Ref. [150] have shown that when the cancellation happens, two-loop contribution to an effective scalar interaction with external gluons are of the same order as the box ones. Since these calculations (boxes from gauge, twist-2 and two-loop) are very involved and only relevant for the case of specific cancellations, we will not take them into account for the calculation of the SI cross section. Moreover, they tend to create a larger suppression of the cross section that is already out of reach of current experiments. As a result, the restrictions that we will present below from DD are not strongly affected by this assumption.

In order to obtain the most up to date limits from DD, we calculated the effective $g_{h\chi_1^{0}\chi_1^{0}}$ coupling from Ref. [20] and used that to compute the SI cross sections. We then compared to the current upper limits on the DM-nucleon SI scattering cross section, where the strongest ones (within the DM mass range we are considering) are those reported by the XENON1T collaboration [152]. We also show the projected sensitivity of DARWIN [153], the most sensitive DD experiment planned for DM at the electroweak scale. However, the expected SI cross section around the DARWIN limit must be taken with a grain of salt since sub-leading corrections might change $\sigma_{SI}$ in that region.

In Fig. 4.5 we display the results for the spin-independent cross section as a function of the DM mass, for the regions $M_\Sigma < -m_{\chi_1^{0}}$ (left) and $M_\Sigma > -m_{\chi_1^{0}}$ (right). It follows that XENON1T restricts the coupling to be less than 1.75 if the lower bound on $R_{\gamma\gamma}$ from ATLAS is imposed. The dashed black line in both panels shows the CMS lower bound on $R_{\gamma\gamma}$, which excludes models even further and for the region $M_\Sigma > -m_{\chi_1^{0}}$ imposes $y \leq 1.2$. We also checked the impact of DD results on the other free parameter of the model, $M_\Sigma$, but we found that they place no further restrictions on it as shown in the bottom panels of Fig. 4.5. The prospects coming from the DARWIN experiment correspond to the green solid line, which show that couplings
as small as 0.5 may be probed. It is worth mentioning that the lower limit on the SI cross section is due to the cancellation between the two one-loop corrections to the $h\chi_1^0\chi_1^0$ vertex, hence, in order to have a precise value of $\sigma_{SI}$ in this region a more detailed calculation is necessary.

### 4.3.1 Indirect detection from dwarf spheroidal galaxies

In regions of high DM density such as dwarf spheroidal galaxies (dSphs) or the center of the Milky Way, DM particles may more easily find each other and annihilate into SM particles. The dSphs are particularly interesting because of their proximity to the Milky Way, their high DM to baryon mass-ratio, and their low background, thus making the DM detection via gamma-rays feasible. The Fermi satellite has
searched for gamma rays in dSphs founding no deviations from the expected spectrum, which has lead to upper limits on the thermally averaged DM annihilation cross section [154].

For the DTF, the DM annihilation proceeds in the same channels as the ones in the early Universe, i.e., via $t$ and $u$-channel annihilation into $W^+W^-$ and $ZZ$ bosons. The gauge bosons then decay and produce, for instance, gamma rays that may be detected as an excess in the spectrum. To obtain the constraints we calculated the thermally averaged cross section using the publicly available package micrOMEGAS [117] and used this to compare with limits reported in [154].

The results are shown in Fig. 4.6 where all points shown satisfy the ATLAS $R_{\gamma\gamma}$ constraint and DD bounds as explained in previous sections. As can be seen, the Fermi satellite observation over 15 dSphs imposes stringent limits on the model in a such a way that a large portion of the DM mass range is ruled out. Moreover, stringent limits on the mass of the next-to-lightest fermion also arise, since such particles act as the mediators in the $t$- and $u$-channels of the DM annihilation. For the region where $M_{\Sigma} < -m_{\chi^0_1}$ we find that $86\text{ GeV} < m_{\chi^0_1} < 280\text{ GeV}$ is already ruled out, this also leads to a restriction on $m_{\chi^0_1} > 340\text{ GeV}$ for $m_{\chi^0_1} > 280\text{ GeV}$. On the other hand, for the region where $M_{\Sigma} > -m_{\chi^0_1}$ we find that the diffuse spectrum requires that $m_{\chi^0_1} > 280\text{ GeV}$, $m_{\chi^0_1} > 300\text{ GeV}$ while $M_{\Sigma} \lesssim -230\text{ GeV}$. We also note that points that satisfy the $R_{\gamma\gamma}$ restriction of the CMS experiment are those with the higher $\langle \sigma v \rangle$ this is due to both observables dependence on the mixing angle. For $R_{\gamma\gamma}$ the dependence was already shown in Sec. 4.2. For the diffuse spectrum, the dependence on $\cos \theta$ enters through the vertices of the annihilation channels. In Appendix A this
dependence is shown for the DM interaction with the $W^\pm$ gauge boson and a $Z_2$-odd charged fermion. The expected 15 years and 45 dSphs observation will explore the whole region of the right panel and will leave a very narrow range of $m_{\chi_1^0}$ of $\sim$ 80 GeV un-explored.

### 4.3.2 Indirect detection from gamma-ray lines

Another promising detection channel is DM annihilation into two photons within regions with high DM density. In this case, the photon energies will be closely related to the DM mass leading to a spectrum exhibiting a sharp peak referred as a line-like feature [155]. No other astrophysical source is known to produce such features, thus, they play an important role in DM searches. In this regard, the Fermi [83] and H.E.S.S. [82, 84] collaborations have looked for gamma-ray lines coming from the center of the Milky Way, with no evidence of DM so far. This in turn leads to constraints on the DM $\langle \sigma v \rangle_{\gamma\gamma}$ annihilation into photons.

In the DTF, the DM annihilation into two photons is mediated by heavier $Z_2$-odd fermions interacting with vector and Goldstone bosons. Though the annihilation cross section in this case is loop suppressed, it may be possible to place constraints. In order to calculate the constraints on $\langle \sigma v \rangle_{\gamma\gamma}$ we follow the procedure given in Ref. [156] (the specific calculations we obtain along with the topologies that contribute are given in the Appendix A). After considering all the restrictions coming from collider, DD and ID in the diffuse spectrum, our results show that the Fermi and H.E.S.S. results do not place additional constraints on the model for both $M_{\Sigma} < -m_{\chi_1^0}$ and $M_{\Sigma} > -m_{\chi_1^0}$ regions since $\langle \sigma v \rangle_{\gamma\gamma} \sim 10^{-29}$ cm$^3$/s, which is nearly an order of magnitude lower than the most sensitive results which are presented by the H.E.S.S collaboration in [84]. As a result, this observable does not restrict the parameter space of the model.

### 4.4 DM detection in multicomponent dark sectors

An interesting possibility that has recently taken momentum is for the DM to be composed of different sectors, which is a far more general setting than the usual one DM candidate. For instance, the observed relic density could be the result of WIMP and Axion particles. In this case, it is possible that the sectors do not communicate, and so they behave as two completely independent DM particles, without affecting each other’s relic density and experimental bounds. For this section we will consider the WIMP DM candidate from the DTF to be part of multicomponent DM, that is, we obtain experimental bounds for models where the WIMP’s relic density is less than or equal to the central value reported by the PLANCK collaboration, $\Omega_{\text{Planck}}$ [58].

