Macroeconomic Implications of the Underground Sector: Challenging the Double Business Cycle Approach

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Abstract: Within the literature on business cycles featuring underground activities, there is an approach based on the arguable premise that these are countercyclical. This paper develops a real business cycle model without such an assumption. Preferences are additively separable in formal and underground labor. Further, leisure time is spent on irregular work and non-market activities. Simulations permit examining how the model performs and comparing the results with related findings. Also, computational experiments allow analyzing the effects of taxes, enforcement and tastes for underground labor on aggregate fluctuations. These experiments offer a comprehensive view of the cyclical implications of the shadow economy.

I. INTRODUCTION

There is no consensus regarding how the unofficial sector interacts with the official one over the business cycle. Several studies have estimated time series of underground output and ascertained their comovements with GDP using a variety of methods (e.g., currency demand, electricity use, MIMIC). Following this approach, Bajada (2003) and Giles (1997) provide evidence of a procyclical relation between the two sectors in Australia during the period 1967-95 and New Zealand in 1968-94, respectively. In contrast, Russo (2008) finds that the cyclical component of the US GDP is negativaely correlated with the cyclical component of the hidden output through 1960-2003, suggesting the existence of a 'double business cycle'

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wherein peaks of the regular economy coincide with troughs of the irregular one and vice versa.

Unlike this empirical literature, some business cycle models addressing shadow economic activities rely on the premise that business cycles in the official and unofficial sectors are negatively correlated. These models, mostly represented by Busato and Chiarini (2004) and Russo (2008), highlight a process of inter-sectoral reallocation of labor and production in response to aggregate productivity shocks and tax disturbances. Particularly, Busato and Chiarini (2004) claim this process can resolve some unsatisfactory results concerning the labor market generated by conventional real business cycle models without informality, such as the employment variability puzzle.[2] These studies, however, do not systematically assess the effects of variations in the extent of the underground economy on aggregate fluctuations.

In view of the lack of unambiguous evidence on the cyclicality of the unofficial economy, it is deemed there are no grounds for taking such double business cycle for granted in the development of equilibrium models of aggregate fluctuations. The present paper thus challenges this notion by precluding a countercyclical relation between regular and irregular work effort. That way, the model economy is allowed to deliver its own pattern of comovement between the two sectors, instead of a particular one *ex ante* imposed. The model economy, moreover, may provide comparable results regarding the cyclical behavior of major aggregates such as output, consumption, hours and labor productivity, hence shedding further light on the implications of underground activities for macroeconomic performance.

In line with the criticism above, the model developed in this paper relies to a large extent on that of Busato and Chiarini (2004). The characterization of preferences, however, is its main distinguishing aspect, so that official and unofficial labor are additively separable. Furthermore, leisure time is spent on both irregular work effort and non-market activities. Given these characteristics, empirical evidence on the elasticity of labor supply in the underground sector is used to calibrate the model. Then, simulations are conducted to examine how the model economy reacts to technology and fiscal policy shocks. The model improves significantly on the previous study as far as the comovements of productivity are concerned. Moreover, the unofficial sector turns out to be weakly countercyclical, thus reinforcing the relevance of the present approach.

The paper is organized as follows. Section II develops a model incorporating the aforementioned distinguishing elements, while selection and estimation of its parameters are described in section III. Then, section IV examines the model's ability to reproduce basic facts about U.S. business cycles. Sensitivity analyses are further applied to consider the potential effects of different enforcement structures, tax systems and tastes for irregular labor on the decision to divert resources underground and on macroeconomic fluctuations. This exercise allows one to contrast the moments obtained from the simulations with estimated cross-country correlations of business cycle stylized facts and the extent of the underground sector, and to make inferences about the determinants and aggregate implications of the shadow economy. Section V concludes.

[2] The employment variability puzzle refers to the fact that employment (or total hours worked) is almost as variable as output and strictly procyclical, something difficult to replicate in a standard neoclassical model.

II. THE MODEL ECONOMY

The present model relies to a large extent on that of Busato and Chiarini (2004). Accordingly, there are two sectors in the economy: official and unofficial. The official or regular sector produces everything that is measured in GDP strictly following all the laws and regulations in place. The unofficial or irregular sector, conversely, comprises the production of legal goods by legally registered firms, but either it is not reported to the tax authorities or it is conducted with workers that are not legally registered.

The economy is populated by the government, a large number of identical firms, and a large number of identical households, all of whom are infinitely-lived. The government uses tax revenues to finance a stochastic stream of expenditures and enforces a monitoring system for tax evasion. Firms solve an expected profit maximization problem every period, subject to a technological constraint and to the possibility of being discovered and penalized for producing 'off the books'. Households choose consumption, investment and hours to work on each date and in each sector.

2.1 Firms

Competitive firms in this economy purchase capital and labor services from households to produce a homogeneous good. The representative firm uses two different technologies: one associated with the official sector, and the other with the underground sector. Let y_t^f denote formally-produced output, and y_t^u output produced in the shadows. Technologies are specified as follows:

$$y_t^f = z_t^f k_t^\alpha (l_t^f)^{1-\alpha} \tag{1}$$

and

$$y_t^u = z_t^u l_t^u . \tag{2}$$

Formal output y_t^f is the result of a Cobb-Douglas technology applied to capital, k_t, and regular labor, l_t^f. Given that shadow economic activities are labor intensive, y_t^u is produced using solely underground labor, l_t^u. This amounts to assuming that irregular production has a fixed stock of capital. Finally, z_t^f and z_t^u are sectoral productivity shocks.[3] Total production is defined as $y_t^{tot} = y_t^f + y_t^u$.

Revenues accrued in the official sector, $q_t^f(1-\tau_t^f)y_t^f$, are taxed at the stochastic rate τ_t^f. The representative firm does not pay taxes on revenues accrued unofficially, $q_t^u y_t^u$, where q_t^u is the price of commodities produced off the books. However, the firm may be discovered evading and forced to pay corporate taxes, augmented by a surcharge factor $\varsigma \geq 1$. Note that since the officially-produced and the unofficially-produced goods are identical, they must have the same price in equilibrium $q_t^f = q_t^u \equiv q_t$. For simplicity, this price is imposed along the solution and normalized to unity.

3 Note that the latter shock may be seen as representing several inputs such as managerial skills, creativity, workplace organization, etc. These elements exist in the irregular sector just as they do in regular economic activities, and are capable of rising the corresponding labor productivity.

If the firm is discovered, with probability $\phi \in (0,1)$, revenues are $y_t^D = (1-\tau_t^f)y_t^f + (1-\varsigma\tau_t^f)y_t^u$. With probability $1-\phi$, the firm is not discovered, in which case revenues equal $y_t^{ND} = (1-\tau_t^f)y_t^f + y_t^u$. Thus, total expected revenues at time t are

$$E(y_t) = (1-\tau_t^f)y_t^f + (1-\varphi\varsigma\tau_t^f)y_t^u. \tag{3}$$

The cost of renting capital equals its marginal productivity, r_t. Formal labor cost is represented by the wage paid for hours worked in the official sector, augmented by the fixed payroll tax rate τ^s. Let $w_t^f \equiv (1+\tau^s)w_t$ define such a cost, where w_t is the pre-tax wage. Since the representative firm does not pay social security contributions on hours hired 'under the table', the cost of unofficial labor equals the pre-tax wage, i.e. $w_t^u = w_t$. As with the corporate tax rate, however, the firm faces the probability ϕ of being detected evading and forced to pay payroll taxes, increased by the surcharge factor ς. If the firm is caught dodging either of the two tax liabilities, then it is penalized on both payments.

