# FEATURES OF PHASE WAVE FRONT BINARY ENCODING AND THEIR POTENTIAL UTILIZATION FOR ALIGNMENT PURPOSES

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## ABSTRACT

We make use of binary encoding of a phase wave front for the generation of diffractive elements. Binary encoding enables to introduce a constant phase shift between different diffraction orders. We describe the main features of phase wave front encoding and then suggest to apply them for a design of a diffractive alignment element. The proposed positioning tool is a modified version of the hyperbolic zone plate.

The contribution describes properties of binary encoding of phase wave fronts. Binary encoded elements generate different diffractive orders. Usually it is a drawback leading to the substantial limitation of diffraction efficiency. The paper puts the particular attention to a possibility of introducing an arbitrary mutual phase shift between conjugate diffractive orders. This interesting feature of binary encoding was not distinctly pointed out till now, to our knowledge. In a particular case of phase binary encoding the phase shift between +1 and -1 orders can give rise to their destructive interference. We suggest to utilize this interference for alignment applications. The proposed positioning tool is a modified hyperbolic zone plate. It forms in its focal plane diffractive field with the characteristic central dark spot. According to last investigations such diffractive structures are particularly promising for alignment purpose[1,2]. The phase wave front $\boldsymbol{U}\left(\vec{\boldsymbol{r}}\right)=\mathbf{exp}\left[\boldsymbol{i}\Phi\left(\vec{\boldsymbol{r}}\right)\right]$ can be encoded in a diffractive element by means of the periodical complex function $\boldsymbol{T}\left[\Phi\left(\vec{\boldsymbol{r}}\right)\right]$ with a period of $2\pi$ of a phase argument $\Phi(\vec{r})$ [3]. Module of the transmittance values fulfill the inequality:

$$0 \le \left|\boldsymbol{T}(\Phi)\right| \le 1 \tag{1}$$

Thanks to its periodicity the transmittance can be developed in the following Fourier series:

$$\boldsymbol{T}(\Phi)=\sum_{n}\boldsymbol{A}_n\,\mathbf{exp}(\boldsymbol{in}\Phi) \tag{2}$$

Where $\boldsymbol{A}_n$ are coefficients corresponding to different diffractive orders of the transmittance $\boldsymbol{T}(\Phi)$. $|\boldsymbol{A}_n|^2$ is interpreted as the diffraction efficiency of the n-th order. The first order ($n$=1) reconstructs the field of interest: $\boldsymbol{U}\left(\vec{\boldsymbol{r}}\right)=\mathbf{exp}\left[\boldsymbol{i}\Phi\left(\vec{\boldsymbol{r}}\right)\right]$. The simplest and commonly used is binary encoding, where the function $T(\Phi)$ is of the following particular form:

$$T_1(\Phi)=\begin{cases}1, & -\frac{\pi}{2}+2\pi m<\Phi<\frac{\pi}{2}+2\pi m \\ \alpha, & \text{otherwise.}\end{cases} \quad ; m=0,\pm1,\pm2... \tag{3}$$

The number $\alpha$ is equal to -1 or 0 in the case of phase or amplitude encoding, respectively. The main drawback of binary encoding lies in the low diffraction efficiency of elements fabricated according to this method. The diffraction efficiency $\eta$ of the diffractive structure relates to the first order of interest and is equal to $\eta=|A_1|^2 100\%$ . Binary phase encoding gives rise to the efficiency $\eta=40.53\%$ while amplitude encoding leads to the efficiency four times smaller. Increasing of the diffraction efficiency is possible but requires the application of the troublesome and much more precise kinoform technique[4]. The function $T_1(\Phi)$ described by Eq.(3) is not only one possible in binary encoding. A displacement of an argument $\Phi$ in Eq.(3) by $\Phi_o$ leads to the modified transmittance $T_1\left[\Phi\left(\vec{r}\right)-\Phi_o\right]$ of the diffractive element. The both transmittances $T_1(\Phi)$ and $T_1(\Phi-\Phi_o)$ generate diffractive orders corresponding to the same absolute values of the coefficients $A_n$. The only difference lies in the fact that the transmittance $T_1(\Phi-\Phi_o)$ gives rise to a mutual phase shift $2n\Phi_o$ between conjugate $\pm n$-th orders. Eq.(3) and the periodicity of the function $T_1(\Phi)$ lead to the following connection:

$$A_{-n}=A_n\exp(i2n\Phi_o) \tag{4}$$

Introduction of the mentioned phase shift reveals in the rearrangement of geometrical regions of a diffractive structure relating to the possible values 1 and $\alpha$ of the function $T_1$ (Fig. 1a and Fig. 2a). Carrying on our investigations concerning the application of diffractive elements for positioning purposes we noted surprisingly that possibility of introducing an arbitrary phase shift between conjugate orders in binary encoding can be utilized for a simple fabrication of a diffractive alignment tool. Our approach is based on binary encoding of the following wave front:

$$U_o\left(\vec{r}\right)=\exp\left[\frac{-ik\left(x^2-y^2\right)}{2f}\right]\text{rect}\left(\frac{x}{d},\frac{y}{d}\right) \tag{5}$$

corresponding to the transmittance of the thin hyperbolic lens of the focal length $f$ and a square aperture of the width $d$; $k=2\pi/\lambda$ where $\lambda$ is a wavelength of the light field. When one neglects non important diffraction effects in the $y$ direction then, according to the Fresnel approximation the wave front $U_o\left(\vec{r}\right)$ after propagation along the focal distance $f$ is transformed into the following field $U_1\left(\vec{r}_1\right)$:

$$\boldsymbol{U}_1\left(\vec{\boldsymbol{r}}_1\right)=\frac{\boldsymbol{d}}{\sqrt{2i\lambda f}}\exp(\boldsymbol{ikf})\exp\left(\frac{\boldsymbol{iky}_1^{\ 2}}{4\boldsymbol{f}}\right)\exp\left(\frac{\boldsymbol{ikx}_1^{\ 2}}{2\boldsymbol{f}}\right)\mathrm{sinc}\left(\frac{\boldsymbol{dx}_1}{\lambda\boldsymbol{f}}\right) \tag{6}$$

The above function is equal with an accuracy to a phase factor to the complex amplitude of the focal field formed by the properly oriented cylindrical lens of the width *d* and a focal length *f*. Therefore there is formed a narrow light strip stretched along the $\boldsymbol{OY}_1$ axis in the intensity focal image. Binary encoding of the wave front $\boldsymbol{U}_o\left(\vec{\boldsymbol{r}}\right)$ (Eq.(5)) by means of the function $\boldsymbol{T}_1(\Phi)$ (Eq.(3)) leads to so called the equilateral hyperbolic zone plate (EHZP)[5] (Fig.1a). EHZP generates other orders besides the wave front of interest $\boldsymbol{U}_o\left(\vec{\boldsymbol{r}}\right)$. Particularly -1 order ($\boldsymbol{n}=-1$) forms in the focal plane the field being the same as the wave front $\boldsymbol{U}_1\left(\vec{\boldsymbol{r}}_1\right)$ but additionally rotated by the right angle. Then the both ±1 orders reconstruct in the output focal plane the intensity pattern in a form of a light cross. Because the orders of interest are generated with the same initial phase then constructive interference occurs in the focal plane centre. The interference contribution of nonfocused other orders is negligible therefore the focal intensity distribution is distinguished by the light peak in the central part of the cross[5] (Fig. 1b).