Figure 4.7 shows the ratio $e_{\chi_1^0} = \Omega_{\chi_1^0}/\Omega_{\text{Planck}}$ as a function of $m_{\chi_1^0}$. For the region where $M_{\Sigma} < -m_{\chi_1^0}$, the relic abundance is at most 40% of the observed value.
except for the narrow region where $m_{\chi_0^1} \sim 80$ GeV (where annihilation into weak gauge bosons is kinematically suppressed). On the other hand, in the region where $M_\Sigma > -m_{\chi_0^1}$ there are no models that saturate relic density, and so, the DTF accounts at most 40% of the universe DM content. Where, as explained above, the rest could come from another particle(s) such as an axion or another WIMP, with the restriction that there are no additional DM-DM conversion channels. We must add a comment here, unlike the previous section, we are assuming that the DM arises from a standard cosmology scenario, in that sense, the relic abundance of the WIMP DM is the one calculated through the usual method of solving the Boltzmann’s equation, thus we assume that the WIMP relic density obtained by micrOmegas is the correct one.

Figure 4.7: $\epsilon_{\chi_0^1}$ vs. $m_{\chi_0^1}$ for $M_\Sigma < -m_{\chi_0^1}$ (left panel) and $M_\Sigma > -m_{\chi_0^1}$ (right panel). All points satisfy collider bounds presented in Sec. 4.2.

Now we set out to investigate experimental bounds on the model. For colliders, the restrictions are the same as those presented in Sec. 4.2 since they are independent of the DM abundance. On the other hand, DD and ID rates do depend in the local DM density, and as a result the constraints presented in Sec. 4.3 will be different in this scenario. To quantify this, we used the parameter $\epsilon_{\chi_0^1}$ [157, 158] to re-scale the DD and ID observables. For the case of DD, the expected scattering rate will be re-scaled by $\epsilon_{\chi_0^1}$ which means that the SI cross section is effectively re-scaled to be $\sigma_{SI} = \epsilon_{\chi_0^1} \sigma_{SI - \chi_0^1}$; hence, DD constraints are now relaxed. The results are shown in Fig. 4.8 for $M_\Sigma < -m_{\chi_0^1}$ (left panel) and $M_\Sigma > -m_{\chi_0^1}$ (right panel). The left panel shows that for models that satisfy the lowest ATLAS limit on $R_{\gamma\gamma}$, DD imposes $y \leq 2.1$ while for models that satisfy lowest CMS limits $y \leq 1.9$. On the other hand, in the right panel, for models that satisfy the lowest ATLAS limit on $R_{\gamma\gamma}$, DD imposes $y \leq 2.2$ while for models that satisfy lowest CMS limits $y \leq 0.95$ which means that in this case CMS diphoton decay is more restrictive than DD (even considering DARWIN prospects).

For indirect detection, the situation is far less restrictive because the thermally averaged cross section is re-scaled by a factor of $\epsilon_{\chi_0^1}^2$, thus suppressing it. As a result, ID does not impose additional constraints on the model.
4.5 Summary

To conclude, in this chapter we have considered the DTF model under the custodial symmetry where the relic abundance is saturated either by a non-standard cosmology or due to the presence of other DM particle that does not affect the $\chi_0^1$ relic abundance. Hence we do not impose the relic density constraint and, as a result, the mass of the heavier charged and neutral fermions may lie close to the DM mass, which lifts partly the $R_{\gamma\gamma}$ restriction. Nonetheless, this observable restricts the triplet mass to be negative and Yukawa couplings to be less than 2.25 for the case of $M_\Sigma > -m_{\chi_0^1}$.

On the other hand, electroweak production of charginos and neutralinos at the LHC does not limit the parameter space, when considering CMS restrictions, though it restricts a few models for the case of $M_\Sigma < -m_{\chi_1^0}$. On the other hand, when ATLAS results on compressed spectra are considered, we find that $M_\Sigma < -165$ GeV for the region $M_\Sigma > -m_{\chi_1^0}$. Regarding DD and ID for the non-standard cosmology scenario, we found that Xenon1T results demand a Yukawa coupling $y < 1.75$, whereas the Fermi results imply that the DM mass is in general restricted to be $m_{\chi_1^0} < 280$ GeV except for a narrow region of $m_{\chi_1^0} \sim 80$ GeV when $M_\Sigma < m_{\chi_1^0}$. As an aside comment, gamma-ray line searches do not impose additional restrictions since $\langle \sigma v \rangle_{\gamma\gamma}$ is well below the current experimental sensitivity.

For the scenario of the DM as part of the multi-component dark sectors we found that DD impose a less severe constraint on the Yukawa coupling ($y < 2.2$) while current ID do not have anything to say.
Chapter 5

The Matrix Element Method applied to DM at the LHC

The DM problem greatly motivates the current experimental efforts to discover new BSM physics at the LHC. In many BSM models, dark matter particles are produced at the LHC as the end products of the cascade decays of heavier particles. Generally speaking, the longer the cascade, the more information we have in order to measure particle properties like masses, widths, couplings, etc. Hence, the most challenging cases are actually the simplest ones, such as the ones illustrated in Fig. 2.5. As explained in Ch. 2, for the topologies of Fig. 2.5 it is difficult to obtain the mass of the parent(s) and daughter(s) particle(s) simultaneously using usual kinematic variables such as endpoints, [159] this happens because there is only one observable endpoint. Now, in order to perform the endpoint measurement, one can use any one of several variables, e.g., the transverse mass, $M_T$, of the parent particle [160, 161], or the transverse momentum, $p_T$, of the visible daughter particle for the case of Fig. 2.5(a), or the Cambridge $M_{T2}$ variable [88, 162], the cotransverse mass $M_{CT}$ [163], or their 1D variants [164, 165] for the case of Fig. 2.5(b). However, again, the best one can do is to obtain a relationship between the mass of the parent and the mass of the daughter particle, leaving the overall mass scale undetermined. In particular, standard methods for measuring the $W$ boson mass, such as via properties of the transverse mass spectra like the Jacobian peak [160, 161], or by determining $M_W/M_Z$ by comparison with $Z$ boson events [166], are not sufficient in this scenario. One possibility that has been suggested in the literature to determine the overall mass scale is to go beyond the leading order diagrams of Fig. 5.1 and consider hard initial state radiation (ISR), which provides a kick to the system in the transverse plane. In the presence of ISR, the functional dependence $M_W(M_\nu)$, derived from either $M_T$ for the case of Fig. 5.1(a) [87, 167] or from $M_{T2}$ for the case of Fig. 5.1(b) [164, 168–173] exhibits a kink at the true value $M_\nu^{true}$. To understand how this could work, it is important to see where the kink comes from:

- First, since $M_T$ ($M_{T2}$) defines the boundary region where the parent and daughter mass are consistent with the event, the curve changes event by event. In some events, it will be flatter and some steeper. But an overlay of many events
will show the kink at the mass of the daughter that is consistent with the topology.

• Second, when the parent particle recoils against ISR, the curve will be flatter (steeper) for low (high) upstream momentum thus also generating a kink in a plot of many events.

In Ref. [167], it was shown that when there is no ISR, neither single nor pair production, which are the cases considered here, have a noticeable kink. Thus, even if the first possibility is still viable, the kink does not appear for these topologies. Moreover, the authors reach the conclusion that these topologies only present a clear kink when there is hard ISR. The problem is that demanding hard ISR will imply a significant loss of statistics, which is far from ideal.

On the other hand, measuring other properties such as spin, chirality, and width of the new particles is important in order to uncover the underlying theory. For this reason, methods such as the MEM [37, 38, 173] present an interesting possibility to study events generated by BSM particles at the LHC (or any other collider for that matter). The method consists of obtaining the probability density function that an observed event is the result of a hypothetical process. There are several advantages to this: First, unlike other multivariate analysis used in particle physics, it makes use of the Feynman amplitude \( M \) associated to the hypothesis, second, it takes advantage of all events, thus it uses all the statistics available.