With probability ϕ, the firm is discovered and total costs are $C_t^D = w_t^f l_t^f + (1+\varsigma\tau^s)w_t l_t^u + r_t k_t$. If the firm is not discovered, with probability $1-\phi$, total costs equal $C_t^{ND} = w_t^f l_t^f + w_t^u l_t^u + r_t k_t$. Thus, total expected costs at time t are

$$E(C_t) = w_t^f l_t^f + (1+\varphi\varsigma\tau^s)w_t l_t^u + r_t k_t. \tag{4}$$

The representative firm produces so as to maximize expected profits $E(y_t) - E(C_t)$ each period, taking market prices as given. Firm's behavior is characterized by the first order conditions:

$$r_t = (1-\tau_t^f)\alpha z_t^f k_t^{\alpha-1}(l_t^f)^{1-\alpha}, \tag{5}$$

$$w_t^f = (1-\tau_t^f)(1-\alpha)z_t^f k_t^{\alpha}(l_t^f)^{-\alpha} \tag{6}$$

and

$$w_t = \frac{1-\varphi\varsigma\tau_t^f}{1+\varphi\varsigma\tau^s}z_t^u. \tag{7}$$

These conditions imply that capital and both regular and irregular labor are paid their real marginal products. Setting equations (6) and (7) equal, and taking account of the definitions of w_t^f and w_t^u above, the following arbitrage condition is obtained:

$$\frac{1-\tau_t^f}{1+\tau^s}(1-\alpha)z_t^f k_t^{\alpha}(l_t^f)^{-\alpha} = \frac{1-\varphi\varsigma\tau_t^f}{1+\varphi\varsigma\tau^s}z_t^u. \tag{8}$$

This condition underlines that the firm equates the marginal products of labor across the two sectors each period, taking into consideration that the regular sector pays taxes while the irregular sector aims to escape taxation.

2.2 Households

The representative household chooses consumption and hours to work on each date and in each sector to maximize the present discounted value of utility $E_0\sum_t \beta^t U(c_t, l_t^f, l_t^u)$. Household behavior is represented by adapting Cho and Cooley's (1994) motivation for modeling family labor supply. According to these authors, households make labor supply decisions along both the intensive (hours worked) and the extensive (employment) margins. Here these two dimensions are reinterpreted as representing households' labor supply in the regular and underground sectors. The momentary utility function is assumed to be separable between consumption and leisure (labor) as follows:

$$U(c_t, l_t^f, l_t^u) = \ln c_t - a\frac{(l_t^f)^{1+\gamma}}{1+\gamma} - b\frac{(l_t^u)^{1+\eta}}{1+\eta}. \tag{9}$$

A well-behaved utility function implies that $\alpha, b \geq 0$, $\gamma, \eta > 0$, and that all its components be twice continuously differentiable and increasing. The second term, $a\frac{(l_t^f)^{1+\gamma}}{1+\gamma}$, represents the disutility of working in the formal sector, while the last term, $b\frac{(l_t^u)^{1+\eta}}{1+\eta}$, reflects the idiosyncratic cost of working in the underground economy. This cost may be associated to the lack of any social insurance in this sector.[4] Alternatively, one could interpret the inverse of the elasticity η as positively related to the disutility of working in the shadows. Note that implicit in the choice of this functional form is the absence of adjustment costs for moving labor supply (demand) across sectors, so that labor reallocation is almost completely unconstrained.

Households pay a stochastic tax rate τ_t^w on official labor income and receive a lump-sum transfer T_t from the government. Thus, they face the budget constraint:

$$c_t + i_t = (1-\tau_t^w)w_t l_t^f + w_t l_t^u + r_t k_t + T_t, \tag{10}$$

where i_t denotes investment at time t.[5] Investment, in turn, increases the capital stock according to the state equation:

$$i_t = k_t - (1-\delta)k_{t-1}, \tag{11}$$

where δ denotes the depreciation rate.

With the instantaneous utility function defined as in equation (9), the value function $V(k_t, K_t, A_t)$ of the representative household satisfies

$$V(k_t, K_t, A_t) = \max_{k_{t+1}, l_t^f, l_t^u} \left\{U(c_t, l_t^f, l_t^u) + \beta E_t\left[V(k_{t+1}, K_{t+1}, A_{t+1})\right]\right\},$$

subject to the budget constraint (10) and the law of motion for the capital stock (11). As specified below, A_t is a vector of technology and fiscal policy shocks. Household decisions are characterized by the intra-temporal conditions for labor supply allocation:

$$a(l_t^f)^{\gamma} = \frac{1-\tau_t^w}{c_t}\frac{1-\tau_t^f}{1+\tau^s}(1-\alpha)z_t^f k_t^{\alpha}(l_t^f)^{-\alpha} \tag{12}$$

and

4 Importantly, each sector has its own particular characteristics, which are aimed to capture with regard to household's behavior through the per-period utility function (9).

5 Note that households are paid the pre-tax wage for working in both sectors. Since they are subject to taxation only on official earnings, this implies that the regular wage is lower than the irregular one. This is not an implausible assumption, as Lemieux *et al*. (1994) show using micro data from a survey conducted in Québec City (Canada). These authors develop a model of time allocation supporting their empirical observations. In the context of the present model, households presumably are willing to receive a higher remuneration for not having social insurance in the underground economy.

$$b\,(l_t^u)^{\eta} = \frac{1-\varphi\varsigma\tau_t^f}{1+\varphi\varsigma\tau^s}\frac{z_t^u}{c_t}, \tag{13}$$

and by the Euler equation:

$$1 = \beta E_t\Big(\frac{c_t}{c_{t+1}}\Big)(1 + r_{t+1} - \delta), \tag{14}$$

where $r_t \equiv (1-\tau_t^f)\alpha z_t^f K_t^{\alpha-1}(l_t^f)^{1-\alpha}$ from firm profit maximization (see Equation 5).

2.3 Government

The government produces non-productive services and makes transfer payments each period by collecting taxes on firms' revenues and labor earnings. Government consumption is assumed to follow a stochastic process given by

$$g_t = z_t^g y_t^{tot}, \tag{15}$$

where z_t^g is a random variable. The flow budget constraint is

$$g_t + T_t = \tau_t^f y_t^f + \varphi\varsigma\tau_t^f y_t^u + (\tau^s + \tau_t^w)\,w_t l_t^f + \varphi\varsigma\tau^s w_t l_t^u. \tag{16}$$

Note that the lump-sum transfer T_t is treated as a residual that takes on whatever value is necessary to satisfy the government budget constraint at each point in time, given z_t^g, the productivity shocks and the tax disturbances.

In order to discourage fiscal evasion, the government enforces a monitoring system whereby firms are inspected each period with a fixed probability ϕ and forced to pay contributions to social insurance and the corporate tax rate on the previously concealed activities. The government is assumed to be always able to perfectly identify the amount of underground production and labor at every inspection or, equivalently, that the cost of verifying the amount of hidden production/labor is zero, so that all the proceeds from the taxation of these activities and the fines are effectively revenue for the government.[6] Given that taxes on irregular production/labor are collected only with a certain probability, the government balances its budget in expectation.

