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Intense laser field effects on the linear and nonlinear intersubband optical properties of a semi-parabolic quantum well

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Abstract. By using the compact-density matrix approach, the effect of a nonresonant intense laser field on the linear and nonlinear optical absorptions based on intersubband transitions and the refractive index changes in an asymmetric semiconductor quantum well have been presented. Our results show that the peak position of the absorption coefficient is sensitive to intense laser field, the absorption maximum shifts towards lower energies for increasing intense laser field value. Also we observe as the intense laser field strength increases, the total refractive index change has been increased in magnitude and also shifted towards lower energies. The results indicate that linear and nonlinear optical properties of the low dimensional semiconductor heterostructures can be adjusted in a desired energy range by using intense laser field.

1 Introduction

There is currently a considerable interest in the study of the physical properties of the low-dimensional heterostructures such as quantum wells, wires and dots. In these systems the restriction on the motion of the charge carriers allows us to control the physical properties of the structures. The studies on these systems offer a wide range of potential applications in the development of semiconductor optoelectronics devices [\[1](#page-4-0)[–5\]](#page-4-1).

Recently, the linear and nonlinear optical absorptions based on intersubband transitions and the refractive index changes in semiconductor quantum wells have been presented $[6-11]$ $[6-11]$. The nonlinear effects in these low-dimensional quantum systems can be enhanced more dramatically over those in bulk materials due to the existence of a strong quantum-confinement effect. These nonlinear properties have the potential for device applications in far-infrared amplifiers [\[12\]](#page-4-4), photo detectors [\[13](#page-4-5)], and high-speed electro-optical modulators [\[14\]](#page-4-6). The nonlinear optical properties of the low-dimensional systems generally depend on the asymmetry of the confinement potential [\[15\]](#page-4-7). The second-order optical effects disappear in a symmetric quantum well structure, so finite second-order susceptibilities can only be observed by breaking the symmetry of the heterostructure and/or confinement potential. The tunable asymmetry of the potential, therefore, is expected to yield promising nonlinear optical properties. There are several applications in semiconductor heterojunction devices and in optical systems [\[16](#page-4-8)[,17](#page-4-9)]. Tong and Kiriushcheva have showed that the tunable asymmetry

can be used in reduction of noise in resonance tunneling devices and other devices [\[17\]](#page-4-9). It is obvious that the semi parabolic quantum well (SPQW) is an asymmetric system; therefore the nonlinear optical properties of the system are significantly enhanced [\[18](#page-4-10)[–21\]](#page-4-11). As is well known the effect of a high-frequency intense laser field (ILF) also leads to major modifications in the shape of the confining potential of the quantum well structure [\[22](#page-4-12)[–32\]](#page-4-13). Therefore in lowdimensional semiconductor heterostructures, the problem of estimating the effects of the ILF on the confining potential and corresponding bound states plays an important role in the optoelectronic device modeling. In this context, a considerable amount of work has been devoted to study the nonlinear optical properties of semiconductor nanostructures with different confinement potentials under the ILF [\[33](#page-4-14)[–36](#page-4-15)].

This work is concerned with the theoretical study of the effects of ILF on the linear, third-order nonlinear optical absorption and refraction index change in a SPQW which is given in Figures [1a](#page-1-0) and [1b](#page-1-0) for $\alpha_o = 0$ and $\alpha_o = 80$ Å, respectively. The paper is organized as follows. In Section 2 the theoretical framework is described. Section 3 is dedicated to the results and discussion, and finally, our conclusions are given in Section 4.

2 Theory

The method used in the present study is based on the nonpertubative theory used to describe the atomic behavior under intense, high-frequency laser field conditions [\[37](#page-4-16)[,38\]](#page-4-17). It starts the space-translated version of the semi-classical

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Fig. 1. The variation of the confinement potential profile and squared wave functions corresponding to the first and second energy levels as a function of the normalized position (\tilde{z} = L/a_B , where $a_B = \varepsilon_o \hbar^2 / m^* e^2$ is the effective Bohr radius) for two different laser dressing parameters: (a) $\alpha_o = 0$ and (b) $\alpha_o = 80$ Å.