This method has been successfully used by the D0 and CDF collaborations in the measurement of the top quark mass [40, 89, 174] and to discover single production of top quarks [175, 176]. Moreover, a variation of the method called MELA has been used by the CMS collaboration to search for the Higgs boson [177] and by both ATLAS and CMS collaborations to measure its spin-parity [41].

5.1 The MEM

To understand how the method works, recall that for a given set of kinematic variables \( Y \), the probability that a given event is described by them is:

\[
P(Y \in \Phi) = \int_{\Phi} P_i(y) dy,
\]  

where, in this case, \( y \) is the set of all the four-momenta of the particles assumed to be involved in the process, while \( P_i(y) \) is the probability density function and \( \Phi \) is the phase space volume, that is \( \Phi = \int dy \). This is just the basic application of a probability density function. For the case we are concerned which is an event \( i \), and hypothesis \( Y \), the normalised probability is:
\[ P(Y \in \Phi) = \frac{1}{\sigma_i} \int d\sigma_i(y), \quad (5.2) \]

where \( \sigma_i \) is the cross section corresponding to the hypothesis \( Y \). Though some of the final four-momenta are observed, the integration takes over all possible four-momenta \( p_i \), this is because the true final momenta are not necessarily the same as the reconstructed final momenta, which is due to detector resolution. To account for this, a transfer function is introduced, thus:

\[ P(Y \in \Phi) = \frac{1}{\sigma_i} \int d\sigma_i(y') W(y'|y). \quad (5.3) \]

The transfer function is usually a gaussian or bi-gaussian, however, if the particle is any of the charged leptons belonging to the first two families, or a photon, the transfer function is a delta function since the objet reconstruction is very accurate in that case [89]. The probability is usually written in its final form in a more transparent way as given in [178]:

\[ P(p_{\text{rec}}; \alpha) = \frac{1}{\sigma} \int d\Phi(p_{\text{par}}) dx_1 dx_2 \frac{f(x_1) f(x_2)}{2sx_1x_2} |\mathcal{M}(p_{\text{par}}; \alpha)|^2 \delta^4(p_{\text{initial}} - p_{\text{final}}) W(p_{\text{rec}}, p_{\text{par}}), \quad (5.4) \]

where \( \alpha \) is the set of the hypothesis tested by the method, \( p_{\text{par}} \) is the partonic four-momenta while \( p_{\text{rec}} \) is the reconstructed four-momenta (as explained above). The delta function is included to ensure energy-momentum conservation and the functions \( f(x_1) \) and \( f(x_2) \) are the parton distribution functions (p.d.f) where the momentum dependence has been replaced by the fractions \( x_1 \) and \( x_2 \) of the \( \sqrt{s} \) of the process respecrively. With the probability obtained from 5.4 for each event given a set of hypothesis, a likelihood is obtained as \( L_\alpha = \prod_{i=1}^{N} P(p_{\text{rec}}; \alpha) \), and the best set of hypothesis \( \alpha \) that is in agreement with the events is the one that maximize \( L \). It is common practice to obtain a minimum rather than a maximum, which is obtained as

\[ -\ln L_\alpha = - \sum_{i=1}^{N} \ln(P(p_{\text{rec}}; \alpha)) + N \int P(p_{\text{rec}}; \alpha), \quad (5.5) \]

where the last term is included for cases in which the probability \( P(p_{\text{rec}}; \alpha) \) is not properly normalized [179], this is the so-called “extended likelihood”.
Chapter 5. The Matrix Element Method applied to DM at the LHC

5.1.1 MadWeight

The calculation of $P(p_{\text{rec}}; \alpha)$ is a difficult one. First, it demands integration of the four-momenta over all the allowed values consistent with energy-momentum conservation, that is, from minus to plus infinity. Moreover, since the Feynman amplitude depends on propagators, it peaks for certain values of the momentum, while the transfer function usually peaks at different momenta. To obtain reliable calculations that are computationally not too demanding, we used the publicly available package MadWeight [180]. The package uses MADGRAPH5_aMC@NLO [181] for the calculation of the Feynman amplitude. In order to perform the integration, MadWeight aligns, when possible, the peaks so that they are in the same direction of integration, for that it relies on adaptive Monte Carlo integrator VEGAS [182]. The result of this complicated process is that the user feeds MadWeight with a set of events and the ansatz values for the set of hypothesis. MadWeight returns the weight of the event for the hypothesis $\alpha$, that is $P(p_{\text{rec}}; \alpha)$ and the associated likelihood.

Throughout the following sections, the application to specific scenarios of the MEM via MadWeight will be presented. We will apply this to a problem of BSM physics that involves DM. Our task is to show that it is possible to measure simultaneously several unknown parameters. We start by obtaining at least two such parameters. First, we show how this could be done for the mass and width of a new resonance while fixing chiralities of the couplings to quarks and leptons. Next, we find the chiralities while fixing the mass and width. After this, we show that the method yields good results when attempting to measure four parameters simultaneously. To our knowledge, such ambitious use of the method has not been reported in the literature.

5.2 Formulation of the problem

We begin by considering single production of a new resonance, $W^\pm$, which decays semi-invisibly via a two-body decay into a visible SM particle, $\tilde{\ell}$, and an invisible...
particle, \( \nu \). For definiteness, we shall take \( \bar{\ell} \) to be an anti-lepton (positron or antimuon) and \( \nu \) to be an invisible particle, which can be a BSM dark matter candidate. There are good reasons to consider this, since, the idea of measuring two and even four parameters simultaneously with MEM is an ambitious one, it makes sense to work with particles that are, in general, well reconstructed by the detector. This, in turn, allows us to choose the transfer function to be a delta function, as explained above. The \( W^+ \) resonance can be produced singly, as shown in Fig. 5.1(a), or as part of a \( W^+W^- \) pair, as in the \( s \)-channel\(^1\) diagram of Fig. 5.1(b). As suggested by our notation, this setup includes, but is not limited to the SM production of \( W^\pm \) bosons decaying leptonically. In particular, the process of Fig. 5.1(a) may refer to the production of a charged Higgs scalar [183], a charged slepton in supersymmetry (SUSY) models with \( R \)-parity violation [184, 185], or a new \( W' \) heavy gauge boson [186, 187]. Similarly, the process of Fig. 5.1(b) may be interpreted as the pair-production of (perhaps quarkophobic) charged Higgs bosons [188, 189] or \( W' \) bosons [190], of charginos [191], Kaluza-Klein leptons [192, 193], or sleptons [194, 195]. However, for definiteness, in our simulations below we shall assume that the \( W^\pm \) are spin-1 particles while \( \ell (\bar{\ell}) \) and \( \nu (\bar{\nu}) \) are spin 1/2, as in the SM. We shall parametrize the \( W \) couplings to leptons (quarks) as 

\[
\begin{align*}
g_{\ell R} P_R + g'_{\ell R} (g_{\ell R} P_R + g''_{\ell R} P_L),
g_{\ell L} P_L + g'_{\ell L} (g_{\ell L} P_L + g''_{\ell L} P_R),
\end{align*}
\]

so that the angles \( \phi_\ell \) and \( \phi_q \) encode the information about the chirality of the \( W \) couplings to quarks and leptons, respectively.