2.4 Sources of Aggregate Fluctuations

Sectoral productivity and fiscal policy shocks are formalized as a vector of exogenous state variables that follows an autoregressive process around a mean in logs:

$$A_t = PA_{t-1} + \varepsilon_t, \tag{17}$$

where A_t is a vector $[\ln(z_t^f/z_{ss}^f), \ln(z_t^u/z_{ss}^u), \ln(\tau_t^f/\tau_{ss}^f), \ln(\tau_t^w/\tau_{ss}^w), \ln(z_t^g/z_{ss}^g)]^T$ containing the ratio of the time t value of each state variable (i.e. productivity shocks, stochastic tax rates, and shock on government expenditures) to their steady state values. Likewise, $P = diag(\rho_i)$,

[6] While this assumption greatly simplifies the analysis, it is evidently unrealistic as it ignores enforcement costs.

where $i = f, u, \tau^f, \tau^w, g,$ is a 5 × 5 matrix describing the autoregressive components of each of the five shocks. Lastly, the innovation $\varepsilon = [\varepsilon_f, \varepsilon_u, \varepsilon_{\tau^f}, \varepsilon_{\tau^w}, \varepsilon_g]^T$ is a vector of random variables with zero mean and covariance matrix

$$\Omega = \begin{pmatrix} \sigma_f^2 & \sigma_{fu} & \sigma_{f\tau^f} & \sigma_{f\tau^w} & \sigma_{fg} \\ \sigma_{uf} & \sigma_u^2 & \sigma_{u\tau^f} & \sigma_{u\tau^w} & \sigma_{ug} \\ \sigma_{\tau^f f} & \sigma_{\tau^f u} & \sigma_{\tau^f}^2 & \sigma_{\tau^f \tau^w} & \sigma_{\tau^f g} \\ \sigma_{\tau^w f} & \sigma_{\tau^w u} & \sigma_{\tau^w \tau^f} & \sigma_{\tau^w}^2 & \sigma_{\tau^w g} \\ \sigma_{gf} & \sigma_{gu} & \sigma_{g\tau^f} & \sigma_{g\tau^w} & \sigma_g^2 \end{pmatrix},$$

where $\sigma_{ij} = \theta_{ij}\sigma_i\sigma_j$.

2.5 Equilibrium

A recursive competitive equilibrium for this economy consists of a set of prices $\{w_t, r_t\}_{t=0}^{\infty}$, a value function $V(k_t, K_t, A_t)$, decision rules $\{c_t, i_t, l_t^f, l_t^u, k_{t+1}, K_{t+1}\}_{t=0}^{\infty}$, and policy functions g_t and T_t such that:

- households maximize utility;
- firms maximize profits;
- the government balances its budget;
- individual and aggregate decisions are consistent, i.e. $k_t = K_t$, and
- markets clear, that is, the decision rules satisfy the resource constraint

$$c_t + i_t + g_t = y_t^{tot}. \tag{18}$$

Note that the equilibrium for this model involves an interior solution, as shown in the online appendix.[7]

III. CALIBRATION

The model parameters are calibrated to the U.S. economy with the aim to replicate its annual aggregate fluctuations. The main reason for so proceeding is to represent tax shocks as closely as they take place in reality. Tax rates, as Braun (1994) notes, probably vary little over the course of a year, so the strongest comovements are likely to occur at annual frequencies. Furthermore, the fact that data on the underground economy are difficult to obtain makes calibration at higher frequencies substantially more complicated.

The system of equations used to compute the dynamic equilibria of the model depends on a set of twelve parameters. Five pertain to household preferences (β, α, b, γ and η), five to the tax structure and the institutional context (the probability of a firm being detected ϕ, the surcharge factor ς, the payroll tax rate τ^s, and the steady state values of labor and corporate income tax rates τ_{ss}^w and τ_{ss}^f), and the remaining two parameters to technology (the capital

[7] The appendix is available at https://sites.google.com/site/catalinagrandacarvajal/publications

share α and the depreciation rate δ). In addition to these parameters, one has to characterize the innovations and their interactions.

The values of the discount factor, the depreciation rate and the capital income share are set to 0.95, 0.1 and 0.36, respectively. These three parameter values are commonplace in the existing business cycle literature. In contrast, calibrating the utility parameters α, *b*, γ and η presents the most difficult problem. Adapting again from Cho and Cooley (1994), the relation between formal and underground labor is obtained using the intratemporal first-order conditions for the household as follows:

$$\ln\left(l_t^u\right) = \frac{1}{\eta}\ln\left(\frac{a}{b\left(1-\tau_t^w\right)}\right) + \frac{\gamma}{\eta}\ln\left(l_t^f\right). \tag{19}$$

With γ set to 1, a value often assumed in business cycle studies, this relationship between regular and irregular labor can be estimated so that the other parameters fit three empirical observations: First, about one-third of the time endowment is spent in labor market activity, and hence the steady-state fraction of formal hours of work is assumed to equal 0.33. Second, Schneider's (2005) estimates suggest that the steady-state fraction of underground labor in the U.S. is 0.084.[8] Third, an elasticity of unofficial labor with respect to official labor (γ/η) of about 2.35 was derived from a Norwegian study on labor supply when tax evasion is an option Jørgensen *et al*. (2005).[9] Following this approach, Equation (19) holds for $a = 3.757$, $b = 4.584$, and $\eta = 0.426$.

Table 1: Preferences, Technology and Enforcement Parameters

β	δ	α	a	b	γ	η	ϕ	ς
0.95	0.10	0.36	3.7569	4.5839	1	0.4255	0.015	1.2

As for the enforcement parameters, these are taken from Slemrod and Yitzhaki's (2002) survey on tax evasion and administration. They report that the fraction of tax returns audited in the U.S. is about 1.5%, whereas the statutory penalty for non-criminal evasion is about 20%. Hence, the values of the probability of detection and the surcharge factor used in the baseline model are 0.015 and 1.2, respectively. All the parameters mentioned thus far are summarized in *Table 1*.

Moving on to the tax structure, the values for the tax rates are obtained from the OECD Tax Database. The social security contribution rate τ^s is set to 0.0765, which corresponds to the rate in place since 1990. It is worth noting here that the assumption that the payroll tax rate takes a constant value, instead of displaying a stochastic nature, is aimed to consider this particular feature of employer's taxation in the United States. In contrast, the parameters used

8 The estimate presented here is an average of Schneider's covering the period 1989-2003

9 It is worthy of note that there are not many empirical analyses of labor supply in the underground sector. In addition to the study presently used, Frederiksen *et al*. (2005) jointly estimate labor supply in the taxed and untaxed sectors for male Danish workers. Extensive searches have yielded no works of this sort for the US at all. Consequently, one shall qualify the procedure pursued here by recognizing that workers in Scandinavian countries face relatively high marginal tax rates on wage income compared to workers in most other OECD countries.

for corporate and personal income tax rates indicate steady-state levels, as these tax rates are much more likely to exhibit changes over time. The value of the corporate tax rate used, τ_{ss}^{f} = 0.392, stands for the combined federal and state statutory corporate income tax rate, while the chosen labor income tax rate, τ_{ss}^{w} = 0.224. represents the combined federal and sub-national government income tax (plus employee social security contributions) as a percentage of gross wage earnings. For further details, see OECD Center for Tax Policy and Administration (2010).

In the same vein, the steady-state value of the share of government expenditures in total output is estimated by taking the average of the ratio of government consumption expenditures and gross investment to GDP during 1960-2006. Series on these aggregates are featured by the Bureau of Economic Analysis. Furthermore, the steady-state value of official productivity is normalized to unity so that, using the arbitrage condition for labor in the tax evading sector (Equation 8), one obtains that the steady-state value of unofficial productivity is 0.4464. This value suggests that productivity in the informal sector is low relative to productivity in the formal sector, a feature extensively documented in the literature (see La Porta and Shleifer 2008).

