The mirage of luminal modified gravitational-wave propagation

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Gravity-Matter coupling frame

Calculation frame	Jordan (JMC)	Einstein(EMC)
Jordan	$\mathcal{L}_{\rm JMC} = \sqrt{g_{\rm J}}\left[\Omega^2 R_{\rm J} + L_{\rm J}^{\rm MG} + L_{\rm J}^{\rm matter}(g_{\rm J})\right]$	$\mathcal{L}_{\rm EMC} = \sqrt{g_{\rm J}}\left[\Omega^2 R_{\rm J} + L_{\rm J}^{\rm MG} + L_{\rm J}^{\rm matter}(\Omega^2 g_{\rm J})\right]$
Einstein	$\mathcal{L}_{\rm JMC} = \sqrt{g_{\rm E}}\left[R_{\rm E} + L_{\rm E}^{\rm MG} + L_{\rm E}^{\rm matter}(\Omega^{-2} g_{\rm E})\right]$	$\mathcal{L}_{\rm EMC} = \sqrt{g_{\rm E}}\left[R_{\rm E} + L_{\rm E}^{\rm MG} + L_{\rm E}^{\rm matter}(g_{\rm E})\right]$

A modified gravity theory is defined by two ingredients

- Free Lagrangian of the metric and the additional MGT degrees of freedom
- Interaction Lagrangian with matter coupling
- 2 Different families for each free Lagrangian: JMC and EMC
- The interaction Lagrangian affects the source term for Gws
- Ω is the real effective Planck mass, not $\mathbf{M_*}=\Omega/c_T$

General Relativity GW propagation in Jordan and Einstein frame :
mirage of modified luminal propagation

Einstein | Jordan

$$g_{\rm E} = \Omega^2 g_{\rm J} \ , \ a_{\rm E} = \Omega\, a_{\rm J}$$

$$\mathcal{L}_{\rm GR} = a_{\rm E}^2 \left[h'^2 - (\nabla h)^2 \right]$$

$$\mathcal{L}_{\rm GR} = \Omega^2 a_{\rm J}^2 \left[h'^2 - (\nabla h)^2 \right]$$

$$h'' + 2\mathcal{H}_{\rm E} h' - \nabla^2 h = 0$$

$$h'' + 2\mathcal{H}_{\rm J}\left(1 - \frac{\Omega'}{\mathcal{H}_{\rm J}\Omega}\right) h' - \nabla^2 h = 0$$

$$\mathcal{H}_{\rm J}\left(1 - \frac{\Omega'}{\mathcal{H}_{\rm J}\Omega}\right) = \mathcal{H}_{\rm E}$$

$$\mathcal{H}_{\rm J} = a'_{\rm J} / a_{\rm J}$$

- Gws are conformally invariant, but the friction term-scale factor relation no
- The friction term is the same function of space and time in the two frames
- The scale factor-friction relation is not in itself and indicator of gravity modification

Redshift scale factor relation

$$\begin{aligned} P_\mu P^\mu &= P^\mu P^\nu g^{\mathrm{J}}_{\mu\nu} \\ &= E^2 - \delta_{ij} p^i p^j a_{\mathrm{J}}^2 \\ &= E^2 - \delta_{ij} p^i p^j \Omega^{-2} a_{\mathrm{E}}^2 \end{aligned} \longrightarrow \quad E = p\, a_{\mathrm{J}} = p\, a_{\mathrm{E}} \Omega^{-1}$$

$$(1+z) = \frac{E(z)}{E(0)} = \left[\frac{a(0)}{a(z)}\right]_{\mathrm{J}} = \frac{\Omega(z)}{\Omega(0)} \left[\frac{a(0)}{a(z)}\right]_{\mathrm{E}}$$

Einstein frame formulation of generic MGTs

- There is no physical frame: any theory can be studied in any frame, without any observable consequence
- The interaction between GWs and the detectors is currently computed assuming minimal coupling between matter and metric, since detectors were built to test GR
- Local observational constraints on gravity imply that $\Omega(z=0)\approx 1$, but at higher redshift the non minimal coupling of matter to the Einstein frame metric can be tested to constrain $\Omega(z)$ with other observables
- The Friedman equation is modified, due to the time varying effective Planck mass, so the scale factor evolution can be also modified, in particular $a(z)$, but the from of the GW propagation equation is the same that in GR if $c=c_T$, because the gravity modification is just an effective DE fluid in the Einstein frame

