Entanglement properties of quantum polaritons

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Exciton polaritons are coupled states of matter and light, originated by the strong interaction between an optical mode and semiconductor excitons. This interaction can be obtained also at a single-particle level, in which case it has been shown that a quantum treatment is mandatory. In this work we study the light-matter entanglement of polaritons from a fully quantum formalism including pumping and dissipation. We find that the entanglement is completely destroyed if the exciton and photon are tuned at the resonance condition, even under very low pumping rates. Instead, the best condition for maximizing entanglement and purity of the steady state is when the exciton and photon are out of resonance and when incoherent pumping exactly compensates the dissipation rate. In the presence of multiple quantum dots coupled to the light mode, matter-light entanglement survives only at larger detuning for a higher number of quantum dots considered.

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I. INTRODUCTION

The interaction of radiation and matter, pushed to the extreme level of strong coupling between single qubits, is nowadays a solid reality implemented in many structures and provides a key requirement for coherent control and manipulation of quantum states [1–3]. After its realization in atomic and superconductor systems [4,5], the technology of semiconductor nanostructures has reached the capability to sustain strongly confined optical and electronic modes [6], bridging the era of quantum information processing to solid-state devices [7–9].

In optical nanocavities, light-matter interaction is often described by the physics of polaritons, or dressed states, as the strong coupling of a single fermionic two-level system with a single bosonic photon mode [10,11]. The paradigm of strong coupling between exciton and photon, the Jaynes-Cummings ladder, is a fully quantum model with outstanding predicting power when combined with the nonconservative processes characteristic of specific implementations [12–16]. Even when the entanglement has a local nature, the need of including dissipation is of a fundamental character, as much as the pure Hamiltonian dynamics itself, and its effect on the entangled nature of polaritons arises as a natural question [17–21].

The aim of this work is to study the entanglement properties of the steady state of a microcavity–quantum dot system in a master equation formalism [22]. Entanglement and purity are measured through Peres' negativity [23,24] and linear entropy [25], respectively. Two incoherent effects dominate the dissipative dynamics of the system: at low temperature, photon leakage through the microcavity mirrors is the largest energy-loss channel [26], and in order to reach a nontrivial steady state, an incoherent exciton pumping is needed [27,28]. Their effect is to increase the entropy of the system and completely cancel out the quantum correlations when exciton and photon are at resonance, while negativity is maximum at a fixed finite detuning. It results also that, in the case of multiple quantum dots mutually coupled to the cavity, quantum coherence between the matter dipoles is preserved only if the incoherent pumping is reduced at least 1 order of magnitude with respect to the photon leakage.

The rest of the paper is organized as follows: In Sec. II we present the model and the theoretical formalism employed to approach the problem. Then, in Sec. III we show the results of the computed quantum properties of a single quantum dot–microcavity system, measured through linear entropy as a quantifier of the degree of purity and negativity as a measurement of the degree of entanglement between matter and light. In Sec. IV we study the particular problem of the entanglement between two quantum dots coupled through the mode of the electromagnetic field, a particular instance of the many-quantum-dots extension introduced at the end of the previous section. Finally, in Sec. V we make an overview and conclude.

II. THEORETICAL FRAMEWORK

In order to keep the model as simple as possible, we directly address the quantum properties of the crucial density matrix and neglect, for example, the interaction with phonons, exciton dephasing, and exciton-exciton interactions in the case of multiple quantum dots. We model a quantum dot (QD) embedded in a nanocavity as a two-level system (ground $|G\rangle$ and excited $|X\rangle$ states). Its interaction with a single electromagnetic mode of frequency ω_C, in the dipole and rotating wave approximations, is described by the Hamiltonian ($\hbar = 1$)

$$\hat{H} = \omega_C \hat{a}^\dagger \hat{a} + (\omega_C - \Delta)\hat{\sigma}^\dagger \hat{\sigma} + g(\hat{a}\hat{\sigma}^\dagger + \hat{a}^\dagger \hat{\sigma}). \qquad (1)$$