Schrödinger equation for a particle moving under the combined forces of potential and the laser field derived by Kramers in the general context of quantum electrodynamics [\[39\]](#page-4-18). Within this approach the electron motion in the presence of an electromagnetic field can be described in an accelerated frame that oscillates in the phase to the field [\[37](#page-4-16)[,40\]](#page-4-19). For simplicity, we assume a monochromatic electromagnetic field with angular frequency Ω . For linear polarization and for wavelengths large enough in order the non-relativistic dipole approximation to be valid, the vector potential of the radiation field in laboratory frame reads $\mathbf{A}(t) = A_o \cos(\Omega t) \hat{\mathbf{e}}$, where $\hat{\mathbf{e}}$ is the unit vector. By applying the time-dependent translation $\mathbf{r} \to \mathbf{r} + \alpha(t)$ the semi-classical Schrödinger equation in the momentum gauge, describing the interaction dynamics in the laboratory frame of reference, was transformed by Kramers as follows [\[39\]](#page-4-18),

$$
-\frac{\hbar^2}{2m^*}\nabla^2\varphi(\mathbf{r},t) + V(\mathbf{r} + \alpha(t))\varphi(\mathbf{r},t) = i\hbar\frac{\partial\varphi(\mathbf{r},t)}{\partial t} \tag{1}
$$

where $V(\mathbf{r})$ is atomic binding potential, $V(\mathbf{r}+\alpha(t))$ is the 'dressed' potential energy and

$$
\vec{\alpha}(t) = \alpha_o \sin(\Omega t) \hat{e}, \quad \alpha_o = \frac{eA_o}{m^* c\Omega} \tag{2}
$$

represents the quiver motion of a classical electron in the laser field. In this approximation, the influence of the highfrequency laser field is entirely determined by the "dressed

potential" $V(\mathbf{r} + \alpha(t))$ [\[39](#page-4-18)],

$$
\alpha_o = (I_o^{1/2}/\Omega^2)(e/m^*)(8\pi/c)^{1/2}
$$
 (3)

where e and m^* are the absolute value of the electric charge and effective mass of an electron, c is the velocity of the light and A_0 is the amplitude of the vector potential and I_o is the intensity of ILF.

Following the Floquet approach [\[38](#page-4-17)[,39\]](#page-4-18), the spacetranslated version of the Schrödinger equation, equation (1) , can be cast in the equivalent form of a system of coupled time independent differential equations for the Floquet components of the wave function φ , containing the (in general complex) quasi-energy E . An iteration scheme was developed to solve this, for the zeroth Floquet component φ_o the system reduces to the following time-independent Schrödinger equation [\[38](#page-4-17)[–41\]](#page-4-20).

$$
\left(-\frac{\hbar^2}{2m^*}\nabla^2 + V(\mathbf{r}; \alpha_o)\right)\varphi_o = E\varphi_o \tag{4}
$$

where $V(\mathbf{r}; \alpha_o)$ is the 'dressed' confinement potential which depends on w and I_0 only trough α_0 [\[37\]](#page-4-16).

In the absence of the laser field, we introduce the functional form of a semi-parabolic confining potential $V(z)$ as,

$$
V(z) = V_o [\theta(z - L) + \theta(-z)] + \frac{V_o}{L^2} z^2 \Theta((L - z)(z))
$$
 (5)

where V_o is the conduction band offset at the interface, L is the well width, Θ is the Heaviside unit step function which is satisfies $\Theta(z)=1 - \theta(-z)$ and θ is the unit step function. We choose the z-axis along the growth direction. By applying the above described dressed potential theory to our particular SPQW system, we write down the timeindependent Schrödinger equation in one dimensional case for an electron inside a SPQW in the presence of an intense high-frequency laser field (the laser-field polarization is along the z direction), is given by

$$
-\frac{\hbar^2}{2m^*}\frac{\partial^2\psi(z)}{\partial z^2} + V(\alpha_o, z)\psi(z) = E\psi(z)
$$
 (6)

where $\psi(z)$ is the wave function, m^* is the effective mass and $V(\alpha_o, z)$ is the 'dressed' confinement potential which is given by the following expression;

$$
V(\alpha_o, z) = V_o [\theta (-z - \alpha_o) + \theta (z - \alpha_o - L)]
$$

+
$$
\frac{V_o}{L^2} \left(\frac{\alpha_o^2}{2} + z^2\right) \Theta ((z + \alpha_o) (L + \alpha_o - z))
$$