Effective field theory confirmation

Jordan Frame — Einstein frame

1210.0201, 1304.4840 — 1407.8439, 1705.09290

$$\mathcal{L}^{\rm eff}_{\rm DE} = \sqrt{g_{\rm J}}\left[\Omega^2 R_{\rm J} + L^{(2)}_{\rm J}\right] \qquad \mathcal{L}^{\rm eff}_{\rm DE} = \sqrt{g_{\rm E}}\left[R_{\rm E} + L^{(2)}_{\rm E}\right]$$

$$\mathcal{L}^{\rm eff}_{h} = \frac{a^2_{\rm J}\Omega^2}{c^2_{\rm T}}\left[h'^2 - c^2_{\rm T}(\nabla h)^2\right] \qquad \mathcal{L}^{\rm eff}_{h} = \frac{a^2_{\rm E}}{c^2_{\rm T}}\left[h'^2 - c^2_{\rm T}(\nabla h)^2\right]$$

Propagation equation

Jordan Frame Einstein frame

$$h'' + 2\mathcal{H}_J\left(1 - \frac{c_T'}{\mathcal{H}_J c_T} - \frac{\Omega'}{\mathcal{H}_J\Omega}\right)h' - c_T^2\nabla^2 h = 0 \qquad h'' + 2\left(\frac{a_E'}{a_E} - \frac{c_T'}{c_T}\right)h' - c_T^2\nabla^2 h = 0$$

$$h'' + 2\frac{\alpha'}{\alpha}h' - c_T^2\nabla^2 h = 0$$

$$\alpha = \frac{a_E}{c_T} = \frac{\Omega a_J}{c_T} \qquad \mathcal{H}_J\left(1 - \frac{\Omega'}{\mathcal{H}_J\Omega}\right) = \mathcal{H}_E$$

GW and EM luminosity distance

$$h = \chi/\alpha \longrightarrow \chi_k'' + \left(c_{\rm T} k^2 - \frac{\alpha''}{\alpha}\right)\chi_k = 0 \longrightarrow h_k \propto \frac{1}{\alpha}$$

For GW emitted by binary $$h_k \propto \frac{1}{\alpha\, r} \propto \frac{1}{d_{\rm L}^{\rm GW}}$$

For EM wave $$d_{\rm L}^{\rm EM} = r(1+z)$$

The matter-gravity coupling has no effect on GW propagation in vacuo

$$\alpha = \frac{a_{\rm E}}{c_{\rm T}} = \frac{\Omega a_{\rm J}}{c_{\rm T}} \longrightarrow d_{\rm L}^{\rm GW}|_{\rm E} = d_{\rm L}^{\rm GW}|_{\rm J} = r\,\frac{\alpha(\eta_o)}{\alpha(\eta_s)} = \frac{c_T(\eta_s)}{c_T(\eta_o)} d_{\rm L}^{GR}$$

Observable relations derived using EFT

	GR	MGT - Einstein frame	MGT - Jordan frame
Scale factor	$a_{\rm E}$	$a_{\rm E}$	$a_{\rm J} = (M_* c_{\rm T})^{-1} a_{\rm E}$
Coupling	$g_{\rm E}$	$g_{\rm E}$	$g_{\rm J}$
$(1+z)$	$\frac{a_{\rm E}(0)}{a_{\rm E}(z)}$	$\frac{a_{\rm E}(0)}{a_{\rm E}(z)}$	$\frac{a_{\rm J}(0)}{a_{\rm J}(z)} = \frac{M_*(z) c_{\rm T}(z) a_{\rm E}(0)}{M_*(0) c_{\rm T}(0) a_{\rm E}(z)}$
$d_L^{\rm GW}$	$d_{\rm GR}^{\rm GW} = r \frac{a_{\rm E}(0)}{a_{\rm E}(z)}$	$d_{\rm MGT}^{\rm GW} = \frac{c_{\rm T}(z)}{c_{\rm T}(0)} d_{\rm GR}^{\rm GW}$	$d_{\rm MGT}^{\rm GW} = \frac{c_{\rm T}(z)}{c_{\rm T}(0)} d_{\rm GR}^{\rm GW}$
$d_L^{\rm EM}$	$d_{\rm GR}^{\rm EM} = r \frac{a_{\rm E}(0)}{a_{\rm E}(z)}$	$d_{\rm MGT}^{\rm EM} = d_{\rm GR}^{\rm EM}$	$d_{\rm MGT}^{\rm EM} = r \frac{a_{\rm J}(0)}{a_{\rm J}(z)} = \frac{M_*(z) c_{\rm T}(z)}{M_*(0) c_{\rm T}(0)} d_{\rm GR}^{\rm EM}$
$\frac{d_{\rm L}^{\rm GW}}{d_{\rm L}^{\rm EM}}$	1	$\frac{c_{\rm T}(z)}{c_{\rm T}(0)}$	$\frac{M_*(0)}{M_*(z)}$