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The detuning Δ is the difference between cavity-mode and exciton energies, g is the matter-light coupling constant, $\hat{\sigma} = |G\rangle\langle X|$ is the QD ladder operator, and $\hat{a}^\dagger$ ($\hat{a}$) is the usual creation (annihilation) operator of the cavity mode. The Hamiltonian (1) commutes with the excitation number $N = N_{\mathrm{ph}} + N_{\mathrm{ex}} = \hat{a}^\dagger\hat{a} + \hat{\sigma}^\dagger\hat{\sigma}$; hence, it only causes transitions between matter-light states of the same excitation manifold. In the absence of interaction with the environment, polaritons are defined as the energy eigenstates of H and are explicitly given by

$$|n+\rangle = \sin\Phi_n\,|G,n\rangle + \cos\Phi_n\,|X,n-1\rangle,$$
$$|n-\rangle = \cos\Phi_n\,|G,n\rangle - \sin\Phi_n\,|X,n-1\rangle, \qquad (2)$$

where $\{|n\rangle\}$ denotes the Fock number states of the field and $\tan 2\Phi_n = 2g\sqrt{n}/\Delta$. We include the loss of photons (κ) and the continuous pumping of excitons (P) in the master equation for the density operator $\hat{\rho}$ of the system:

$$\frac{d\hat{\rho}}{dt} = i[\hat{\rho},\hat{H}] + \frac{P}{2}(2\hat{\sigma}^\dagger\hat{\rho}\hat{\sigma} - \hat{\sigma}\hat{\sigma}^\dagger\hat{\rho} - \hat{\rho}\hat{\sigma}\hat{\sigma}^\dagger) + \frac{\kappa}{2}(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}), \qquad (3)$$

where we have made the Born-Markov approximation. The basic assumption behind our approach, which focuses on the steady state $\hat{\rho}_{ss}$ of the equation of motion (3), is that polariton lifetime is much longer than the time required to reach the asymptotic solution [29,30]. Although polariton lifetimes in microcavities are usually short, they have been increased in recent years with the improvement of experimental techniques, reaching quality factors above 300 000 [31]. (The parameters employed in this work are thought for a cavity with Q factor of 200 000.) On the other hand, it has been reported in the literature that thermalization times in microcavities are typically of around 40 ps [32]; this means that the model is valid for polaritons with lifetimes of ~ 50 ps (a Q factor of around 76 000), an achievable situation nowadays. The steady state $\hat{\rho}_{ss} = \hat{\rho}(\kappa,P,g,\Delta)$ of the system is a function of the dissipative rates κ and P, the matter-light coupling constant g, and the detuning Δ. Unless stated otherwise, the steady-state solution $\hat{\rho}_{ss}$ of (3) is obtained by setting $\omega_C = 1$ eV, $g = 0.1$ meV, and $\kappa = 5\times10^{-3}$ meV, while Δ and P are varied in ranges similar to those of current experiments.

III. LIGHT-MATTER ENTANGLEMENT AND PURITY

Linear entropy and negativity are employed to quantify purity and light-matter entanglement, respectively. The former, defined as $S_L(\hat{\rho}) = 1 - \mathrm{Tr}(\hat{\rho}^2)$, vanishes for pure states and is maximum for maximally mixed states. The latter is defined as $\mathcal{N}(\hat{\rho}) = 2\sum_{\lambda<0}|\lambda|$, where λ denotes the eigenvalues of the partial transpose of $\hat{\rho}$.

Negativity and linear entropy are depicted in Fig. 1 for different detuning conditions ($\Delta = 0, \Delta = g, \Delta = 2g, \Delta = 3g$, $\Delta = 7g$, and $\Delta = 10g$) and as a function of pumping power P. As expected from a physical point of view, the maximum of the negativity occurs at $P = \kappa$, when gain and losses exactly compensate. The dependence on the detuning shows instead a nonmonotonic behavior, with a maximum value of $\mathcal{N} \approx 0.32$ for $\Delta \approx 3g$, corresponding to the minimum of the linear entropy, $S_L \approx 0.29$. Fixing the condition of $P = \kappa$, we plot in Fig. 2 the behavior of the linear entropy and the negativity as a function of the detuning.

