Mass scales and stability of the proton in $[SU(6)]^3 \times Z_3$

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We prove that the proton is stable in the gauge model $[SU(6)]^3 \times Z_3$ which unifies nongravitational forces with flavors, broken spontaneously by a minimal set of Higgs fields and vacuum expectation values down to $SU(3)_C \otimes U(1)_{EM}$. We also compute the evolution of the gauge coupling constants and show how agreement with precision data can be obtained.

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Recently we proposed [1] a grand unification model of forces and flavors based on the gauge group $G=[SU(6)]^3 \times Z_3$. Our aim has been to provide some clues for the explanation of the intriguing fermion mass spectrum and mixing parameters.

The fermion content of our model includes in a single irreducible representation of $G$ the three families of known fermions, each family being defined by the dynamics of the $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{Y_{(a-b-c)}}$ gauge group. This last group is the left-right symmetric (LRS) extension of the $SU(3)_C \otimes SU(2)_L \otimes U(1)_{Y_s}$ standard model (SM).

Explicitly, $G \equiv SU(6)_L \otimes SU(6)_C \otimes SU(6)_R \times Z_3 \times \mathbb{Z}_3$, where $SU(6)_C$ is a vectorlike group which includes three hadronic and three lepton colors. $SU(6)_C$ includes as a subgroup the $SU(3)_C \otimes U(1)_{Y_{(a-b-c)}}$ of the LRS model. $SU(6)_L \otimes SU(6)_R$ is the left-right symmetric flavor group which includes the $SU(2)_L \otimes SU(2)_R$ gauge group of the LRS model.

The 105 gauge fields (GF's) and the 108 Weyl fermions fields in $G$ are explicitly depicted in Ref. [1]. Let us describe here some of them: The 105 GF's can be divided in two sets: 70 of them belonging to $SU(6)_L \otimes SU(6)_R$ and 35 associated with $SU(6)_C$. The first set includes $W^\pm_L$ and $Z^0_L$, the GF's of the known weak interactions, plus the GF's associated with the postulated right weak interaction, plus the GF's of the horizontal interactions, etc. All of them are $SU(3)_C$ singlets and have electrical charges 0 or ± 1. The second set includes the eight gluon fields of $SU(3)_C$; nine leptoquark GF's, $X_i$, $Y_i$, and $Z_i$, $i = 1, 2, 3$ with electrical charges $-2/3, 1/3$, and $-2/3$, respectively, another nine leptoquark GF's charge conjugated to the previous ones, six diquark GF's, $P^a_0$, $P^0_a$, and $P_\alpha^0$, $a = 1, 2$, with electrical charges as indicated, and three GF's associated with diagonal generators in $SU(6)_C$, where $B_{Y_{(a-b-c)}}$, the GF's associated with the $U(1)_{Y_{(a-b-c)}}$ factor in the LRS model, is one of them.

The known fermion fields are included in $\psi(108)_L = Z_3 \psi(6, 1, 6) = \psi(6, 1, 6)_L + \psi(1, 6, 6)_L + \psi(6, 6, 1)_L$, with quantum numbers with respect to $(SU(3)_C, SU(2)_L, U(1)_{Y_s})$ given by

\begin{align*}
\psi(6, 6, 1) & \equiv \psi^\alpha_\alpha : 3(3, 2, 1/3) \oplus 6(1, 2, -1) \oplus 3(1, 2, 1), \\
\psi(1, 6, 6) & \equiv \psi^A_\alpha : 3(\bar{3}, 1, -4/3) \oplus 3(\bar{3}, 1, 2/3) \oplus 6(1, 1, 2) \oplus 9(1, 1, 0) \oplus 3(1, 1, -2), \\
\psi(6, 1, 6) & \equiv \psi^\alpha_\alpha : 9(1, 2, 1) \oplus 9(1, 2, -1).
\end{align*}

As is clear, $a, b, \ldots, A, B, \ldots = 1, \ldots, 6$ label $SU(6)_L$, $SU(6)_R$, and $SU(6)_C$ tensor indices, respectively.