To maintain symmetry in the model, and since there is no evidence about the persistence of the productivity shocks in the irregular sector (ρ_u), this parameter is assumed equal to ρ_f. The values for these parameters, in turn, are borrowed from the work of Benhabib *et al*. (1991) on home production, but adjusted to take account of the difference in frequencies studied.[10] Hence, the autocorrelation coefficients for the productivity shocks are $\rho_f = \rho_u = 0.95^4$. Also following Benhabib *et al*. (1991), the standard deviation of the productivity shocks is fixed to 0.007 and the correlation of the shocks between the two sectors used is 0.66.[11]

Finally, the parameters characterizing the distributional properties of fiscal policy shocks (i.e., persistence, standard deviations and correlations between innovations), as well as the interaction of these shocks with formal technology, are calibrated with some estimates obtained by Braun (1994). This author employs historical data to develop a statistical model of the government's feedback rule during the postwar period (1956-1980) using the Generalized Method of Moments. The values of the parameters characterizing the structure of shocks are presented in *Table 2*.

IV. MODEL EVALUATION

This section compares the performance of the present model with actual data and with selected alternative approaches. After discussing the results, some computational experiments assessing the cyclical implications of changes in the model parameters are conducted. The analyzed parameters pertain to the tax and enforcement structure, as well as households' tastes regarding work effort in the underground sector. Some inferences derived from the sensitivity analyses may be contrasted with data showing how business cycle stylized facts vary across countries with the size of the shadow economy (see Granda-Carvajal 2010), further evaluating the relevance of this and other related models and highlighting some promising extensions.

10 The model by Benhabib *et al*. (1991) is calibrated to match fluctuations at quarterly frequencies

11 I checked the robustness of the benchmark model to changes in the latter parameter using zero (0) and -0.66 as correlations. The results of these checks are available upon request.

Table 2: Parameter Values for Structure of Shocks

Parameter	Description	Value	Source
ρ_f, ρ_u	Persistence of sectoral productivity shocks	0.814	BRW (1991)
ρ_{τ}^{f}	Persistence of corporate tax rate shocks	0.786	Braun (1994)
ρ_{τ}^{w}	Persistence of labor income tax rate shocks	0.95	Braun (1994)
ρ_g	Persistence of government expenditures shocks	0.702	Braun (1994)
σ_f, σ_u	Standard deviation of sectoral productivity shocks	0.007	BRW (1991)
$\sigma_{\tau}f$	Std. dev. of corporate tax rate shocks	0.186	Braun (1994)
$\sigma_{\tau}f$	Std. dev. of labor income tax rate shocks	0.049	Braun (1994)
σg	Std. dev. of government expenditure shocks	0.036	Braun (1994)
θ_{fu}	Correlation of sectoral productivity shocks	0.66	BRW (1991)
$\theta_{f\tau}^{f}$	Corr. b/w official TFP and corporate tax shocks	-0.454	Braun (1994)
$\theta_{f\tau}^{w}$	Corr. b/w official TFP and labor income tax shocks	0.022	Braun (1994)
θ_{fg}	Corr. b/w official TFP and govt. expenditures	-0.533	Braun (1994)
$\theta_{\tau}^{f}{}_{\tau}^{w}$	Corr. b/w corporate and labor income tax shocks	0.122	Braun (1994)
$\theta_{\tau}^{f}{}_{g}$	Corr. b/w corporate tax and govt. exp. shocks	0.355	Braun (1994)
$\theta_{\tau}^{w}{}_{g}$	Corr. b/w labor income tax and govt. exp. shocks	0.073	Braun (1994)

Note: BRW (1991) refers to Benhabib *et al*. (1991).

4.1 Moments and Comparisons

To analyze how well the model accounts for aggregate fluctuations, a number of simulated moments are compared with the stylized facts characterizing the cyclical behavior of the U.S. economy during the period 1960-2006. The data used to obtain the stylized facts were taken from the National Income and Product Accounts calculated by the Bureau of Economic Analysis, in what regards real GDP and the expenditure components (i.e. consumption, investment, government expenditures), and from the International Economic Database, which features an index of total hours in the manufacturing sector (base year=1992). The dataset was compiled from web-based versions.

Before computing any statistics, both the actual time series and the generated series are logged and detrended using the Hodrick-Prescott filter with a smoothing parameter of 100. Detrending the series in this way facilitates comparison with McGrattan *et al*. (1997) and Busato and Chiarini (2004). Another advantage of such a procedure is that standard deviations can be interpreted as mean percentage deviations from the trend. After filtering the series, second moments are calculated from each of them. The relative volatility of each variable to output (official and aggregate) is calculated as the ratio of the standard deviation of the two variables, whereas the correlations involve each variable and output or formal hours.

Tables 3 and *4* display the relative ability of three different models to match the major stylized facts of the business cycle in the U.S. Note that formal consumption is distinguished from total consumption by making use of the national accounts identity.[12] This model predicts

[12] In a closed economy, the accounting identity $y^f = c^f + i^f + g$ holds. Since investment only takes place in the formal sector, the present model assumes that $i \equiv i^f$ and hence formal consumption can be obtained as $c^f = y^f - i - g$.

fairly well the volatility and the cyclicality of investment, average hours and labor productivity. Indeed, it improves quite significantly on Busato and Chiarini (2004) as far as the comovements of the latter variable with output and total (formal) hours are concerned. This observation cannot be highlighted more, given that the model reproduces recent tendencies in labor market dynamics with considerable accurateness.[13] Yet it fails to replicate the cyclical properties of government consumption, and understates the properties of private consumption as well. This is in contrast to both of the mentioned studies, especially McGrattan *et al*. (1997), which mimics the corresponding empirical moments somewhat closely. Despite these shortcomings, one should observe that formal consumption is more volatile than total consumption, as it certainly might be taken as an indication of model's success in resembling the data.

Table 3: Relative Standard Deviations Across Models

	Data	Model forecast		MRW('97)	BC ('04)
	$\sigma(x)/\sigma(y^f)$	$\sigma(x)/\sigma(y^f)$	$\sigma(x)/\sigma(y^{tot})$	$\sigma(x)/\sigma(y^f)$	$\sigma(x)/\sigma(y^{tot})$
GDP	1.00000	1.00000	1.11557	1.0000	1.86
Total output	–	0.89640	1.00000	–	1.00
Consumption	–	0.33150	0.36981	–	0.80
* Formal	0.89129	0.50095	0.55884	0.8298	–
Investment	4.08135	5.18734	5.78685	2.4628	6.64
Govt expend	1.49125	0.85712	0.95617	2.3085	–
Hours	1.92521	1.18151	1.31806	0.7447	1.10
Productivity	1.20065	1.40436	1.56666	–	2.00

Notes: MRW('97) refers to McGrattan *et al*. (1997); BC ('04) refers to Busato & Chiarini (2004).

Table 4: Correlations Across Models

	Correl. with y^f			Correl. with y^{tot}	
	Data	Model	MRW('97)	Model	BC ('04)
GDP	1.0000	1.0000	1.00	0.9975	0.95
Consumption	–	0.3897	–	0.3278	0.69
* Formal	0.8795	0.3754	0.91	0.3112	–
Investment	0.8425	0.9319	0.66	0.9522	0.98
Govt expend	0.3542	0.8770	0.40	0.8835	–
Hours	0.8479	0.9607	0.70	0.9754	0.73
Productivity	-0.5268	-0.9600	–	-0.9713	0.08
Correl. with hours					
	Data	Model	BC ('04)		
Productivity	-0.8973	-0.9931	0.04		

Notes: MRW('97): McGrattan *et al*. (1997); BC ('04): Busato & Chiarini (2004).