The (normalized) kinematic distributions of the leptons in the final state will depend on five model parameters:

\[
\{ M_W, M_\nu, \Gamma_W, \phi_q, \phi_\ell \},\tag{5.6}
\]

where \( M_W \) (\( M_\nu \)) is the mass of the parent (daughter) particle, \( \Gamma_W \) is the width of the parent, and

\[
\tan \phi_q \equiv \frac{g_{\ell R}'}{g_{\ell R}}, \quad \tan \phi_\ell \equiv \frac{g_{\ell L}'}{g_{\ell L}}, \tag{5.7}
\]

so that the angles \( \phi_q \) and \( \phi_\ell \) encode the information about the chirality of the \( W \) couplings to quarks and leptons, respectively.

Given this setup, our main goal will be to measure simultaneously several of the unknown parameters while fixing others. As already discussed in the introduction, this is not a trivial task. Measuring the mass splitting is relatively straightforward (see Fig. 5.2 below), but fixing the mass scale and width, for instance, requires subtle measurements of the relevant kinematic distributions. We shall make use of the MEM, which is ideally suited for our purposes.

\( ^1 \)We focus on the photon-mediated \( s \)-channel diagram for simplicity: since the \( W \) is charged, it must couple to photons, so that the diagram of Fig. 5.1(b) is guaranteed to exist. In principle, there can be additional \( t \)- and \( u \)-channel pair-production diagrams, but this requires that the \( W \) couples to quarks as well, in which case the single production from Fig. 5.1(a) should dominate. There could also be \( s \)-channel diagrams mediated by \( Z \) or other more exotic gauge bosons, but this possibility also involves additional assumptions. All of those complications can be easily incorporated in the analysis and the MEM method would still work, but our discussion would become more opaque.
5.2.1 Initial assumptions

Since we are considering the difficult task of measuring simultaneously the quantities of Eq. 5.6, we will start by taking initial assumptions: First, because the method is a hypothesis test, we are not considering how to find those events that come from the topologies of Fig. 5.1. Instead, we assume that the events from those topologies, especially for the case of a BSM scenario, have already been isolated, and they will be our input events. Thus, our focus will turn to how to test the different hypothesis for the parameters of Eq. 5.6. Second, in order to have more control of the many variables of the problem, we will approach this only at the parton level. This is a first approach to the application of MEM for finding simultaneously many parameters, and thus it has been simplified. In order for our findings to be used at collider experiments, a more thorough approach should be considered. In particular, the method should be, in the future, applied taking into account the background, hadronic processes and full detector simulations.

5.2.2 Single production

We first consider single W production from Fig. 5.1(a). The spin and color averaged squared matrix element for the process $u \bar{d} \rightarrow W^+ \rightarrow \bar{\ell} \nu$ is given by

$$\langle |M|^2 \rangle = \frac{4|V_{ud}|^2}{3[(s - M_{WW})^2 + (\Gamma_W M_W)^2]} \times \left[ \left\{ (g^q_L)^2 (g^f_L)^2 + (g^q_R)^2 (g^f_R)^2 \right\} (p_u \cdot p_\tau)(p_\tau \cdot p_\nu) \\
+ \left\{ (g^q_R)^2 (g^f_L)^2 + (g^q_L)^2 (g^f_R)^2 \right\} (p_\tau \cdot p_\tau)(p_u \cdot p_\nu) \right], \tag{5.8}$$

where the parton-level center-of-mass energy squared is $s = (p_u + p_d)^2$ and we have generalized to the case of arbitrary fermion couplings to the W-like intermediate resonance. $V_{ud}$ is the analog of the CKM matrix element, and is the SM CKM matrix element if we are considering production and decay of the SM W boson. Moreover, for simplicity, we only consider the initial states $u$ and $\bar{d}$ in this calculation. If we take $p_1$ to be the momentum of the incident parton with positive $z$ momentum, $p_2$ to be the momentum of the incident parton with negative $z$ momentum, and define

$$F_1 = f_u(x_1)f_{\bar{d}}(x_2), \quad F_2 = f_u(x_2)f_{\bar{d}}(x_1), \quad k_1 = (p_1 \cdot p_\tau)(p_2 \cdot p_\nu), \quad k_2 = (p_1 \cdot p_\nu)(p_2 \cdot p_\tau), \tag{5.9}$$

we find that the likelihood for a particular event is proportional to

$$(F_1 + F_2)(k_1 + k_2) + \cos 2\phi_f \cos 2\phi_q (F_1 - F_2)(k_1 - k_2). \tag{5.10}$$
5.2. Formulation of the problem

We see that only the second term depends on the helicity of the couplings. As expected, this term will not contribute in the absence of a longitudinal boost (when \( F_1 = F_2 \)) or if the lepton is emitted perpendicular to the beamline in the rest frame of the \( W \) (when \( k_1 = k_2 \)). We also note that \( k_1 \) and \( k_2 \) depend only on \( p_\ell z \), not on \( p_\ell T \), so when determining the \( p_\ell T \) distribution we get no contribution from the second term, as the contribution from this term from each point with a given value of \( (p_\ell T, p_\ell z) \) is cancelled by the contribution of the term with \( (p_\ell T, -p_\ell z) \). So it will be the \( p_\ell z \) distribution rather than the \( p_\ell T \) distribution that will give us sensitivity to the chirality of couplings, as we will see in more detail below.

In our subsequent analyses we generate events with \textsc{MadGraph5 AMC@NLO} [181] version 2.5.5 for the parameter point with \( M_W = 1000 \) GeV, \( M_\nu = 500 \) GeV, \( \Gamma_W = 50 \) GeV at \( \sqrt{s} = 13 \) TeV without applying selection criteria (cuts) or detector simulation to the events. We use \textsc{MadWeight5} [43] for computation of the weights in MEM calculations using \( \delta \)-function transfer functions, and have verified that the \textsc{MadWeight} results can be reproduced using (5.8), where appropriate.

We note that there are two relevant observables: the transverse momentum \( p_\ell T \) and the longitudinal momentum \( p_\ell z \) of the lepton, as the only visible particle in the final state is the lepton, with fixed (zero) mass. The third momentum degree of freedom corresponds to an azimuthal angle, which cannot have a non-trivial distribution in the absence of some very unexpected physics (or detector effects) breaking the azimuthal symmetry.

**Measurement of the mass “difference”**. One quantity, related to the difference of the squared masses of the \( W \) and the \( \nu \), can be easily measured from the endpoint of the distribution of the \( W \) transverse mass \( M_{WT} \). In the absence of ISR, this quantity can also be measured from the kinematic endpoint of the, \( P_\ell T \) (lepton transverse momentum) distribution, \( \mu \), as

\[
\frac{M_W^2 - M_\nu^2}{2M_W} = \text{constant} \equiv \mu. \tag{5.11}
\]

The endpoint, corresponding to the position of the Jacobian peak, is well known to exist for the case of the SM \( W \) [160, 196, 197], so there is no surprise that it exists in the case at hand. We note that \( \mu \), the maximum value of lepton \( p_T \) is also the 3-momentum of the lepton in the center of mass (CM) frame (still in the absence of ISR). Eq. (5.11) allows us to fix one of \( M_W \) and \( M_\nu \), once we have measured the other. Thus, in what follows, we shall focus on measuring the orthogonal mass degree of freedom, i.e., the overall mass scale. From now on we shall always choose the test masses to satisfy the relation (5.11). In other words, we shall vary one of the two masses, e.g., \( M_\nu \), and then compute the other mass from

\[
M_W = \mu + \sqrt{\mu^2 + M_\nu^2}. \tag{5.12}
\]

In Fig. 5.2, we show the lepton \( p_\ell T \) distributions in single \( W \) production for different
Chapter 5. The Matrix Element Method applied to DM at the LHC

Figure 5.2: Unit-normalized lepton $P_{T\ell}$ distributions for single $W$ production in the limit of $\Gamma_W = 0$. The invisible particle mass $M_\nu = 500$ GeV, and the mass of the parent particle is varied as shown.