The analysis done in Ref. [1] shows that the most economical set of Higgs fields (HF's) and vacuum expectation values (VEV's) which breaks the symmetry from $G$ down to $SU(3)_C \otimes U(1)_{EM}$ and at the same time produces what we called the modified horizontal survival hypothesis is formed by

\begin{equation}
\phi_1 = \phi(675) = \phi_{[A, B]} + \phi_{[\alpha, \beta]} + \phi_{[a, b]} + \phi_{[a, \beta]} (1)
\end{equation}

with VEV's in the directions $[a, b] = [1, 6] = [-2, 5] = \ldots$.
$-[3,4]$, $[A, B]$ similar to $[a, b]$ and $[\alpha, \beta]=[5,6],$

$$\phi_2 = \phi(1323) = \phi_{2,(a,b)} + \phi_{2,(A,B)} + \phi_{2,(\alpha, \beta)}$$  \hfill (2)

with VEV's in the directions $\{a, b\} = \{1, 4\} = \{2, 3\}$, $\{A, B\}$ similar to $\{a, b\}$ and $[\alpha, \beta]=[4,5],$

$$\phi_3 = \phi'(675) = \phi_{3,(a,b)} + \phi_{3,(A,B)} + \phi_{3,(\alpha, \beta)}$$  \hfill (3)

with VEV's such that $\langle \phi_{3,(a,b)} \rangle = \langle \phi_{3,(\alpha, \beta)} \rangle = 0$, and $\langle \phi_{3,(A,B)} \rangle \equiv M_R,$

$$\phi_4 = \phi(108) = \phi_{4,a} + \phi_{4,b} + \phi_{4,A}$$  \hfill (4)

with VEV's such that $\langle \phi_{4,a} \rangle = \langle \phi_{4,b} \rangle = 0$ and $\langle \phi_{4,A} \rangle \equiv M_Z$, with values different from zero only in the directions $\langle \phi_2^2 \rangle = \langle \phi_3^2 \rangle = \langle \phi_3^2 \rangle = \langle \phi_4^2 \rangle = \langle \phi_4^2 \rangle = \langle \phi_5^2 \rangle = \langle \phi_5^2 \rangle = M_Z \sim 2 \times 10^2 \text{GeV}.$

In Eqs. (1)–(3), the symbols $\{\cdot, \cdot\}$ and $[\cdot, \cdot]$ indicate symmetrization and antisymmetrization, respectively, of the indices inside the brackets. The mass hierarchy suggested in Ref. [1] is $\langle \phi_2 \rangle > \langle \phi_1 \rangle \simeq \langle \phi_2 \rangle \gg M_Z \sim 10^2 \text{GeV}.$

For the renormalization group equation (RGE) analysis which follows, we adopt the working conditions known as "the survival hypothesis" [2] and "the extended survival hypothesis" [3]. The survival hypothesis claims that [2] at each energy scale, the only fermion fields which are relevant are those belonging to chiral representations of the unbroken symmetries. The extended survival hypothesis claims that [3] at each energy scale the only scalars which are relevant are those that develop VEV's at that scale and at lower mass scales. Both hypothesis are satisfied if a particular selection of scalar fields and VEV's is made, and appropriate terms in the scalar potential and Yukawa Lagrangian are included.

When the symmetry is broken in two steps, $G \rightarrow G_L \otimes G_C \otimes G_R \otimes \cdots \rightarrow SU(3)_C \otimes SU(2)_L \otimes SU(1)_Y \otimes SU(3)_C \otimes U(1)_{EM}$, where $M = \{\phi_1\} + \{\phi_2\} + \{\phi_3\}$ and $M_Z = \{\phi_4\}$, the one loop running coupling constants of the standard model satisfy

$$\alpha_{i}^{-1}(M_Z) = \alpha_{i}^{-1}(M) - b_i^3 \ln(M/M_1), \tag{5}$$

where $\alpha_i = g_i^2/(4\pi)$, $i = 1, 2, 3$ refers to $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$, respectively, and