-
$$
\frac{V_0}{\pi L^2} \Theta (z + \alpha_o) \theta (\alpha_o - z) \left[\left(\frac{\alpha_o^2}{2} + z^2 - L^2\right) \times \arccos\left(\frac{z}{\alpha_o}\right) - \frac{3z}{2} \sqrt{\alpha_o^2 - z^2} \right]
$$

-
$$
\frac{V_o}{\pi L^2} \Theta (-z + \alpha_o + L) \theta (z + \alpha_o - L)
$$

$$
\times \left[\left(\frac{\alpha_o^2}{2} + z^2 - L^2\right) \arccos\left(\frac{L - z}{\alpha_o}\right) \times \left(\frac{3z + L}{2}\right) \sqrt{\alpha_o^2 - (L - z)^2} \right].
$$
 (7)

With the analytical expression for the dressed potential in equation [\(7\)](#page-1-2) which is valid for all values of α_o and all points of z, we obtain bound-state energy levels under intense laser field conditions. We should point out that for a square quantum well with width L ; a closed-form expression for the dressed potential valid for all $\alpha_o > 0$ is presented by Lima et al. [\[31\]](#page-4-21). But, they have restricted theirs attention to the points with $|z| < (L+\alpha_o)$ whereas, we derive here an analytical expression for the dressed potential (Eq. (7)) is valid for all z values.

To calculate the changes of the refractive index and absorption coefficients corresponding to the optical transitions between two subbands we have used the density matrix approach method $[42,43]$ $[42,43]$. The entire system is simultaneously irradiated by a nonresonant ILF with frequency Ω and a light field of frequency ω . We consider an optical radiation of angular frequency ω applied to the system with polarization along the growth direction z such as

$$
E(t) = E_o \cos(\omega t) = \tilde{E}e^{i\omega t} + \tilde{E}e^{-i\omega t}.
$$
 (8)

The electronic polarization $P(t)$ and susceptibility $\chi(t)$ caused by the optical field $E(t)$ can be defined through the dipole operator M , and the density matrix ρ as

$$
P(t) = \varepsilon_o \chi(\omega) \tilde{E} e^{-i\omega t} + \varepsilon_o \chi(-\omega) \tilde{E} e^{i\omega t}
$$

= $\frac{1}{V} \text{Tr}(\rho M)$ (9)

where V is the volume of the system, ε_o is the permittivity of the free space, and Tr stands for the trace. The susceptibility $\chi(\omega)$ is related to the change in the refractive index as follows [\[44\]](#page-4-24)

$$
\frac{\Delta n(\omega)}{n_r} = \text{Re}\left(\frac{\chi(\omega)}{2n_r^2}\right) \tag{10}
$$

where n_r is the refractive index of the system. By using the same density matrix formalism, the linear and third-order nonlinear refractive index changes are given as follows [\[11\]](#page-4-3)

$$
\frac{\Delta n^{(1)}(\omega)}{n_r} = \frac{1}{2n_r^2 \varepsilon_o} |M_{21}|^2 \sigma_V \left[\frac{E_{21} - \hbar \omega}{(E_{21} - \hbar \omega)^2 + (\hbar \Gamma_{12})^2} \right],
$$
\n(11)