$W$ masses; $M_W = 800$ GeV (green dotted), $M_W = 1000$ GeV (red solid), and $M_W = 1200$ GeV (blue dashed) in the $\Gamma_W = 0$ limit. The plots were generated using a Monte Carlo simulation. The mass of the invisible particle is set to $M_\nu = 500$ GeV. Here we consider only left-handed fermionic couplings to the $W$ boson. The black dashed line gives the theoretical prediction for the true lepton $P_{T\ell}$ distribution (in the absence of cuts) which is given by

$$\frac{1}{\sigma} \frac{d\sigma}{dp_{T\ell}} = \frac{3}{4 + 3\rho} \frac{p_{T\ell}}{\mu^2 - p_{T\ell}^2} \left( 2 - \frac{p_{T\ell}^2}{\mu^2} + \rho \right),$$

(5.13)

where $\rho \equiv 2M_\nu^2 / (M_W^2 - M_\nu^2)$. Therefore when the lepton $P_{T\ell}$ reaches its maximum value, $\mu$, the longitudinal momentum $p_z$ goes to zero and we obtain the well known Jacobian peak in the distribution (5.13). In principle, this equation indicates the $P_{T\ell}$ spectrum depends on both $M_W$ and $M_\nu$ via the quantities $\rho$ and the endpoint, $\mu$. However in practice, the dependence on $\rho$ tends to be subtle, so in practical situations, we may only be able to measure $\mu$, and of course, a given value of $\mu$ corresponds to any $M_W$ and $M_\nu$ satisfying eq. (5.11). One of the important points of this work is that in addition to measuring $\mu$ (from a kinematic endpoint), by utilizing the MEM, we can simultaneously also obtain (a) the mass scale, i.e., $M_W$ itself; (b) the width $\Gamma_W$. On the other hand, if we fix the mass scale and the width, we can obtain the chirality of the couplings (5.7). Moreover, we find that at least for the case of single production it is possible to obtain information of all parameters of Eq. 5.6.

Measurement of the mass scale $M_W$. The mass scale is notoriously difficult to measure; even in this very simple topology it cannot be determined from kinematic endpoint measurements alone (unless we require hard ISR). Instead we have to rely on subtle effects. The two tools at our disposal are the distributions of the measured lepton $P_{T\ell}$ and $P_{T\ell z}$. Interestingly, the shapes of both of these distributions encode information about the mass scale, as illustrated in Fig. 5.3. As seen in the left panel, the $P_T$ distribution is rather weakly sensitive to the mass scale. However, the right
5.2. Formulation of the problem

Figure 5.3: Unit-normalized distributions of the lepton transverse momentum $P_T$ (left) and longitudinal momentum $P_z$ (right), for different values of $M_W$ and $\Gamma_W = 0$ obtained using analytical expressions and, in the case of $P_z$, pdfs from LHAPDF [198]. The mass $M_\nu$ of the invisible particle has been fixed from the measurement (5.11).

panel in Fig. 5.3 shows that the longitudinal momentum does contain information about the mass scale which can potentially be observed.

As a proof of principle, we perform an exercise to find the mass scale by simply fitting to $P_{\ell z}$ templates generated for different mass spectra obeying the relation (5.11). That is, we generate a hundred thousand events for different values of $M_W$ and $M_\nu$ while keeping $\mu$ constant, and find the $P_{\ell z}$ distributions of the leptons. These distributions will be considered, in a way, the theoretical distribution. We also generate ten thousand events with $M_W = 1000$ GeV and $M_\nu = 500$ GeV (our study point). The $P_{\ell z}$ distributions obtained will be considered the experimental ones. Now that we have the "theoretical" and "experimental" distributions, we bin the measured $P_{\ell z}$ distribution and the one from the templates and then we calculate $\chi^2$. A similar method has been used by the LIGO collaboration for the case of gravitational waves [199].

The advantage of the template method is that it avoids the time-consuming integrations over the invisible momenta, which are needed for the MEM. The result is shown in Fig. 5.4, where we plot the $\chi^2 / d.o.f.$ for several hypothesized values of $M_\nu$ (with $M_W$ calculated from (5.12)). The right panel of Fig. 5.4 shows results from the same exercise, but for the case of $d\bar{u} \rightarrow W^- \rightarrow \ell \nu$. The minima of the $\chi^2$ curves for the case of $W^+$ production, shown in the left panel Fig. 5.4, is at $M_\nu = 524$ GeV. On the other hand, for the case of $W^-$ production, shown in the right panel Fig. 5.4, is at $M_\nu = 487$ GeV. The difference between the results for $W^+$ and $W^-$ production could come from the statistical nature of the measurement. Still, both results are near the true value, $M_\nu = 500$ GeV this suggests that the template method in principle works. However, the MEM will be more sensitive, as (1) it uses the correlations among $P_{\ell T}$ and $P_{\ell z}$ in the data and (2) incorporates the dependence on the remaining parameters in eq. (5.6), which makes it possible to do a simultaneous measurement of several parameters.

**Measurement of the width $\Gamma_W$.** The width effects will manifest themselves in
two places. First, there will be some smearing of the $P_{\ell T}$ endpoint \cite{200} as illustrated in Fig. 5.5.\footnote{The same effect would be observed in the $M_T$ distribution \cite{201}.} However, the $P_{\ell T}$ distributions resulting from different choices of $M_W$, $M_\nu$, and $\Gamma_W$ will be relatively similar provided $M_W$ and $M_\nu$ give the same position of the Jacobian peak $\mu$ (following eq. (5.11)). Given this criterion, the distributions will tend to be more similar if the masses and the width satisfy the relation

\[
\frac{\Gamma_W}{\Gamma_W^{\text{true}}} = \frac{1 + \left(\frac{M_\nu^{\text{true}}}{M_W^{\text{true}}}\right)^2}{1 + \left(\frac{M_\nu}{M_W}\right)^2},
\]

which follows from demanding a similar distribution in the “endpoints” obtained from “off-shell” $W$ bosons in the different scenarios. Second, the width will also affect the lepton $P_z$ distribution, although to a much smaller extent than the mass scale.

**Simultaneous measurement of the mass scale and the width.** In Fig. 5.6, we examine how well we can simultaneously measure $M_\nu$ and $\Gamma_W$ by determining the $\chi^2$ fit to the one-dimensional $P_{\ell T}$ distribution for the study point with $M_W = 1000$ GeV.
5.2. Formulation of the problem

\[ \chi^2 \text{ fit to the one-dimensional } P_{\ell T} \text{ distribution for a data sample of 10,000 events.} \]

The fitted parameters are \( M_\nu \) and \( \Gamma_W/M_W \), with \( M_W \) computed from (5.11). The study point (\( \times \)) has \( M_W = 1000 \text{ GeV, } M_\nu = 500 \text{ GeV, and } \Gamma_W = 50 \text{ GeV.} \)

The dashed line marks the flat direction (5.14). The color bar indicates the \( \chi^2/d.o.f. \) (we used 100 bins).