$$b_i^3 = \left\{ \begin{array}{ll}
\frac{1}{3} C_i \text{(vectors)} - \frac{2}{3} C_i \text{(Weyl fermions)} & \frac{1}{3} C_i \text{(scalars)} / 4\pi
\end{array} \right. \tag{6}$$

with $C_i(\cdots)$ the index of the representation to which the $(\cdots)$ particles are assigned. For a complex field the value of $C_i \text{(scalars)}$ should be doubled. With the normalization of the generators of $G$ such that $\alpha_1(M) = \alpha_2(M) = \alpha_3(M)$, the relationship

$$\alpha_{EM} = \frac{1}{3} \alpha_2 \sin^2 \theta_W = \frac{1}{2} \alpha_1 \cos^2 \theta_W, \tag{7}$$

where $\theta_W$ is the weak mixing angle, is valid at all energy scales. This last equation implies also that

$$3\alpha_{EM}^{-1} = 14\alpha_1^{-1} + 9\alpha_2^{-1}. \tag{8}$$

Equations (5)–(8) give straightforwardly

$$\frac{3}{23} \alpha_{EM}^{-1}(M_Z) = \alpha_2^{-1}(M) + \left( b_3^3 - \frac{14}{23} b_3^3 - \frac{9}{23} b_3^3 \right) \ln \left( \frac{M_1}{M_Z} \right) \tag{9}$$

and

$$\sin^2 \theta_W (M_Z) = 3 \alpha_{EM}(M_Z) \left[ \alpha_2^{-1}(M_Z) + b_3^3 \ln \left( \frac{M_1}{M_Z} \right) \right] \nonumber$$

where $b_3^3 = (11 - 4)/2\pi$, $b_2^3 = [\frac{22}{9} - \frac{5}{9} (3 - n_1^2)]/2\pi$, $b_1^3 = -[\frac{3}{2} (3 - n_1^2) + \frac{23}{24}] / 2\pi$, $N_H = 9$ is the number of low-energy Higgs fields doublets in $\{\phi_4\}$, and $n_1^0 = 2$, $n_1^0 = 27/14$ are related to the number of fermion fields which decouple from $\psi(108)_L$ according to the survival hypothesis and the Appelquist-Carrazone theorem [4] $[n_1^0 = n_2^0 = 0$ when all the fermion fields in $\psi(108)_L$ contribute to $b_3^3].$

Substituting in the last two equations the experimental values $[5] \sin^2 \theta_W (M_Z) = 0.233, \alpha_{EM}(M_Z) = 127.9,$ and $\alpha_3 (M_Z) = 0.122$ we get from Eq. (9) $\ln (M/M_Z) = 6.3$, while from Eq. (10) $\ln (M/M_Z) = 1.1$ which are widely incompatible solutions. Therefore the model with only two mass scales is excluded.

When the symmetry is broken in three steps: $G \rightarrow G_L \otimes G_C \otimes G_R \otimes \cdots \rightarrow SU(3)_C \otimes SU(2)_L \otimes SU(1)_{EM}$ where $M \gg M_I \gg M_Z = \phi_4$, the one loop running coupling constants of the SM satisfy

$$\alpha_i^{-1}(M_Z) = \alpha_i^{-1}(M) - b_i^3 \ln(M/I) - b_i^3 \ln(M/M_1). \tag{11}$$

It is easy then to show that

$$\frac{3}{23} \alpha_{EM}^{-1}(M_Z) = \alpha_2^{-1}(M) + \left( b_3^3 - \frac{14}{23} b_3^3 - \frac{9}{23} b_3^3 \right) \ln \left( \frac{M_I}{M_Z} \right)$$

$$+ \left( b_3^3 - \frac{14}{23} b_3^3 - \frac{9}{23} b_3^3 \right) \ln \left( \frac{M_I}{M_1} \right) \tag{12}$$

and

$$\sin^2 \theta_W (M_Z) = 3 \alpha_{EM}(M_Z) \left[ \alpha_2^{-1}(M_Z) + b_3^3 \ln \left( \frac{M_I}{M_Z} \right) \right]$$

$$+ \left( b_3^3 - \frac{14}{23} b_3^3 - \frac{9}{23} b_3^3 \right) \ln \left( \frac{M_I}{M_1} \right) \tag{13}$$

where $b_{i}^3, i = 1, 2, 3$ are the same as above, but $b_i^3, i = C, Y, L$ depend upon the structure of the subgroup $G_L \otimes G_C \otimes G_R \otimes \cdots \rightarrow G_I.$