$$
\frac{\Delta n^{(3)}(\omega)}{n_r} = -\frac{\mu c}{4n_r^3 \varepsilon_o} |M_{21}|^2 \frac{\sigma_V I}{\left[(E_{21} - \hbar \omega)^2 + (\hbar \Gamma_{12})^2 \right]^2} \times \left[4 \left(E_{21} - \hbar \omega \right) |M_{21}|^2 - \frac{\left(M_{22} - M_{11} \right)^2}{\left(E_{21} - \hbar \omega \right)^2 + (\hbar \Gamma_{12})^2} \left\{ (E_{21} - \hbar \omega) \right\} \times \left[E_{21} (E_{21} - \hbar \omega) + (\hbar \Gamma_{12})^2 \right] - (\hbar \Gamma_{12})^2 \left(2E_{21} - \hbar \omega \right) \right] \tag{12}
$$

where σ_V is the carrier density in the system, μ is the permeability of the system, $E_{ij} = E_j - E_i$ is the energy difference between two electronic states, M_{ij} = $|\langle \psi_i|e z|\psi_j\rangle |(i,j=1,2)$ is the matrix elements of the dipole

moment, Γ_{12} is the relaxation rate which is equals to the inverse relaxation time T_{12} , c is the speed of the light in free space and I is the incident optical intensity which is defined as $I = \frac{2n_r}{\mu c} |E(\omega)|^2$. The total change of the refractive index can be written as

$$
\frac{\Delta n(\omega)}{n_r} = \frac{\Delta n^{(1)}(\omega)}{n_r} + \frac{\Delta n^{(3)}(\omega)}{n_r}.
$$
 (13)

The linear and third-order nonlinear optical absorption coefficients are given as follows [\[11](#page-4-3)[,45](#page-4-25)]

$$
\alpha^{(1)}(\omega) = \omega \sqrt{\frac{\mu}{\varepsilon_R} \frac{|M_{21}|^2 \sigma_V \hbar T_{12}}{(E_{21} - \hbar \omega)^2 + (\hbar \Gamma_{12})^2}} \tag{14}
$$

$$
\alpha^{(3)}(\omega) = -\omega \sqrt{\frac{\mu}{\varepsilon_R} \left(\frac{1}{2\varepsilon_o n_r c} \right)}
$$

$$
\times \frac{|M_{21}|^2 \sigma_V \hbar \Gamma_{12}}{[(E_{21} - \hbar \omega)^2 + (\hbar \Gamma_{12})^2]^2} \left[4 |M_{21}|^2 - \frac{(M_{22} - M_{11})^2 [3E_{21}^2 - 4E_{21} \hbar \omega + \hbar^2(\omega^2 - \Gamma_{12}^2)]}{E_{21}^2 + (\hbar \Gamma_{12})^2} \right]
$$
(15)

where $\varepsilon_R = n_r^2 \varepsilon_o$ is the real part of the permittivity. In addition, the total optical absorption coefficient is given by

$$
\alpha(\omega) = \alpha^{(1)}(\omega) + \alpha^{(3)}(\omega). \tag{16}
$$

3 Results and discussions

 (11) $1/0.2$ ps⁻¹, $n_r = 3.2$, $\sigma_V = 3.0 \times 10^{16}$ cm⁻³, $\mu =$
 (11) $4\pi \times 10^{-7}$ H m⁻¹. We assume an uniform effective mass We have firstly solved numerically the Schrödinger equation to investigate intense laser field effects on the linear, third-order nonlinear optical absorption coefficients and refractive index changes in a semi-parabolic $GaAs/Al_xGa_{1-x}As quantum well with $L = 100 \text{ Å}$ width$ and finite barrier height $V_0 = 228$ meV as a prototype. We have used the following physical parameters which are suitable GaAs/Al_xGa_{1-x}As heterostructures [\[11](#page-4-3)]: Γ_{12} = $4\pi \times 10^{-7}$ H m⁻¹. We assume an uniform effective mass $m^* = 0.067m_o$ throughout the heterostructures, where m_o is the electron rest mass.