[13] Among other changes in labor market dynamics during the US postwar period, Galí and van Rens (2010) document a sharp drop in the cyclicality of labor productivity dating back to 1984. The correlation of productivity with output, which used to be strongly positive, fell to a level close to zero, while the correlation of productivity with labor input, which was zero or slightly positive in the earlier period, became negative. These changes overall coincided with the so-called Great Moderation.

Lastly, the simulations yield a correlation of unofficial and official output of –0.3129. This result is comparable to some extent to the correlation found by Busato and Chiarini (2004), about –0.96, and provides further support to the approach pursued in this paper. It reinforces the contention that imposing a negative comovement between both types of production –as implicit in the double business cycle approach– is unnecessary, since an economy may display this characteristic outcome without taking such *a priori* connection for granted. In effect, regular and irregular work effort barely comove in the model economy (their cyclical correlation is –0.0746), which is to be expected given these two variables are separable in utility. Hence a model wherein labor supply choices across sectors are independent is not incompatible with a particular pattern of cyclicality in the shadow economy.

4.2 Sensitivity Analysis

In general, the mechanisms driving the results of the present model are not substantially different from those of RBC models with household production (Benhabib *et al*., 1991; McGrattan *et al*., 1997). Changes in private agents' behavior induced by relative productivity differentials between the formal and the informal sector, as well as by distortionary taxation, explain the response of the economy to exogenous shocks on absolute sectoral productivity and fiscal policy. Yet the possibility that both firms and households evade taxes by diverting resources to another market sector might have some distinguishable cyclical implications. One could conjecture that structural characteristics pertaining to the tax systems, the strength of institutions and/or individuals' preferences towards underground work effort may lead to differing cyclical properties of macroeconomic aggregates. The sensitivity analyses conducted in the following aim to corroborate this conjecture.

The present examination considers the effect of changes in a number of parameter values on the volatility and comovements of six variables: formal output and consumption, investment, government expenditures, and formal labor input and its productivity. While standard deviation stands as the measure of output variability, the relative standard deviation –that is, the ratio of the standard deviation of the variable in question to the standard deviation of output– accounts for the volatility of the remaining variables. The parameters, seven in total, are organized into three main categories: tax policies, enforcement structure, and preference for underground work effort. In each case, the assessment is supplemented by some graphs illustrating the major patterns found.

4.2.1 Sensitivity to tax policies

Tax policies deal with the types of taxes considered individually in the model: payroll tax (τ^s), corporate income tax (τ^f_{ss}) and personal income tax (τ^w_{ss}). Note that these taxes play different roles in the economy since, while the social security contributions rate is modelled as a constant parameter, the other tax rates are made subject to stochastic disturbances. This is why, for the purposes of the present analysis, changes in the latter rates involve modifying their steady-state values. In addition, all three tax rates are jointly adjusted in what is called the 'average tax rate'. The four parameters are varied by percentages with respect to the benchmark values.

One might expect each tax to affect the variability and the comovements of the variables in a somewhat different manner. For comparative purposes, *Figures 1* and *2* contrast the impacts of the different tax rates on the cyclical properties of macro aggregates. Changes in the corporate tax rate (long-dashed dotted line) and the average tax rate (solid line) appear to influence macroeconomic fluctuations the most, as the two figures show volatility and cyclicality of each variable changing more prominently with variations in these tax rates. This is unlike the personal income tax rate (dashed line) and the social security contributions rate (dotted line), whose changes barely deviate each variable's moments from the benchmark ones. Hence both tax rates do not exert a substantial effect on business cycle fluctuations. The following analysis, consequently, focuses on the cyclical implications of variations in the average tax rate.

A higher burden of taxation and social security contributions reduces the expected return on investment and consumption, while increasing their variance. This relation is largely reflected in *Figure 1*, which shows the standard deviations of output, consumption and investment rise with generalized increments in taxes. Furthermore, government expenditures appears to become less volatile the higher is the tax burden. This pattern is mainly driven by reductions in the variability of work effort, as households attempt to smooth utility in view of increasing cyclical fluctuations in income and consumption. The relative standard deviation of labor productivity follows a similar tendency.

Moving on to the comovements, investment and the return on capital become less procyclical as the average tax rate increases, while consumption and government expenditures turn more so. These patterns of cyclical behavior are portrayed in *Figure 2*, which also suggests that the correlation of labor with formal output remains highly positive despite increments in the burden of taxation and social security contributions. In contrast, labor productivity stands as a strongly countercyclical variable, and comoves negatively with labor as well.

4.2.2 Sensitivity to Enforcement Parameters

As related to the model economy, the enforcement structure comprises the probability of a firm being discovered (ϕ) and the penalty surcharge on concealed tax payments (ς). While the detection probability takes a range of values between zero and one, feasible values for the fine are scanned using a sort of bankruptcy constraint on underground production. *Figure 3* shows how the volatility and the comovements of a number of macroeconomic variables change as the audit probability increases. Since the moments of all the variables exhibit similar patterns as penalty rates rise, graphs pertaining to this parameter are not included. The chosen values, along with the resulting moments and the steady-state shares of both regular and irregular labor, are found on *Tables B.1* and *B.2* in the appendix.

As firms are discouraged from diverting resources underground, the economy becomes more vulnerable to the disturbances affecting the formal sector. This might lead to a rise in the volatility of hours and government expenditures, as *Figure 3* confirms. Note that fluctuations in formal labor are apparently enough to mitigate the effects of sectoral productivity and fiscal policy shocks on consumption decisions. This is why formal output and consumption exhibit less fluctuations over the business cycle with firms facing higher audit probabilities or tougher penalties. Also, this decrease in the variability of output and its private components