Equation (13) indicates that $G_I$ cannot be $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{Y(B-L)}$ because if that were the case $b_3^3 = b_1^3$ and $b_2^3 = b_3^3$, and since the first entry of the right-hand side of Eq. (13) has an experimental value of 0.192, $M_I \sim 3 M_Z$ is required to be satisfied and, therefore, there is a very small value for the masses of the gauge bosons associated with $SU(2)_R$. With the minimal set of Higgs fields $G_I$ contains therefore flavor-changing

\[ \text{(continued)} \]
neutral currents and thus $M_f$ has to be greater than 100 TeV. The first two terms of the right-hand side of Eq. (13) have then a lower bound of 0.36, and the experimental value for $\sin^2 \theta_W(M_Z)$ requires that $(b_1^2 - b_2^2) < 0$ which is not satisfied by the minimal set of Higgs fields and VEV’s [1].

We are therefore led to consider introducing a minimum change in the set of HF’s and/or VEV’s such that the new set properly breaks the symmetry, guarantees the survival hypothesis, produces appropriate values for $\sin^2 \theta_W(M_Z)$, and satisfies the mass hierarchy $M \gg M_f \gg M_Z \sim 10^2$ GeV. An analysis of Table I shows that a symmetry-breaking pattern in three steps with $G_L = SU(6)_L \otimes SU(4)_C \otimes U(1)_Y \otimes \cdots$ produces consistent results provided we introduce the following changes. First, add a new set of Higgs fields

$$\phi_2' = \phi_2'(1323) = \phi_2'(A,B) + \phi_2'(a,b) + \phi_2'(a,b)$$

with VEV’s in the directions $(a,b) = (3,6) = -\{4,5\}$, $(A,B)$ similar to $(a,b)$ and $(\alpha,\beta) = \{5,5\}$, and second, orient the VEV’s such that

$$\langle \phi_2'(a,b) \rangle = \langle \phi_2'(A,B) \rangle = 0.$$  

For this particular choice of VEV’s we have that

$$b_1 = (132 - 2x12 - 107)/4 \pi$$

$$b_2 = (132 - 2x12 + 107)/4 \pi$$

where the extended survival hypothesis [3] was taken into account for the HF’s.

As can be seen, the Higgs fields play a fundamental role in Eqs. (12) and (13). Notice also that the symmetry-breaking pattern is achieved with $M = \langle \phi_3 \rangle$, $M_f = \langle \phi_1 \rangle + \langle \phi_2 \rangle + \langle \phi_2' \rangle$, $M_Z = \langle \phi_4 \rangle$, and that $\phi_3$ plays no role in the evolution of the gauge coupling constants.

Substituting now in Eqs. (12) and (13) the experimental values for $\sin^2 \theta_W(M_Z)$, $\alpha_E(M_Z)$, $\alpha_3(M_Z)$ we obtain the equations

$$1.10 = 1.02 \ln \left( \frac{M_f}{M_Z} \right) - 2.26 \ln \left( \frac{M_f}{M_L} \right),$$

$$7.83 = 1.25 \ln \left( \frac{M_f}{M_Z} \right) - 1.82 \ln \left( \frac{M_f}{M_L} \right),$$

which for $M_Z = 91$ GeV have the solutions $M_f \sim 10^9$ GeV and $M \sim 10^{12}$ GeV. These results are in good agreement with those obtained from the analysis of the generational seesaw mechanism in this model [6].