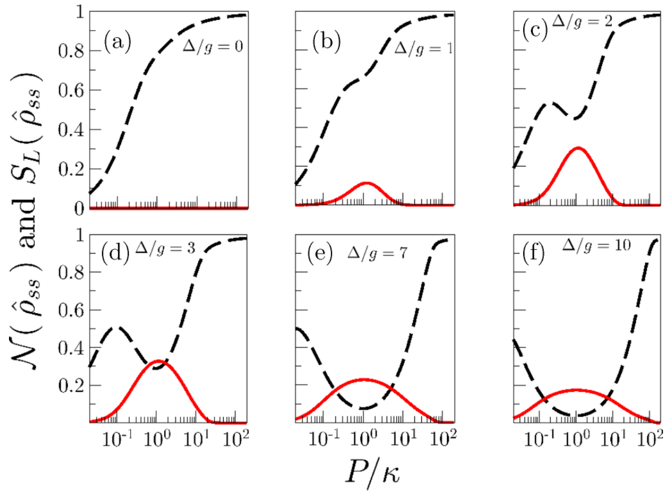

FIG. 1. $\mathcal{N}(\hat{\rho}_{ss})$ (continuous red line) and $S_L(\hat{\rho}_{ss})$ (dashed black line) as a function of the incoherent exciton pumping P for $\kappa = 5\times10^{-3}$ meV and (a) $\Delta = 0$, (b) $\Delta = g$, (c) $\Delta = 2g$, (d) $\Delta = 3g$, (e) $\Delta = 7g$, and (f) $\Delta = 10g$. Note that for $\Delta \neq 0$ the maximum of $\mathcal{N}(\hat{\rho}_{ss})$ always corresponds to a local minimum of $S_L(\hat{\rho}_{ss})$ when $\kappa = P$.

The excitation number ($\hat{N}$) symmetry associated with the Hamiltonian (1) is broken in the time evolution provided by the master equation (3), in the sense that the asymptotic state of the system cannot be labeled with a single eigenvalue of $\hat{N}$.

From Fig. 2 it is possible to identify three different regimes: if the subsystems are in resonance (or near this condition), the interaction, combined with dissipation, produces a very mixed state $S_L \approx 0.75$, and therefore no entanglement is achievable under these conditions [33]. $\mathcal{N}$ then vanishes in the interval $-g \lesssim \Delta \lesssim g$. A diagonalization of the density matrix shows that the steady state of the system is a noncoherent superposition of entangled and separable states, as reported in the Supplementary Information [34]. On the other hand, for

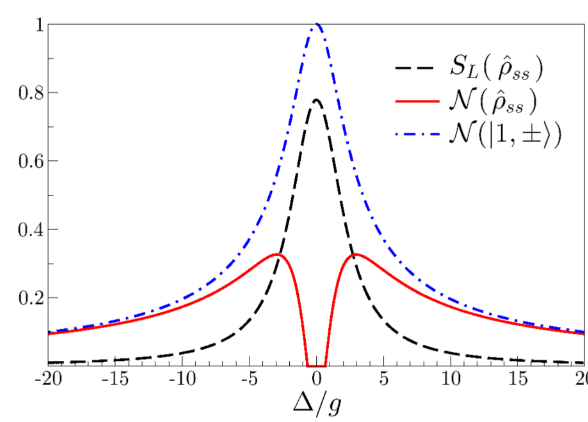

FIG. 2. Linear entropy (dashed black line) and negativity (continuous red line) of the steady state of the system $\hat{\rho}_{ss}$ and negativity of polaritons of the excitation manifold Λ_1 (dashed-dotted blue line) as a function of Δ for $P = \kappa = 5\times10^{-3}$ meV.

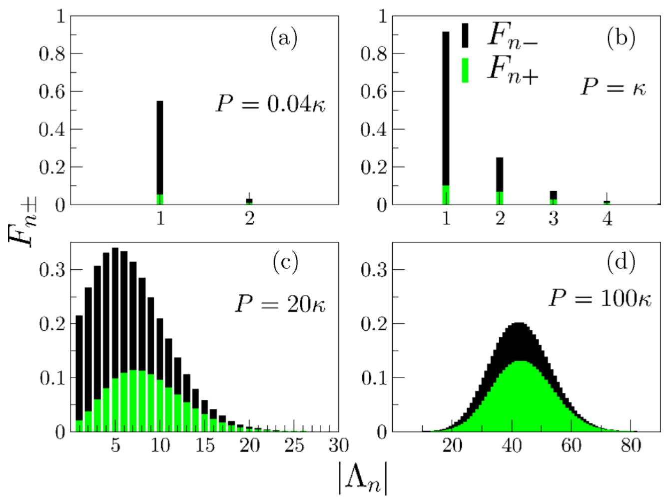

FIG. 3. Sequence of nonzero fidelities $F_{n\pm}$ between the steady state $\hat{\rho}_{ss}$ and the Λ_n–lower (black)/upper (green) polaritons $|n,\pm\rangle$ for $\kappa = 5\times 10^{-3}$ meV, $\Delta = 3g$ and (a) $P = 0.04\kappa$, (b) $P = \kappa$, (c) $P = 20\kappa$, and (d) $P = 100\kappa$. $|\Lambda_n|$ denotes the excitation number of the polariton manifold Λ_n.