GeV, \( M_\nu = 500 \text{ GeV, and } \Gamma_W = 50 \text{ GeV, which is indicated on the plot with the } \times \text{ symbol, again, we used the template method previously explained. The dashed line marks the relatively flat direction in the } \chi^2 \text{ which is described by eq. (5.14). Notice then that it is possible to find the flat direction, yet it is not possible to find the actual value using } \chi^2. \)

On the other hand, as explained in the previous two sections, one can apply the MEM to obtain a measurement of a single parameter, for example the mass scale, as shown in the left panel of Fig. 5.7, or of the width, \( \Gamma_W \), as shown in the right panel of Fig. 5.7. In either case, one has to make an ansatz for the second parameter,

\[ \begin{align*}
\text{the width, and the mass scale, respectively. The ansatz may or may not be correct,} \\
\text{which motivates the } \textit{simultaneous} \text{ measurement of the two parameters, as shown in} \\
\text{Fig. 5.8. The input values of the parameters were } M_W = 1000 \text{ GeV, } M_\nu = 500 \text{ GeV,}
\end{align*} \]
which results in $\mu = 375$ GeV. The $W$ width was 5% of its mass, i.e., $\Gamma_W = 50$ GeV, and we only considered left-handed couplings, i.e., $s_q^l = s_R^l = 0$.

**Measurement of the chirality of the couplings.** Having measured the two masses and the width, we want to show that if we know those parameters, it is possible to measure the chirality of the couplings. Fig. 5.9 shows a simultaneous fit to the chirality of the couplings to quarks and leptons, for fixed $M_W = 1000$ GeV and $\Gamma_W = 50$ GeV (the nominal values measured in Fig. 5.8). As one would expect

---

3In practice, the chiralities should also be measurements. We note that in the case when the BSM signal of Fig. 5.1(a) is due to a $W'$ gauge boson decaying to a SM neutrino, the chirality of the couplings can in principle also be determined by studying the $W' - W$ interference effects in the $M_T$ distribution [202].
from eq. (5.10), contour lines in the left plot of Fig. 5.9 are given by
\[
\cos(2\varphi_\ell) \cos(2\varphi_q) = \text{constant.} \quad (5.15)
\]
Fig. 5.9 reveals that the chirality of the couplings can be measured very well (up to the degeneracy described by eq. (5.15)). This is because the \( P_{lz} \) distribution is very sensitive to the chirality, as shown in the right panel of Fig. 5.9, where we plot the \( P_{lz} \) distribution for the correct mass spectrum and the correct \( \Gamma_W \), but for three different choices of the couplings: \( \varphi_q = \varphi_\ell = 0^\circ \) (blue); \( \varphi_q = \varphi_\ell = 45^\circ \) (green); and \( \varphi_q = 0^\circ \) and \( \varphi_\ell = 90^\circ \) (red). The lesson to be learned here is that one should not attempt to do mass measurements from the shapes of the kinematic distributions unless one is sure about the chiralities of the couplings; if the chiralities are a priori unknown, then one should fit the masses, widths and chiralities simultaneously [203]. Table 5.1 summarizes the main dependences of the parameters (5.6) on the two observables \( P_{lT} \) and \( P_{lz} \).

<table>
<thead>
<tr>
<th>P_{lT} ( \checkmark )</th>
<th>P_{lz} ( \sim )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 mass splitting</td>
<td>( \checkmark )</td>
</tr>
<tr>
<td>2 mass scale</td>
<td>( \sim )</td>
</tr>
<tr>
<td>3 width</td>
<td>( \sim )</td>
</tr>
<tr>
<td>4 chirality</td>
<td>( \times )</td>
</tr>
</tbody>
</table>

\textbf{Table 5.1:} The extent to which various observables depend on the input parameters (5.6). \( \checkmark \) (\( \sim \), \( \times \)) indicates strong (weak, no or almost no) dependence.

**Simultaneous measurement of all parameters.** After the preliminary exercises shown in the previous sections, we now attempt to simultaneously measure the relevant parameters (5.6) using the MEM.

As already shown in eqs. (5.10) and (5.15), we cannot extract the individual chiralities \( \varphi_\ell \) and \( \varphi_q \), but only the relative chirality \( \varphi_{rel} \)
\[
\cos^2 \varphi_{rel} = \frac{\cos(2\varphi_\ell) \cos(2\varphi_q) + 1}{2}. \quad (5.16)
\]
As for the remaining three parameters, \( M_\nu, M_W, \) and \( \Gamma_W \), they will all share a common source of uncertainty coming from the overall mass scale, causing their measured values to be highly correlated. In order to reduce the covariance between the parameters being measured, we choose to reparametrize them in terms of the daughter mass \( M_\nu \), the parent mass parameter \( \mu \) from eq. (5.11), and the width parameter \( \gamma \)
\[
\gamma = \Gamma_W \left(1 + \frac{M_\nu^2}{M_W^2}\right), \quad (5.17)
\]
which appears in eq. (5.14).

From eqs. (5.11) and (5.17) we see that when \( M_\nu = 0 \), \( 2\mu \) and \( \gamma \) are identically
equal to $M_W$ and $\Gamma_W$ respectively. With this choice, we expect that $2\mu$ and $\gamma$ will be measured relatively well, while the mass scale uncertainty will only be manifested in the determination of $M_\nu$.

These expectations are confirmed in Fig. 5.10, which shows the results from our simultaneous measurement of the four parameters $M_\nu$, $2\mu$, $\gamma$ and $\cos^2 \varphi_{\text{rel}}$. For our purpose, we use simulated data samples of 1000 events each, and in each case we find the “measured” values of $M_\nu$, $2\mu$, $\gamma$, and $\cos^2 \varphi_{\text{rel}}$ by maximizing the likelihood. Fig. 5.10 shows the unit-normalized distributions of the measured values of each parameter from 100 such pseudo-experiments.

The sample mean and the standard deviation of the measured values (with the true values quoted in parentheses) are as follows

\[
M_\nu = 495 \pm 42 \text{ GeV} \quad (500 \text{ GeV}) \quad (5.18)
\]
\[
2\mu = 750 \pm 4 \text{ GeV} \quad (750 \text{ GeV}) \quad (5.19)
\]
\[
\gamma = 62.2 \pm 6.5 \text{ GeV} \quad (62.5 \text{ GeV}) \quad (5.20)
\]
\[
\cos^2 \varphi_{\text{rel}} = 0.499 \pm 0.045 \quad (0.5) \quad (5.21)
\]

We see that the mass difference $\mu$ is very well constrained, while the measurement of the mass scale $M_\nu$ is less precise, as expected from the toy exercises performed in the lead-up to this analysis.

**The case of pair production.** Now let us consider pair production as in the second diagram of Fig. 5.1. We generate $q\bar{q} \rightarrow W^+W^- \rightarrow 2\ell + \text{MET}$ events using
5.2. Formulation of the problem

MADGRAPH5_aMC@NLO at $\sqrt{s} = 13$ TeV, again without cuts or detector simulation. We set $M_W = 1000$ GeV, $M_\nu = 500$ GeV, and $\Gamma_W$ to 5% of the parent mass. For simplicity we keep only the $s$-channel diagram as $t$ and $u$-channels are more model-dependent. Also we do not include the $Z$-boson in the $s$-channel diagram because of the presence of $W'$-like particle which may not participate in the weak interaction.

As in Fig. 5.2, the mass “splitting” between the $W$ and the $\nu$ can be easily measured, this time from the endpoint of the distribution of $M_{T2}$ instead of $M_T$ [88]. The same combination of masses (5.11) will be constrained.