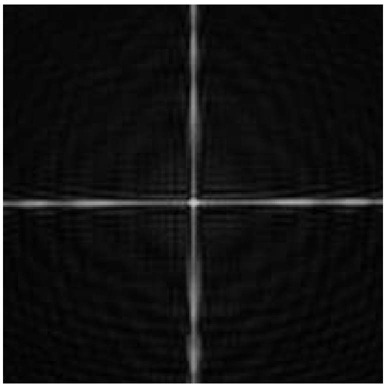

**Fig. 1a**

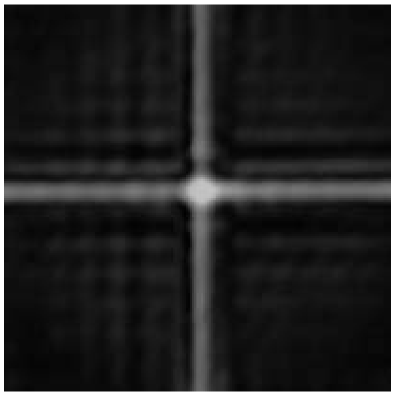

**Fig. 1b**

**Figure No.1** a Enlarged central part of the amplitude EHZP corresponding to the encoding function $\boldsymbol{T}_1(\Phi)$ described by Eq.(3) y Fig 1b Centre of a focal pattern of the amplitude EHZP. The experimental parameters were as follows: *f*=475 mm, *d*= 9mm, λ=0.6328 µm.

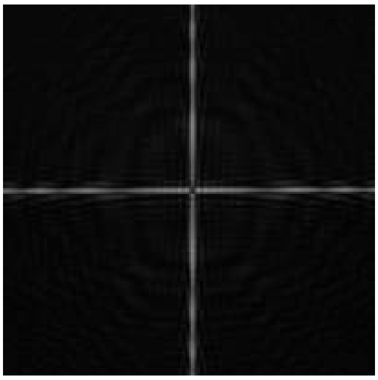

**Fig. 2a**

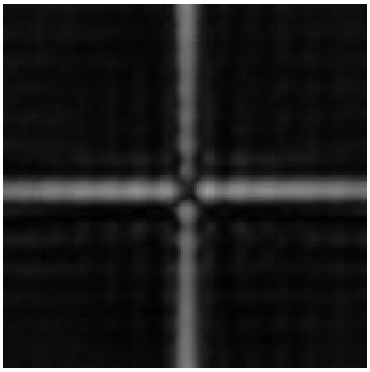

**Fig. 2b**

**Figure No. 2a** Enlarged central part of the amplitude MEHZP corresponding to the encoding function $\boldsymbol{T}_1\left(\Phi - \Phi_o\right)$, where $\Phi_o = \dfrac{\pi}{2}$ y Fig 2b Centre of a focal pattern of the amplitude MEHZP. The experimental parameters were the same as in the case of Fig. 1b.

## CONCLUSIONS

According to the last investigations, particularly promising for positioning purposes seem be diffractive elements generating light field with the central dark spot of possible smallest characteristic dimensions[1-2]. Such structures enable to measure displacements from the desired position by an infinite proportional change of the irradiance[2] in contrary to conventional elements with a central peak, where change of irradiance is close to one. Therefore when two alignment elements generate diffractive patterns of the same characteristic dimensions but the first element forms the central light peak and the second one creates the central dip then the latter structure is potentially better suited for positioning purposes. Fortunately described features of binary encoding of phase wave fronts makes possible the proper modification of EHZP. The proposed modification lies in introducing of a phase shift π between generated +1 and -1 diffractive orders. Due to Eq.(4) one can gain this effect when binary encoding of the wave front $U_o\left(\vec{r}\right)$ is performed according to the function $\boldsymbol{T}_1\left(\Phi - \Phi_o\right)$, where $\Phi_o = \pi/2$ (Fig. 2a). Destructive interference between +1 and -1 diffractive orders occurs in a central part of the focal plane of the modified equilateral hyperbolic zone plate (MEHZP). Then black spot appears in the centre of the output light cross (Fig.2b).

Besides this property the proposed MEHZP exhibits some additional features being potentially useful for alignment purposes:

(a) A cross-like diffractive pattern can be helpful in the three point method[6]. Light cross arms indicate reference axis so one can easier estimate the translation of detector or the

positioning element from a desired position. The diffractive structure forming the similar cross-like pattern is using successfully for the alignment of the Stanford linear accelerator[7].
(b) In a case of MEHZP an output focal image is formed by the both +1 and -1 diffractive orders simultaneously. Therefore in spite of the destructive interference in the centre of the pattern, the phase binary zone plate utilizes about 80% of the incident light energy for the cross-like image formation. Usually so high diffraction efficiency demands the application of the precise kinoform technique with at least four steps of the phase quantization[3,4].

(c) Earlier suggested diffractive positioning elements forming the central dip[1,2] are characterized by the symmetry in respect to their center. This symmetry is a crucial demand during the fabrication.

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