large detunings, the interaction strength diminishes and the matter-light coupling is not enough to generate entanglement. The quantum dot is saturated due to the absence of interaction and exciton pumping, driving the system into the not entangled, pure state $|X0\rangle$. The entropy takes low values in this regime but the negativity decreases as well. The best condition is then the one in which the entropy is low enough to avoid the demolition of the quantum properties of the system, and the interaction is high enough to generate nonseparable states. We find that this condition is maximized for $\Delta \approx 3g$, as shown in Fig. 2. At this detuning condition, there is a maximum in the negativity that coincides with a local minimum in linear entropy. Although the density matrix is not pure, the system has a probability of 84% of being in the nonseparable state $0.29\,|G1\rangle - 0.96\,|X0\rangle$.

Figure 3 compares the steady state of the system with pure polaritonic states $|n,\pm\rangle$ [defined by Eq. (2)] using the sequence of nonzero fidelities $F_{n\pm} = \sqrt{\langle n \pm |\hat{\rho}_{SS}| n\pm\rangle}$. For small values of P [Fig. 3(a)] ρ_{G0G0}, the population of the state without polaritons is much larger than the other populations. In this case only F_1 does not vanish; it is relatively small, though. For $P = \kappa$ [Fig. 3(b)], F_1 increases up to more than 0.91, while the remaining fidelities are still small. This indicates a concentration of the population in the first manifold and a state similar to a pure polariton. As P increases, $F_{n\pm}$ is nonzero for larger excitation numbers, but their values decrease from 0.91 at $P = \kappa$ to 0.2 at $P = 100\kappa$. Changing the detuning, the fidelity of the steady state is always higher for the eigenstate with a higher excitonic component: for negative (positive) detuning, higher fidelities are found for the upper (lower) polariton state.

So far we have only considered interaction between a single quantum dot and a cavity mode, but the model is valid for a system with many quantum dots, provided the QD density is low enough to avoid direct interaction between excitons. This is the case in which the dots are coupled through the photonic mode only. Figure 4 shows linear entropy and negativity for the multiple-quantum-dots case as a function of the detuning between the exciton energy and the cavity mode. At this point we focused exclusively on the set of parameters for which $\mathcal{N}$ is maximum for each number of QDs. As it can be seen, the detuning for maximum entanglement varies for each number of QDs, but the dependence of the negativity with the detuning does not change qualitatively. Low detunings are still not suitable to generate nonseparable states because they maximize the degree of mixedness, and for large detunings, due to the absence of interaction, quantum dots saturate and hence the system is driven to a separable state. The competition between these two effects remains for systems with many noninteracting QDs. While a direct comparison is not possible given that negativity has different maximum values for systems with Hilbert spaces of different size, the detuning for maximum entanglement in each specific number of QDs has been rigorously found.

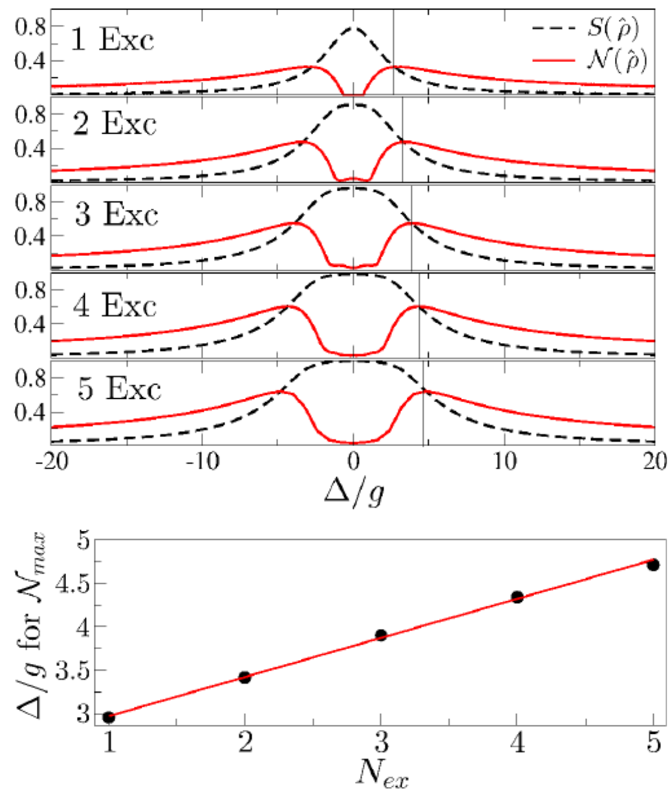

FIG. 4. Upper panel: Negativity (continuous red line) and linear entropy (dashed black line) as a function of Δ for systems with different number of quantum dots (Exc). For all the cases, each QD interacts with the cavity mode with the same strength. Lower panel: detuning for maximum light-matter entanglement as a function of the number of quantum dots.