In analogy to Fig. 5.5, in Fig. 5.11 we show the $M_{T2}$ distribution for several values of the width, $\Gamma_W$, illustrating the smearing of the kinematic endpoint. The figure suggests that there is sensitivity to the width, but the measurement is challenging since the effect is concentrated in the region near the endpoint, $M_{T2} \sim 950 - 1200$ GeV. We point out that the above measurements can also be performed using the $M_{CT}$ variable [163] — it similarly has a well defined kinematic endpoint, which will be partially smeared by the width effects.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5_11.png}
\caption{The same as Fig. 5.5, but for the case of pair production, where we study the $M_{T2}$ distribution instead of $P_T$. In the lower panel we show the bin-by-bin ratio of the number of events for different widths, normalized to the case of $\Gamma_W = 50$ GeV.}
\end{figure}

Just like the case of single production, for the measurement of the mass scale one cannot rely on endpoint measurements alone, and needs to utilize the shapes of the relevant kinematic distributions. Fig. 5.12 depicts several variables whose distributions show sensitivity to the overall mass scale.

Just like in Fig. 5.3, we only consider mass spectra which obey the relation (5.11) and therefore satisfy the measured $M_{T2}$ (or $M_{CT}$) kinematic endpoint. More specifically, we vary the mass $M_\nu$ of the invisible particle as shown in each panel, then choose the parent mass $M_W$ from eq. (5.12). The main effect of the mass scale is to provide a different boost of the parent particles: lighter (heavier) $W$’s will be produced with a higher (lower) boost. While the parent boost itself is unobservable, its effects are reflected in the kinematic distributions of the visible decay products. For example, when the $W$’s are highly boosted, we would expect the leptons to be back to back, and have a large invariant mass, as well as higher (on average) values of their transverse and longitudinal momenta. These expectations are confirmed by
Chapter 5. The Matrix Element Method applied to DM at the LHC

Figure 5.12: The same as Fig. 5.3, but for the case of pair production. We showcase several variables whose distributions are sensitive to the overall mass scale: the dilepton invariant mass $m_{\ell\ell}$ (upper left), the transverse lepton momentum $P_{\ell T}$ (upper right), the larger of the two longitudinal lepton momenta (in absolute value), $\text{max}(|P_{\ell z}|, |P_{\bar{\ell} z}|)$ (lower left), and the other longitudinal lepton momentum with its sign chosen as $\text{sgn}(P_{\ell z}P_{\bar{\ell} z}) \text{min}(|P_{\ell z}|, |P_{\bar{\ell} z}|)$ (lower right).

Fig. 5.12, in which we show distributions of the dilepton invariant mass $m_{\ell\ell}$ (upper left panel), the transverse lepton momentum $P_{\ell T}$ (upper right panel), and the longitudinal lepton momenta (lower two panels). We note that the boost effect is seen better in the distribution of the larger of the two longitudinal lepton momenta (in absolute value), $\text{max}(|P_{\ell z}|, |P_{\bar{\ell} z}|)$, which is shown in the lower left panel. The other longitudinal lepton momentum, $\text{min}(|P_{\ell z}|, |P_{\bar{\ell} z}|)$, is then plotted in the lower right panel, with the sign chosen so that it is positive (negative) when the two longitudinal lepton momenta have equal (opposite) signs.

Fig. 5.12 demonstrates that the kinematic distributions of the visible particles in principle do contain information about the mass scale, which can then be extracted from a fit to those (one-dimensional) distributions, as was done in Fig. 5.4. However, the MEM will have better sensitivity, as it takes into account the correlations among the different variables.

Parameter measurements in the case of pair production. In analogy to single production, we now apply the MEM method to measure simultaneously the mass scale and the width (Fig. 5.13) and the chirality of the lepton couplings (Fig. 5.14). As expected, the MEM method is quite successful in determining simultaneously two of the parameters in (5.6). In particular, the $+$ symbol in Fig. 5.13 denotes the result from our fit, which is close to the input parameter values (marked with $\times$). In
5.3. Summary

In this chapter, we have presented methods to measure masses, widths, and chiralities of couplings, especially for the case of simple topologies where little information is available.

In particular, we showed that with our application of the MEM and under our assumptions, it is possible to simultaneously measure and obtain a value that is
very close, for (a) the mass scale and the width of the decaying resonance, (b) the chiralities of the couplings (up to a degeneracy). We applied this for the case of single and pair production of $W$'s. Moreover, by analyzing sets of 100 events in single production, we were able to obtain a close value for the parameters $\gamma$, $\mu$, $M_\nu$ and $\phi_{\text{rel}}$ simultaneously, to our knowledge, this has never been done before. We also show how the only two observables $P_{\ell z}$ and $P_{\ell T}$ depend on the parameters of Eq. 5.6. For the mass scale we find that it is strongly dependent on $P_{\ell z}$ while weakly dependent on $P_{\ell T}$, whereas the width has a subtle dependence on both observables. In the case of chiralities, we find that it only depends on $P_{\ell z}$, which tell us that one should only fit them once the mass scale and mass splittings are known. In order to be certain that all these conclusions are applicable to the case of the LHC (or any other hadron collider), a more realistic approximation should be taken, that is, one should include events at the hadron level, background events, and full detector reconstruction.
Chapter 6

Conclusions

In this thesis, we have focused on the DM problem, how it could be solved and how it can be constrained.

In Chapter 3 we presented a simple extension of the SM where we enlarge the fermion sector with a Majorana triplet and a vector-like doublet while the scalar sector is extended with an inert doublet and a real triplet. All the new fields are odd under an additional $Z_2$ symmetry while the SM fields are even. This global symmetry renders the DM stable and ensures that neutrino masses are zero at tree-level. The model’s DM candidate is the lightest neutral scalar or fermion. In the case of scalar DM, the phenomenology is similar to that of the inert doublet or the inert triplet, as long as $\mu \ll v$, which is favored by EWPT. In the case of fermionic DM, when the custodial limit is considered, it is possible to have DM candidates with masses $\sim 100$ GeV. This scenario is appealing due to the prospects of being probed by future experiments, while also evading constraints from DD due to the absence of SI and SD interactions at tree level. Nonetheless, the fermionic sector generates large suppression on the Higgs diphoton decay rate, thus being in disagreement with the values measured by ATLAS and CMS. Nonetheless, the scalar sector may enhance the decay rate as long as the sum of the couplings of the charged scalars to the Higgs are smaller than zero. Additionally, the interaction of the new particles with the SM fields generates Majorana neutrino masses at the one-loop level. In the model, all possible realizations of the $d = 5$ Weinberg operator are present, and we show that it is possible to satisfy neutrino physics while also having sensible values of the Yukawa couplings related to the one-loop neutrino masses.

In Chapter 4 we consider only the fermion sector of the previous model. In this case, we study the model both under a non-standard cosmology scenario and multicomponent dark sector scenario. By doing this, we depart from the relic density constraint and we study how the model’s free parameters are constrained with different experiment. First, we focus on constraints from electroweak production at colliders and the measurement of the Higgs diphoton decay rate. We find that the later is by far the most restrictive of the two observables. Then, for each scenario (non-standard cosmology and multicomponent dark matter) we study the restrictions arising from DD and ID in the diffuse and line-like spectrum. For the case of non-standard cosmology, we find that DD imposes limits only on the Yukawa
coupling $y$, while ID in the diffuse spectrum leads to strong constraints on the DM mass. On the other hand, the $\langle \sigma v \rangle_{\gamma\gamma}$ of the model is currently out of reach of the experiments that look for gamma-ray lines. For the case of multi-component DM, we found that currently the model is only constrained by DD.