Figure 1: Sensitivity of Macroeconomic Volatility to Tax Rates

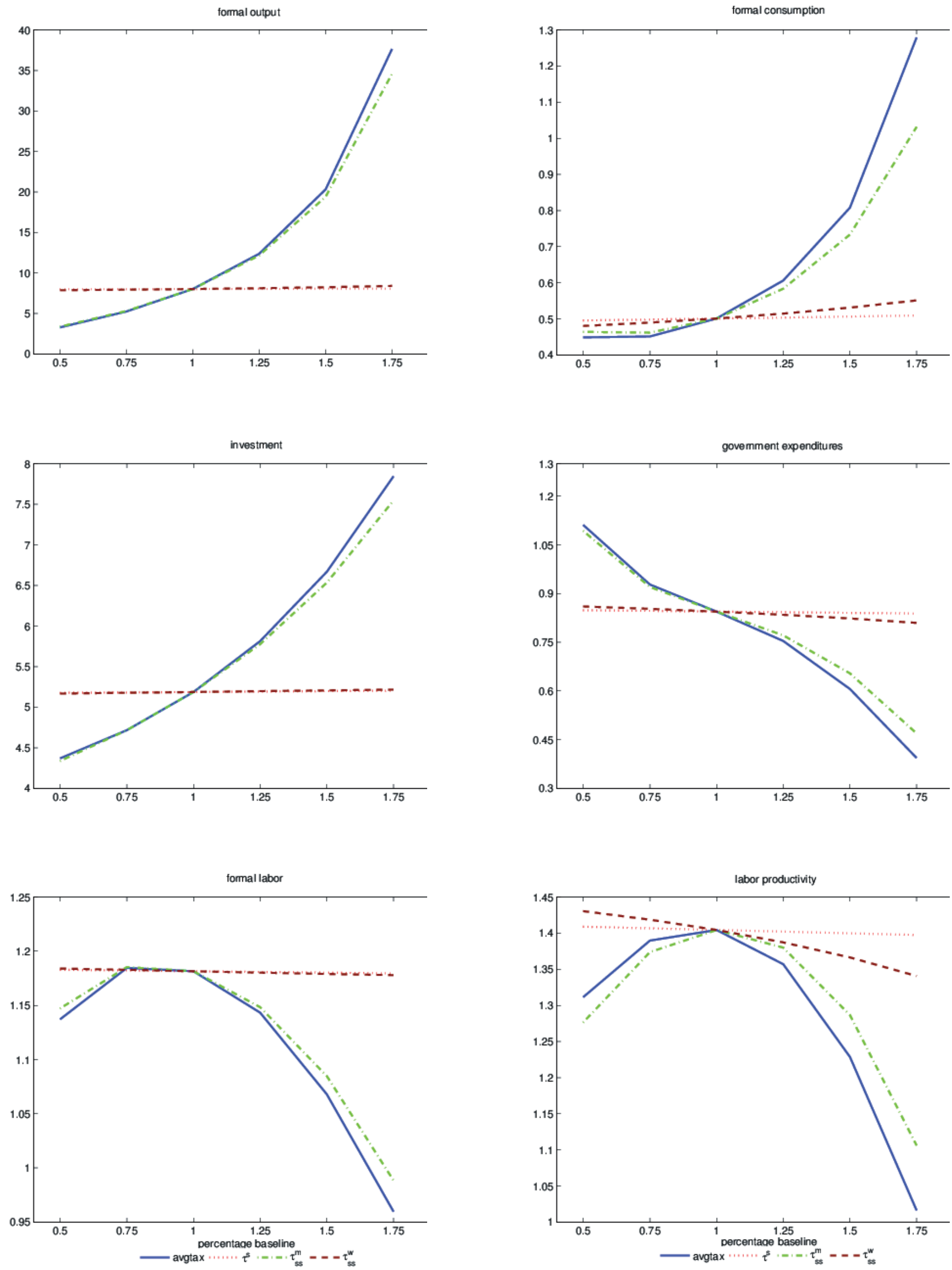
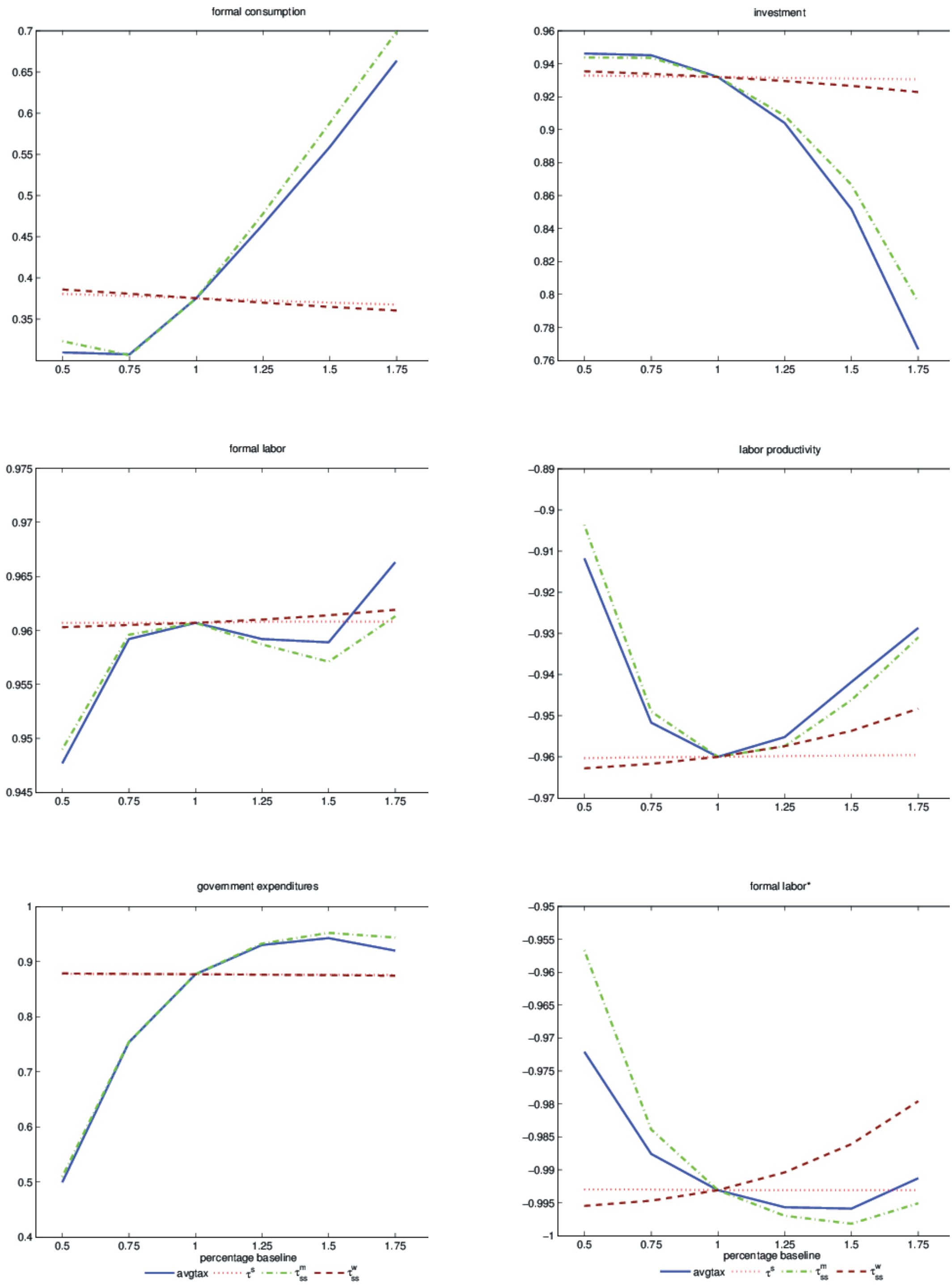


Figure 2: Sensitivity of Comovements to Tax Rates



* Correlation with labor productivity

may reflect that agents are able to assess the proceeds of their activities more accurately once they remain official altogether.[14]

4.2.3 Sensitivity to Preference Parameters

The preference parameters considered in this study mainly refer to the inverse elasticity of underground labor supply, or the elasticity of unofficial labor with respect to official labor ($1/\eta$). To conduct the present analysis, several elasticities are derived based on conditional and unconditional estimates obtained by Jørgensen *et al*. (2005), as described in Section III and displayed on *Table B.7* in the appendix. Increases in the inverse elasticity of underground labor supply reflect a higher responsiveness of irregular work effort to shocks on wages and/or taxes, which in turn might lead to a rise in the volatility of both official and unofficial hours.

Figure 4 shows how macroeconomic volatility behaves as the inverse elasticity of underground labor supply increases. Since the higher variability of work effort contributes to lessen the effects of sectoral productivity and fiscal policy shocks on production and consumption decisions, it is not surprising that formal output, consumption and investment fluctuate less over the business cycle as $1/\eta$ goes up. Note, though, that hours are somewhat more volatile than output and fluctuate increasingly within a rather narrow range. These two characteristics are indicative of the stronger responsiveness of underground labor and that households' choices between the two types of work effort are independent to some degree, as implicit in their separability in preferences.