IV. QD-QD ENTANGLEMENT

A problem that naturally arises is whether or not the electromagnetic field is able to generate entanglement between two quantum dots. This is the simplest system able to present nonlocal entanglement, a necessary condition for sustaining any quantum information protocol. To do so, a partial trace over the degree of freedom of the light is performed, and the

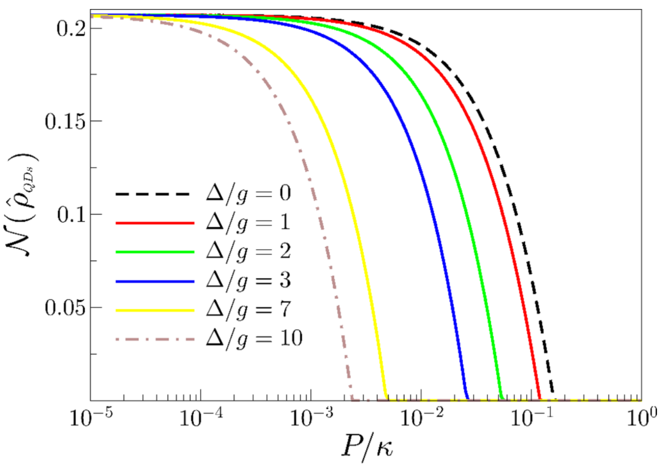

FIG. 5. Negativity of the reduced system of QDs in the steady state as a function of the incoherent pumping rate for different values of the matter-light detuning. Contrary to the case of QD-light entanglement, resonance is the most suitable condition to find a nonvanishing entanglement. In any case, the incoherent pumping has to be lower than the photon leakage rate.

negativity of the reduced operator of the QDs is calculated. This is

$$\hat{\rho}_{QD-QD} = \sum_n \langle n| \hat{\rho}_{ss} |n\rangle , \tag{4}$$

where $|n\rangle$ are the Fock states for the electromagnetic field and $\hat{\rho}_{ss}$ is the full density operator of the steady state. We perform this operation for the case in which there are two quantum dots interacting with the cavity mode. Figure 5 shows the QD-QD entanglement quantified through negativity as a function of the pumping rate for different values of the light-matter detuning. In this case the condition $\kappa = P$ is not the most suitable anymore; in fact, there is a critical value of P for which the entanglement fully vanishes. This value is lower than the photon leakage rate. On the other hand, the figure also shows that the resonance condition enhances the QD-QD entanglement.

V. OVERVIEW AND CONCLUSIONS

We studied the properties of purity and entanglement of a microcavity–quantum dots system in the strong-coupling regime by taking into account dissipative mechanisms. In the case of a single quantum dot we found the conditions for which the matter-light entanglement is maximized in the system. Matter and light should be out of resonance and the incoherent pumping rate should exactly compensate the photon loss. This maximization is due to a competition of two effects: near resonance, the exchange of energy between the quantum dot and the cavity mode is enhanced, but the dissipation generates a mixture in the steady state. In a large detuning condition, the degree of purity of the steady state is enhanced, but the exchange of energy rate is not enough to generate nonseparable states. The value of Δ where these two effects compensate each other is around $\Delta \sim 3g$.

In the case of many quantum dots, the best matter-light entanglement is reached when every quantum dot is pumped with a rate equal to the cavity losses. The most suitable condition is still $\Delta \neq 0$, although the exact value of the detuning slightly varies with the number of excitons. In this case the light is entangled with all the quantum dots, understood as one matter system.

Finally, we focused into the case of two quantum dots coupled through the cavity mode. We found that incoherent pumping strongly works against the coherence of the reduced matter system (QD-QD). The maximum QD-QD entanglement is obtained for an incoherent pumping at least 1 order of magnitude smaller than the photon leakage rate, in which case the system reaches an entanglement of 20%. The exciton energy should also be in resonance with the cavity mode, giving a criteria to identify two different regimes. If $\kappa = P$ and $\Delta \neq 0$ the system is in a state with light-matter quantum correlations; on the other hand, if the incoherent pumping is at least 1 order of magnitude below the dissipation rate and the cavity mode has the same energy of the excitons, the system loses light-matter correlations and the quantum dots are entangled between them via their interaction with the field.

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