In Chapter 5 we consider the problem of DM at the LHC. We focus on the case of $W'$ which then decays to a heavy $\nu$ (the DM) and a SM charged lepton, this for single and pair production. Because there are only two significant observables, $P_{\ell z}$ and $P_{\ell T}$, it is difficult to determine many parameters simultaneously that are relevant to the underlying theory just using kinematics. Hence, under initial assumptions, we use the MEM to simultaneously measure the mass of the DM and the width of the $W'$, while we assume we know the chiralities of the couplings and then we use to method to find the chiralities, while we assume we know the DM mass and the width. For mass and width, we find that the method yields a value that is very close to the input value for both single and pair production, while the mass of the $W'$ can be obtained through kinematics, all under our initial assumptions. On the other hand, we show that the MEM can be used to simultaneously measure the chiralities of the $W'$ couplings up to a degeneracy, under our initial assumptions. We also show that, under our assumptions, it is possible to measure simultaneously $M_{\nu}, \mu, \gamma$ and $\cos^2 \varphi_{\text{rel}}$. In order for this method to be applied to the LHC for our particular BSM scenario, a more realistic approximation should be taken, that is, one should include events at the hadron level, background events, and full detector reconstruction.
Appendix A

Calculation of $\langle \sigma v \rangle_{\gamma\gamma}$ for the DTF model

In this appendix we give show the explicit calculation to obtain the thermally averaged cross section for the annihilation of DM in the DTF model into two photons (e.g. gamma-ray lines). The procedure was obtained with the results presented in Ref. [206].

The thermally averaged cross section is given by

$$\langle \sigma v \rangle = \frac{1}{4} \frac{|B|^2}{32\pi m_{\chi_0}^2},$$

(A.1)

where $B = B_W + B_S$. Here $B_W$ and $B_S$ denote the contributions coming from the charged gauge bosons ($W$) and scalars (Goldstone bosons, $S$) running in the loop, respectively (see Fig. A.1). They read

$$B_i = \frac{\alpha}{\pi} \left( x_1 \frac{C_0(0,1,-1,r_{\text{even}}^2,r_{\text{even}}^2,r_{\text{odd}}^2)}{(r_{\text{odd}}^2 - r_{\text{even}}^2)(1 + r_{\text{odd}}^2 - r_{\text{even}}^2)} + x_2 \frac{C_0(0,1,-1,r_{\text{odd}}^2,r_{\text{odd}}^2,r_{\text{even}}^2)}{(r_{\text{even}}^2 - r_{\text{odd}}^2)(1 - r_{\text{odd}}^2 + r_{\text{even}}^2)} \right) + x_3 \frac{C_0(0,4,0,r_{\text{even}}^2,r_{\text{even}}^2,r_{\text{even}}^2)}{(1 + r_{\text{odd}}^2 - r_{\text{even}}^2)} + x_4 \frac{C_0(0,4,0,r_{\text{odd}}^2,r_{\text{odd}}^2,r_{\text{odd}}^2)}{(1 - r_{\text{odd}}^2 + r_{\text{even}}^2)} \right).$$

(A.2)

Here $r_{\text{even(odd)}} = m_{\text{even(odd)}} / m_{\chi_0}$ with the label even (odd) indicating that the particle is $Z_2$ even (odd) and $C_0(r_{1}^2,r_{2}^2,r_{3}^2,r_{4}^2,r_{5}^2,r_{6}^2)$ is the usual Passarino-Veltman function [207]. In the case of the charged Goldstone boson the mass $m_{\text{even}} = m_W$. On the other hand, the factors $x_i$ are different depending if the mediator is a scalar or a vector boson:

**Scalars**

$$x_1 = \sqrt{2} r_{\text{even}}^2 (r_{\text{even}}^2 - r_{\text{odd}}^2 - 1) (g_{Ls}^2 + g_{Rs}^2),$$

(A.3)

$$x_2 = \sqrt{2} r_{\text{even}}^2 (r_{\text{even}}^2 - r_{\text{odd}}^2 - 1) (g_{Ls}^2 + g_{Rs}^2) + 4 \sqrt{2} r_{\text{odd}} (r_{\text{even}}^2 - r_{\text{odd}}^2 - 1) (g_{Ls} g_{Rs}),$$

$$x_3 = 0,$$

$$x_4 = -2 \sqrt{2} r_{\text{odd}} (g_{Ls}^2 + g_{Rs}^2) + 2 g_{Ls} g_{Rs},$$
Appendix A. Calculation of $\langle \sigma v \rangle_{\gamma\gamma}$ for the DTF model

**Figure A.1:** Topologies that lead to the annihilation of DM into two photons. The external straight lines represent the DM particles, whereas the internal ones represent a charged $Z_2$ odd fermion (shown with a cyan solid line), gauge boson or Goldstone boson (shown with a black solid line). The external wavy lines represent the photons (the gamma-rays).

Where $g_{LS} = g_{RS} = -y \cos \theta / \sqrt{2}$ for the lightest $Z_2$-odd charged fermion and $g_{LS} = g_{RS} = y \sin \theta / \sqrt{2}$ for the heaviest $Z_2$-odd charged fermion.

**Vector Bosons**

\[ x_1 = 2\sqrt{2}((r_{even}^4 + 4 r_{odd}^2 - r_{even}^2(1 + r_{odd}^2))(g_{Lw}^2 + g_{Rw}^2)), \]
\[ -8 r_{odd}(1 - r_{even}^2 + r_{odd}^2)g_{Lw} g_{Rw}, \]
\[ x_2 = -2\sqrt{2}(r_{odd}^2(-3 - r_{even}^2 + r_{odd}^2)(g_{Lw}^2 + g_{Rw}^2) + 8 r_{odd} g_{Lw} g_{Rw}), \]
\[ x_3 = 8\sqrt{2}(-1 + r_{even}^2)(g_{Lw}^2 + g_{Rw}^2), \]
\[ x_4 = 4\sqrt{2} r_{odd}(r_{odd}(g_{Lw}^2 + g_{Rw}^2) - 4 g_{Lw} g_{Rw}), \]

Where $g_{Lw} = g_{Rw} = -g_L \sin \theta / 2$ for the lightest $Z_2$-odd charged fermion and $g_{Lw} = g_{Rw} = -g_L \cos \theta / 2$ for the heaviest $Z_2$-odd charged fermion.
Bibliography


[121] The ATLAS collaboration. “Measurement of fiducial, differential and production cross sections in the $H \to \gamma \gamma$ decay channel with 13.3 fb$^{-1}$ of 13 TeV proton-proton collision data with the ATLAS detector”. In: (2016).


<table>
<thead>
<tr>
<th>Number</th>
<th>Author(s)</th>
<th>Title</th>
<th>In:</th>
<th>Volume/Issue</th>
<th>Page Number</th>
<th>DOI</th>
<th>arXiv</th>
<th>arXiv Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>179</td>
<td>Juan Cruz Estrada Vigil</td>
<td>“Maximal use of kinematic information for the extraction of the mass of the top quark in single-lepton t anti-t events at D0”</td>
<td>PhD thesis, Rochester U., 2001</td>
<td></td>
<td></td>
<td>10.2172/1421397</td>
<td>URL: <a href="http://lss.fnal.gov/cgi-bin/find_paper.pl?thesis-2001-07">http://lss.fnal.gov/cgi-bin/find_paper.pl?thesis-2001-07</a></td>
<td></td>
</tr>
</tbody>
</table>