Friction term and dark energy relation in JMC theories

- In the Jordan frame the friction term is not independent from the effective
dark energy associated to the MGT, but the relation is theory dependent
- Other cosmological observations constrain DE, and consequently the friction, so in general LCDM parametrization cannot be used independently of friction
- Let's consider the example of Horndeski theories, 2210.12174

$$H^2 = \rho_{\rm m} + \rho_{\rm DE} \qquad h''_A + [2 + \alpha_M(t)]\,\mathcal{H} h'_A + k^2 c_T^2 h_A = 0, \qquad \alpha_M \equiv \frac{d\ln(M_*/M_P)^2}{d\ln a} = \frac{2}{\mathcal{H}}\frac{M'_*}{M_*}$$

$$\rho_{\rm DE} \equiv -\frac{1}{3}G_2 + \frac{2}{3}X\left(G_{2X} - G_{3\phi}\right) - \frac{2H^3\phi' X}{3a}\left(7G_{5X} + 4XG_{5XX}\right) + H^2\left[1 - (1-\alpha_{\rm B})M_*^2 - 4X\left(G_{4X} - G_{5\phi}\right) - 4X^2\left(2G_{4XX} - G_{5\phi X}\right)\right]$$

$$M_*^2 \equiv 2\left(G_4 - 2XG_{4X} + XG_{5\phi} - \frac{H\phi'}{a}XG_{5X}\right)$$

$$p_{\rm DE} \equiv \frac{1}{3}G_2 - \frac{2}{3}X\left(G_{3\phi} - 2G_{4\phi\phi}\right) + \frac{4H\phi'}{3a}\left(G_{4\phi} - 2XG_{4\phi X} + XG_{5\phi\phi}\right) - \frac{(\phi'' - aH\phi')}{3\phi' a}HM_*^2\alpha_{\rm B} - \frac{4}{3}H^2X^2G_{5\phi X} - \left(H^2 + \frac{2H'}{3a}\right)\left(1 - M_*^2\right) + \frac{2H^3\phi' XG_{5X}}{3a},$$

Jordan frame "Friedman equation" caveats

Jordan frame Friedman eq.

$$3M_*^2\mathcal{H}^2 = a^2\left(\rho_{\rm m} + \hat{\mathcal{E}}_{\rm S}\right)$$

$$M_*^2\left(2\mathcal{H}' + \mathcal{H}^2\right) = -a^2\left(p_{\rm m} + \hat{\mathcal{P}}_{\rm S}\right)$$

$$\hat{\mathcal{E}}_{\rm S}' + 3\mathcal{H}\left(\hat{\mathcal{E}}_{\rm S} + \hat{\mathcal{P}}_{\rm S}\right) = \left(\rho_{\rm m} + \hat{\mathcal{E}}_{\rm S}\right)\mathcal{H}\alpha_{\rm M}$$

$$\rho_{\rm m}' + 3\mathcal{H}\left(\rho_{\rm m} + p_{\rm m}\right) = 0\,,$$

$$\hat{\mathcal{E}}_{\rm S} \equiv \mathcal{E}_{\rm S} + 3\left(M_*^2 - M_{\rm Pl}^2\right)\frac{\mathcal{H}^2}{a^2}$$

$$\hat{\mathcal{P}}_{\rm S} \equiv \mathcal{P}_{\rm S} - \frac{1}{a^2}\left(M_*^2 - M_{\rm Pl}^2\right)\left(2\mathcal{H}' + 2\mathcal{H}^2\right)$$

Effective Jordan frame Friedman eq.

$$3M_{\rm Pl}^2\mathcal{H}^2 = a^2\left(\rho_{\rm m} + \mathcal{E}_{\rm S}\right)$$

$$M_{\rm Pl}^2\left(2\mathcal{H}' + \mathcal{H}^2\right) = -a^2\left(p_{\rm m} + \mathcal{P}_{\rm S}\right)$$

$$\rho_{\rm m}' + 3\mathcal{H}\left(\rho_{\rm m} + p_{\rm m}\right) = 0$$

$$\mathcal{E}_{\rm S}' + 3\mathcal{H}\left(\mathcal{E}_{\rm S} + \mathcal{P}_{\rm S}\right) = 0\,,$$

M_* is hidden inside the effective DE fluid to facilitate the modification of existing LCDM numerical codes, 2210.12174, 1605.06102
This, and other common functions,
makes the friction and DE interdependent

GW propagation in General Relativity

Field equations: $$G_{\mu\nu} = T^{DE}_{\mu\nu} + T^{mat}_{\mu\nu} = T^{eff}_{\mu\nu} \; ;$$

Linearized l.h.s $$h''_A + 2\mathcal{H}h'_A + \nabla^2 h_A = a^2 \Pi^{eff}_A$$

r.h.s. Is not expanded, it just comes from **Einstein's tensor structure** (TT tensor part), and is valid ad any order, can **include higher order effects** neglected by the quadratic action for h, at least **cubic** action is required to obtain the **space/polarization** dependent anisotropic **source term**