V. CONCLUDING NOTES

This paper explores the macroeconomic implications of the existence of an underground sector. Focused on short-term fluctuations, it develops a real business cycle model featuring sectoral productivity and fiscal policy shocks. These shocks affect agents' responses to productivity differentials and tax changes to the extent that they are willing to substitute irregular for regular activities. That way, the unofficial economy can have substantial effects on the behavior of some aggregate variables. The implementation of a few computational experiments further confirms this consideration, allowing one to make inferences regarding the interaction between the determinants of shadow economic activity and business cycle fluctuations.

The present model differs from that of Busato and Chiarini (2004) mainly in the structure of preferences. Although both representations of household behavior are adapted from Cho and Cooley's (1994) family labor supply model, the characterization adopted here makes official and unofficial labor separable in utility. Furthermore, leisure time is spent on both irregular work effort and non-market activities, rather than entirely devoted to the shadow economy. These two features aim not to impose any sort of comovement of regular and irregular production, as opposed to the assumptions introduced by the former authors. Also, they enable one to take account of some empirical findings on the elasticity of underground labor supply (Jørgensen *et al*. 2005).

[14] Using a rather similar framework, Restrepo-Echavarría (2011) shows that countries with large and poorly measured (i.e., unenforced) informal economies tend to exhibit higher consumption volatilities.

Figure 3: Sensitivity of Macroeconomic Volatility to Detection Probability

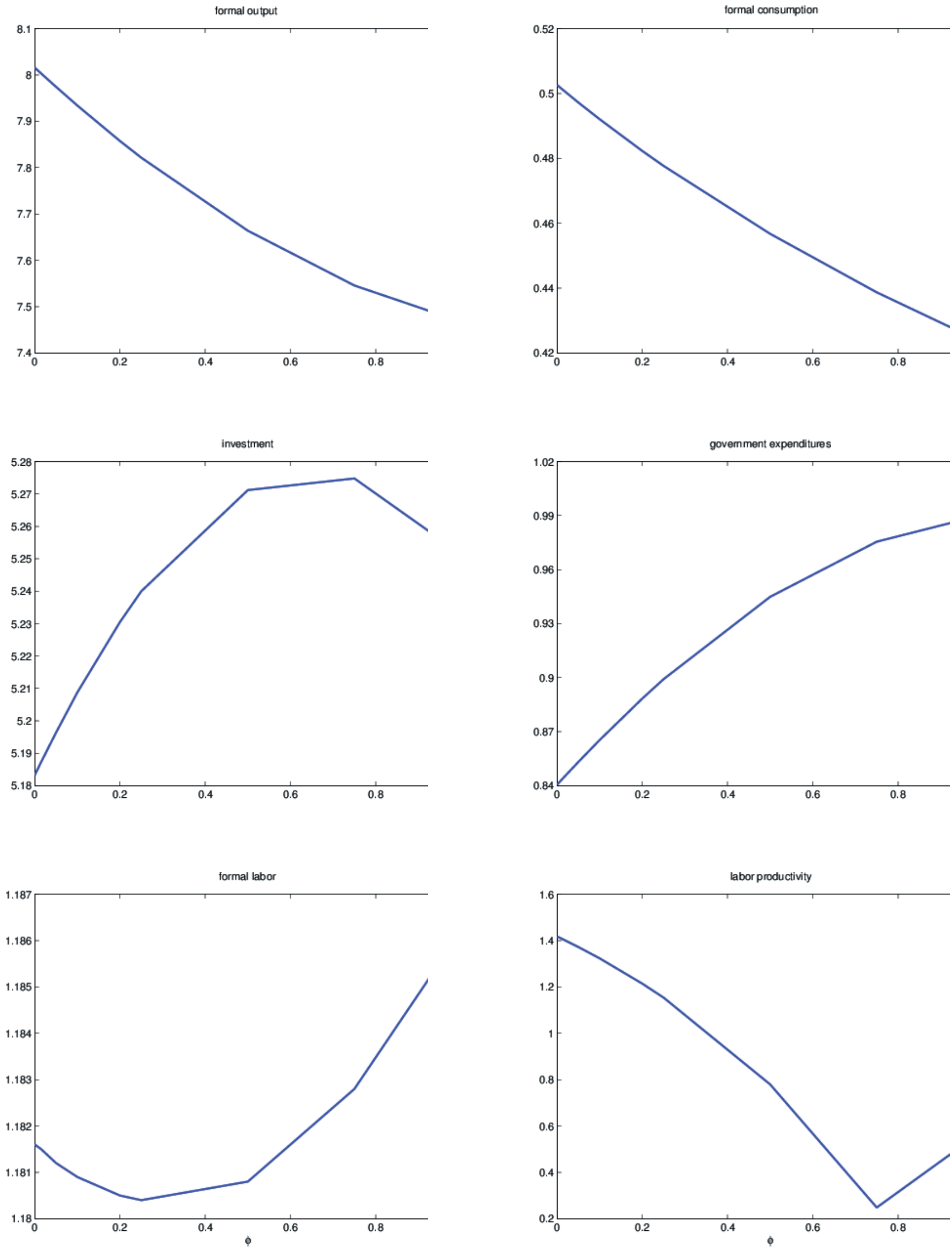
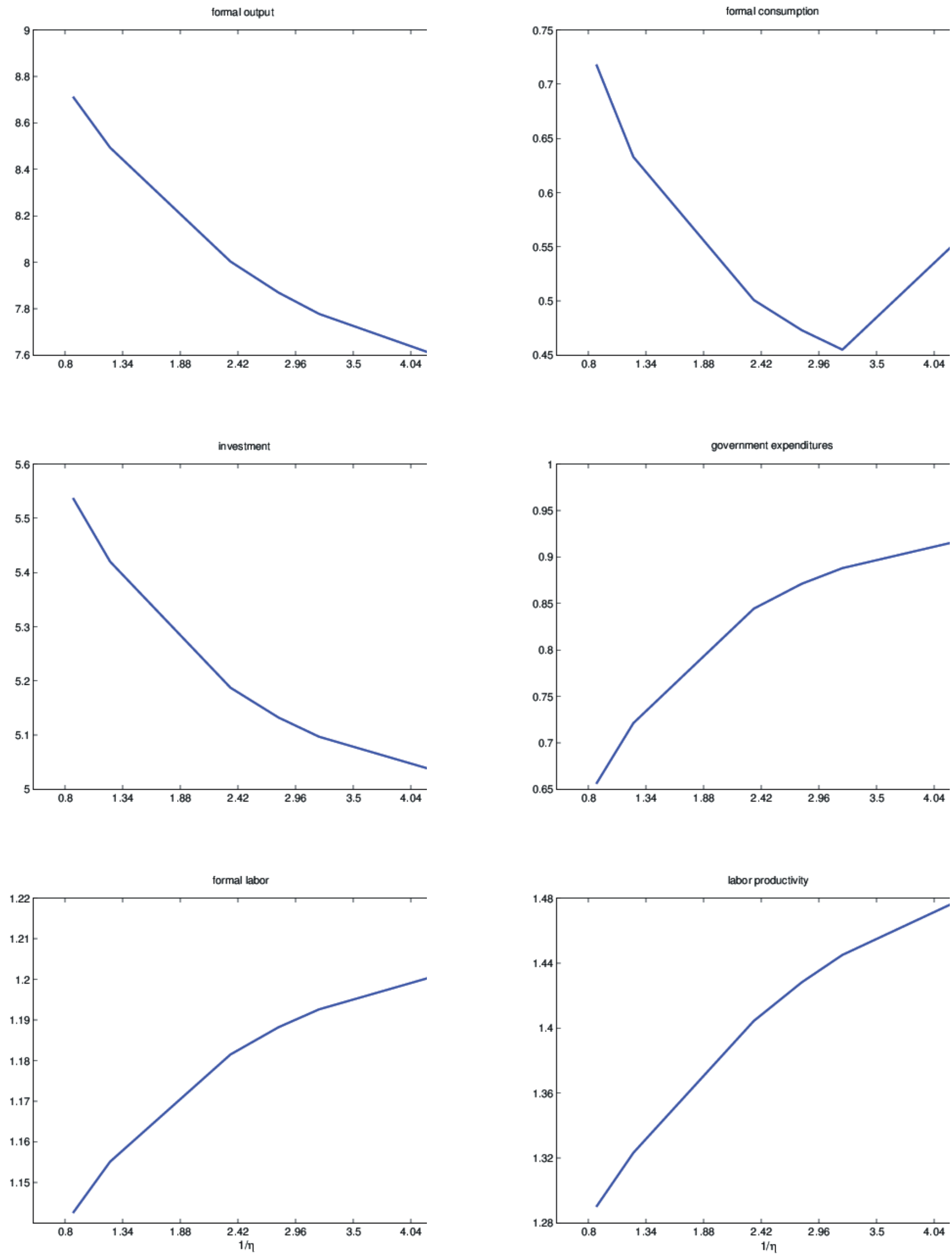