### STABILITY OF THE PROTON

#### A. Baryon number for the particles

The elementary particles in the model are the ones associated with the 105 GF’s, the 108 Weyl fields in $\psi(108)_L$ and the 4104 HF’s in $\phi_1$, $i = 1-4$ and $\phi_2'$. Now, all the elementary particles in our model have a well-defined Baryon Number $B$. Let us note the following.

1. The GF. The 70 GF’s associated with $SU(6)_L \otimes SU(6)_R$ have $B = 0$. For $SU(6)_C$ we have that the 9 leptoquarks have $B$ equal to $1/3$, and the other 17 GF’s have $B = 0$ (including the 9 gluon fields).

2. The Weyl fermion fields. The quark fields in $\psi(6,6,1)_L$ have $B = 1/3$, the quark fields in $\psi(1,6,6)_L$ have $B = -1/3$ and all the other fields in $\psi(108)_L$, $\psi(108)_R$ have $B = 0$.

3. The HF. $B$ for the 4104 HF’s of the model is given in Table II.

#### B. Baryon number as a symmetry of the model

In the subspace of the fundamental representation of $SU(6)_C$ $B$ can be associated with the $6 \times 6$ diagonal matrix $B = \text{Diag}(1,3,1,3,1,3,0,0,0)$. This matrix does not correspond to a generator of $SU(6)_C$ neither of $G$. Now, the full Lagrangian $\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}$ has a $U(1)_B$ global symmetry, where $\chi$ is a constant whose magnitude depends solely on the label of the SU(6)_C representation. For example $\chi = 1$ for $\psi(6,6,1)$, $\chi = 0$ for $G(1,35,1)$, $\chi = -2$ for $\psi(1,15,15)$, etc. Conventionally normalized, the $U(1)_B$ generator may be written in the fundamental representation of SU(6)_C as $\chi = \text{Diag}(1,1,1,1,1,1)/\sqrt{12}$, which is not an element of the Lie algebra of $G$ either.

On the other hand, in the Lie algebra of $G$ there is a generator, an element of the SU(6)_C subalgebra, of the form

<table>
<thead>
<tr>
<th>$\langle \phi \rangle$</th>
<th>SU(4)_C</th>
<th>SU(6)_C</th>
<th>SU(6)_C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle \phi_1,\phi_2 \rangle$</td>
<td>0</td>
<td>fourteen 15's</td>
<td>56</td>
</tr>
<tr>
<td>$\langle \phi_3,\phi_4 \rangle$</td>
<td>1</td>
<td>five 4's</td>
<td>16</td>
</tr>
<tr>
<td>$\langle \phi_1,\phi_2,\phi_3 \rangle$</td>
<td>15</td>
<td>four 15's</td>
<td>112</td>
</tr>
<tr>
<td>$\langle \phi_1,\phi_2,\phi_5 \rangle$</td>
<td>14</td>
<td>fourteen 21's</td>
<td>112</td>
</tr>
<tr>
<td>$\langle \phi_1,\phi_2,\phi_3,\phi_4 \rangle$</td>
<td>21</td>
<td>four 21's</td>
<td>32</td>
</tr>
<tr>
<td>$\langle \phi_1,\phi_2,\phi_3,\phi_5 \rangle$</td>
<td>0</td>
<td>fourteen 21's</td>
<td>112</td>
</tr>
<tr>
<td>$\langle \phi_1,\phi_2,\phi_3,\phi_6 \rangle$</td>
<td>84</td>
<td>fourteen 10's</td>
<td>0</td>
</tr>
<tr>
<td>$\langle \phi_1,\phi_2,\phi_3,\phi_7 \rangle$</td>
<td>126</td>
<td>ten 21's</td>
<td>56</td>
</tr>
<tr>
<td>$\langle \phi_1,\phi_2,\phi_3,\phi_8 \rangle$</td>
<td>0</td>
<td>three 6's</td>
<td>3</td>
</tr>
</tbody>
</table>
TABLE II. Baryon number of the 4104 Higgs fields.