In Figure [2,](#page-3-0) we show the variation of the linear $\alpha^{(1)}(\omega)$, third-order nonlinear $\alpha^{(3)}(\omega)$ and total $\alpha(\omega)$ absorption coefficients as function of the incident photon energy $\hbar\omega$ in a semi-parabolic GaAs/Al_xGa_{1-x}As quantum well for an optical intensity of $I = 0.4$ MW/cm². This figure clearly shows that the peak position of the absorption coefficients are sensitive to intense laser field, the absorption maximum shifts towards lower energies for increasing laser dressing parameter α_o . Since the energy difference between the ground state and the first excited state $\Delta E = E_2 - E_1$ decreases as laser dressing parameter α_o increases. Then as α_o increases the peak position of the absorption coefficients decreases. Also it is seen that, the

Fig. 2. (Color online) The variations of the linear, third-order nonlinear and total absorption coefficients as a function of the photon energy for different values of the dressing parameter α_o .

Fig. 3. (Color online) The variations of the linear, third-order nonlinear and total refractive index changes as a function of the photon energy for $\alpha_o = 0$.

maximum value of the nonlinear and total absorption coefficients depend on intense laser field, such that with the increase of α_o , the nonlinear coefficient increases, but the linear absorption has small variations with α_o and hence the total absorption coefficient decreases with α_o since the large nonlinear variation is opposite to the sign of the linear variation.

In Figure [3,](#page-3-1) the linear, third-order nonlinear and the total refractive index change are plotted as a function of the incident photon energy $\hbar\omega$ for $\alpha_o = 0$. From this figure it is clearly seen that the large linear change contribution is the opposite in sign of the nonlinear change. Therefore, we observe a reduction in the value of the total refraction index change.

Figure [4](#page-3-2) shows the total refractive index change as a function of the incident photon energy $\hbar\omega$ for three different intense laser field values. We observe from this figure that the total refractive index change $\frac{\Delta n}{n_r}$ is very sensitive to the intense laser field. As the intense laser field strength increases, the total refractive index change has been in-

Fig. 4. (Color online) The variation of the total refractive index changes as a function of the photon energy for $\alpha_o = 0$, $\alpha_o = 40 \text{ Å}$ and $\alpha_o = 80 \text{ Å}.$

Fig. 5. The variations of the two lowest energy levels and intersubband transition energy ΔE as a function of the intense laser field.

creased in magnitude and also shifted towards lower energies. This is mainly due to the increasing the effective quantum well width as the intense laser field increases and as a result the energy difference ΔE decreases with α_o . In order to explain this behavior we give the variations of the ground state E_1 , first excited state E_2 and intersubband transition energy ΔE as a function of the intense laser field in Figure [4.](#page-3-2) The figure clearly indicates the tunability of the intersubband transitions by the applied intense laser field. Also we should point out that in conventional square wells the energy difference between the ground state and first excited state ΔE is blue-shifted with α_o especially for small laser field values, $\alpha_0 < 40$ Å, in contrast to the semi-parabolic well case. This tunability makes the structure an ideal candidate for both infrared and near infrared electro-absorption modulators and detectors. This figure also allows us to understand the variation of the optical absorption coefficients with intense laser field. These

results indicate that linear and nonlinear optical properties of the low dimensional semiconductor heterostructures can be adjusted in a desired energy range by using intense laser field.

4 Conclusions

In this work, by using the compact-density matrix approach, we investigated how the linear, third-order nonlinear optical absorption and refraction index change in a SPQW are affected by a nonresonant intense laser field. Our results show that the peak position of the absorption coefficient is sensitive to intense laser field, the absorption maximum shifts towards lower energies for increasing intense laser field value. Also we observe that the total refractive index change is very sensitive to the intense laser field. As the intense laser field strength increases, the total refractive index change has been increased in magnitude and also shifted towards lower energies. Moreover, we see that in SPQW case the intersubband transition energy decreases with α_o in contrast to conventional square QWs case. The model presented here for optical and electronic properties in a SPQW under nonresonant intense laser field also provides a theoretical foundation for further studies of optoelectronics under intense laser field conditions in other low dimensional semiconductor structures with different confining potentials.

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