Figure 4: Sensitivity of Macro Volatility to Elasticity of Underground Labor Supply



Nevertheless, the functional form employed in the model does not preclude alternative specifications of preferences. In an attempt to reconcile models of macroeconomic fluctuations with microeconomic studies on the irregular sector, both theoretical (see, e.g., Cowell 1985) and empirical (Lacroix & Fortin 1992), further extensions should consider utility functions that are not additively separable. Moreover, the very assumption of a homogeneous commodity could be defied, thus allowing for imperfect substitution in consumption between officially- and unofficially-produced goods. Implementing these suggestions might provide a more realistic portrait of the intricacies associated to underground activities.

Other differences with respect to Busato and Chiarini (2004) deal with the characterization of fiscal policy and tax enforcement. Firms in the present model are audited in order to discourage evasion pertaining to corporate taxes and social security contributions. This is unlike the referred authors, who solely consider monitoring in regard to corporate revenue taxation. Furthermore, the payroll tax rate is allowed here to assume a constant value, rather than a stochastic nature. Once these characteristics are accounted for, the model is calibrated using actual estimations of the distributional properties of government spending and tax disturbances (see Braun, 1994). Doing so attempts to better reflect the reality of tax collection in the United States.

The model is able to replicate the cyclical properties of average hours and labor productivity fairly well. It indeed improves substantially on Busato and Chiarini (2004) as far as the comovements of productivity are concerned. Furthermore, the patterns followed by the volatility and the cyclicality of labor market variables across the computational experiments confirm empirical findings suggesting the absence of an empirical correlation between these variables and the extent of unofficial activities (see Granda-Carvajal, 2010). These results as a whole challenge the argument of the double business cycle approach that opportunities for intratemporal substitution induced by the existence of an irregular sector explain the so-called employment volatility puzzle.

Even though the model understates the variability of consumption, the computational experiments corroborate the existence of a positive connection between the size of the underground sector and the standard deviations of output and its private components. That weak enforcement, high tax rates and low distaste for irregular labor each lead to increased participation in shadow activities and to amplified fluctuations in formal output, consumption and, to a lesser degree, investment suggests a possible rationale for those findings. Furthermore, these results support related evidence such as that provided by Ferreira-Tiryaki (2008), and particularly Restrepo-Echavarría (2011), as to the volatility of consumption. Hence the sensitivity analyses contribute to clarify which cyclical features are actually associated to the extent of the unofficial economy.

Having said this, it is worth noting that the results presented here emphasize the underground sector and the presumption of the double business cycle approach as unnecessary when it comes to explaining certain features of macroeconomic fluctuations. In particular, they allow confirming that these elements per se do not explain the cyclical behavior of labor market variables, but tax disturbances (in a two-sector framework) do. By considering how changes in the determinants of informality affect a broad set of moments and aggregates, the analyses pursued in this paper offer a more comprehensive view of the cyclical implications of the shadow economy.

REFERENCES

Bajada, C. (2003). Business Cycle Properties of the Legitimate and Underground Economy in Australia, *Economic Record*. 79: 397-411.

Benhabib, J., R. Rogerson, and R. Wright (1991). Homework in Macroeconomics: Household Production and Aggregate Fluctuations, *Journal of Political Economy*. 99: 1166-1187.

Braun, R.A. (1994). Tax disturbances and real economic activity in the postwar United States, *Journal of Monetary Economics*. 33: 441-462.

Busato, F. and B. Chiarini (2004). Market and underground activities in a two-sector dynamic equilibrium model, *Economic Theory*. 23: 831-861.

Cho, J.-O. and T.F. Cooley (1994). Employment and hours over the business cycle, *Journal of Economic Dynamics and Control*. 18: 411-432.

Cowell, F.A. (1985). Tax evasion with labour income, *Journal of Public Economics*. 26: 19-34.

Ferreira-Tiryaki, G. (2008). The informal economy and business cycles, *Journal of Applied Economics*. 11: 91-117.

Frederiksen, A., E.K. Graversen, and N. Smith (2005). Tax evasion and work in the underground sector, *Labour Economics*. 12: 613-628.

Galí, J. and T. van Rens (2010). The Vanishing Procyclicality of Labor Productivity, IZA Discussion Paper No. 5099, Institute for the Study of Labor, Bonn.

Giles, D.E.A. (1997). Testing for asymmetry in the measured and underground business cycles in New Zealand, *Economic Record*. 73: 225-232.

Granda-Carvajal, C. (2010). The Unofficial Economy and the Business Cycle: A Test for Theories, *International Economic Journal*. 24: 573-586.

Jørgensen, Ø., T. Ognedal, and S. Strøm (2005). Labor supply when tax evasion is an option, Memorandum No. 06, Department of Economics, University of Oslo.

Lacroix, G. and B. Fortin (1992). Utility-based estimation of labour supply functions in the regular and irregular sectors, *Economic Journal*. 102: 1407-1422.

La Porta, R. and A. Shleifer (2008). The Unofficial Economy and Economic Development, *Brookings Papers on Economic Activity*. 39: 275-352.

Lemieux, T., B. Fortin, and P. Fréchette (1994). The Effect of Taxes on Labor Supply in the Underground Economy, *American Economic Review*. 84: 231-254.

McGrattan, E.R., R. Rogerson, and R. Wright (1997). An Equilibrium Model of the Business Cycle with Household Production and Fiscal Policy, *International Economic Review*. 38: 267-290.

OECD Centre for Tax Policy and Administration (2010). OECD Tax Database, Paris, Organization for Economic Co-operation and Development. Available on the web: http://www.oecd.org/ctp/taxdatabase, accessed August 20, 2010.

Restrepo-Echavarría, P. (2011). Macroeconomic volatility: The role of the informal economy, Unpublished paper, The Ohio State University, Columbus, OH.

Russo, F.F. (2008). Double Business Cycle: The Hidden Economy in the US, Unpublished paper, Department of Economics, Boston University, Boston, MA.

Schneider, F. (2005). Shadow economies around the world: what do we really know?, *European Journal of Political Economy*. 21: 598-642.

Slemrod, J. and S. Yitzhaki (2002). Tax avoidance, evasion, and administration. A.J. Auerbach and M.S. Feldstein (eds.), *Handbook of Public Economics*. Amsterdam: North-Holland: 1423-1470.