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$\alpha, \beta$</th>
<th>$a, b$</th>
<th>$A, B$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_{(2), [a, b]}^{(4, A)}$</td>
<td>$\alpha, \beta = 1, 2, 3$</td>
<td>$a, b = 1, ..., 6$</td>
<td>$A, B = 1, ..., 6$</td>
<td>$-1/3$</td>
</tr>
<tr>
<td>$\phi_{1(3), [a, b]}^{(4, A)}$</td>
<td>$\alpha, \beta = 4, 5, 6$</td>
<td>$a, b = 1, ..., 6$</td>
<td>$A, B = 1, ..., 6$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\phi_{1(3), [a, b]}^{(4, A)}$</td>
<td>$\alpha = 1, 2, 3; \beta = 4, 5, 6$</td>
<td>$a, b = 1, ..., 6$</td>
<td>$A, B = 1, ..., 6$</td>
<td>$1/3$</td>
</tr>
<tr>
<td>$\phi_{2, [a, b]}^{(4, A)}$</td>
<td>$\alpha, \beta = 1, 2, 3$</td>
<td>$a, b = 1, ..., 6$</td>
<td>$A, B = 1, ..., 6$</td>
<td>$2/3$</td>
</tr>
<tr>
<td>$\phi_{2, [a, b]}^{(4, A)}$</td>
<td>$\alpha, \beta = 4, 5, 6$</td>
<td>$a, b = 1, ..., 6$</td>
<td>$A, B = 1, ..., 6$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\phi_{2, [a, b]}^{(4, A)}$</td>
<td>$\alpha = 1, 2, 3; \beta = 4, 5, 6$</td>
<td>$a, b = 1, ..., 6$</td>
<td>$A, B = 1, ..., 6$</td>
<td>$1/3$</td>
</tr>
<tr>
<td>$\phi_{4, A}$</td>
<td>$\alpha = 1, 2, 3$</td>
<td>$a = 1, ..., 6$</td>
<td>$A = 1, ..., 6$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\phi_{4, A}$</td>
<td>$\alpha = 4, 5, 6$</td>
<td>$a = 1, ..., 6$</td>
<td>$A = 1, ..., 6$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\phi_{4, A}$</td>
<td>$\alpha = 1, 2, 3$</td>
<td>$A = 1, ..., 6$</td>
<td>$A = 1, ..., 6$</td>
<td>$-1/3$</td>
</tr>
<tr>
<td>$\phi_{4, A}$</td>
<td>$\alpha = 4, 5, 6$</td>
<td>$A = 1, ..., 6$</td>
<td>$0$</td>
<td></td>
</tr>
</tbody>
</table>

$B' = \text{Diag}(1, 1, 1, -1, -1, -1)/\sqrt{12}$,  

which distinguishes between quarks and leptons in our model. Therefore $B$ can be written as $B = \left[ \chi + B' \right]/\sqrt{3}$.

Since the elementary particles of this model have a well-defined $B$, it is obvious that in the unbroken theory the exchange of particles cannot break $B$. This statement is also true after breaking the symmetry due to the following two facts: The baryon number is not gauged (there is no gauge boson associated to $B$); $\phi_i, i = 1 - 4$, and $\phi_2$ with the VEV's as stated do not break $B$ spontaneously.

That is, $B\langle\phi_i\rangle = B\langle\phi_2\rangle = 0$, $i = 1, 2, 3, 4$. Therefore, $B$ is conserved in our model [7]. Since $B$ is conserved, the proton is perturbatively stable.

Now, the single Goldstone boson associated with the broken orthogonal combination is absorbed by the massive gauge field associated with $B'$. Therefore, there are no physical Goldstone bosons, and there is no extra long range force. This mechanism in which a global symmetry emerges from the simultaneous breaking of a gauge and global symmetry is due to 't Hooft [8] and was implemented in the context of grand unified models in Ref. [9].

Finally we would like to mention that, contrary to baryon number, lepton number is violated in this model due to the fact that the GF's associated with the $U(1)_{\gamma_{(\mu - \nu)}}$ generator is gauged. Therefore, neither $L, (B - L)$ or $(B + L)$ are conserved quantities.

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