Effective action and metric encoding higher order interaction

The effective speed is Frequency/Polarization dependence

2211.05760,2301.05679

$$\frac{a^2}{c_{T,A}^2}\left[h_A'^2 - c_{T,A}^2(\nabla h_A)^2\right] = \frac{\Omega^2\tilde{a}^2}{c_{T,A}^2}\left[h_A'^2 - c_{T,A}^2(\nabla h_A)^2\right]$$

$$ds_A^2 = a^2\left[c_{T,A}d\eta^2 - \frac{\delta_{ij}}{c_{T,A}}dx^i dx^j\right]$$

Higher order effects

Matter coupling | Distance ratio

Jordan

$$d_{L,A}^{GW}(z) = \frac{\tilde{M}_A(0,k)}{\tilde{M}_A(z,k)} d_L^{EM}(z)$$

Einstein

$$\frac{d_L^{GW}}{d_L^{EM}}(z) = \frac{a(z)}{\tilde{\alpha}_A(z)} \frac{\tilde{\alpha}_A(0)}{a(0)} = \frac{\tilde{c}_{T,A}(z,k)}{\tilde{c}_{T,A}(0,k)}$$

Conclusions

- GWs propagating in the vacuo at the speed of light follows GR
- Any apparent modification of the friction term can be removed by moving to the Einstein frame
- For luminal MGTs the modification of the GW-EMW distance ratio is due to the non minimal matter-gravity coupling, and consequent change in scale factor-redshift relation, not to the modification of the GW propagation
- The calculation frame and the matter-gravity frame should be clearly distinguished
- Observable quantities should be used to test MGTs, while the friction term of the GW propagation equation is not invariant under conformal transformations, and can be miss-leading in defining the effects of the modification of gravity
- MGTs with matter coupled to the Einstein frame are an independent (neglected) family with a distinct observational signatures

Difference between ‘calculation’ and matter-gravity coupling frames

- Matter follows the geodesics of the metric to which is minimally coupled

- For Jordan metric matter-gravity coupling matter follows the Jordan metric geodesics, while in the Einstein frame a ‘fifth’ force determines a deviation from the Einstein metric geodesics, but the two descriptions are completely equivalent, at least classically.

- Any MGT with matter coupled to the Jordan metric can be studied in the Einstein frame introducing a non minimal coupling to the Einstein metric

- The Einstein frame formulation of MGTs it allows to apply the results obtained in relativistic cosmological perturbations theory, encoding the effects of the gravity modification in the effective energy-stress-tensor (EST), leading to new source terms in the perturbations equations

- The calculation and matter-gravity coupling metrics should be distinguished

EFT GW propagation equation

- EFT

$$h'' + 2\mathcal{H}_\mathrm{J}\left(1 - \frac{c_\mathrm{T}'}{\mathcal{H}_\mathrm{J} c_\mathrm{T}} - \frac{\Omega'}{\mathcal{H}_\mathrm{J}\Omega}\right)h' - c_\mathrm{T}^2\nabla^2 h = 0$$

- EFT c=1

$$h'' + 2\mathcal{H}_\mathrm{J}\left(1 - \frac{\Omega'}{\mathcal{H}_\mathrm{J}\Omega}\right)h' - \nabla^2 h = 0$$

- Lagrangian

$$\mathcal{L}_\mathrm{GR} = a_\mathrm{J}^2\Omega^2\left[h'^2 - (\nabla h)^2\right] = a_\mathrm{E}\left[h'^2 - (\nabla h)^2\right]$$

- The EFT confirms the model independent argument

What defines General Relativity and its modifications?

- Einstein's equations are derived from the Hilbert action + matter action
- In GR the metric is minimally coupled to matter fields
- The matter lagrangian determines the form of the energy-stress-tensor (EST)
- The Hilbert action is the free gravity action, describing gravity in the vacuum, i.e. in absence of interaction of the graviton with other fields
- In MGTs there are additional fields, which in the Einstein frame can be considered additional matter fields, changing the definition of vacuum
- Depending on the MGT matter can be coupled to the Einstein or Jordan frame
- The effects of MGTs on GW propagation in the Einstein frame are due to the interaction of these fields with the graviton.

$$\mathcal{L}^{\rm eff}_{\rm DE} = \sqrt{g_{\rm J}}\left[\Omega^2 R_{\rm J} + L^{(2)}_{\rm J}\right] \qquad \mathcal{L}^{\rm eff}_{\rm DE} = \sqrt{g_{\rm E}}\left[R_{\rm E} + L^{(2)}_{\rm E}\right]$$