A Thesis submitted for the degree of Doctor

ANALYSIS OF THE PION PHOTOPRODUCTION PROCESS ON PROTON UP TO 1.7 GeV

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1. Introduction

In the study of the properties of nucleon resonances, the production of mesons (π, η, etc.) by hadron-induced reactions like beams of stable *baryons* such as *protons*, *deuterons*, and *alpha-particles* is an important tool that has been extensively used. The other type of beams are beams of *mesons* which are the most traditionally used reactions for the study of nucleon resonances, in particular, the scattering of *pions* has substantially contributed to the experimental data base. However this sort of reactions are complicated since the initial and final states are dominated by the *strong* interaction and, in the case of baryons, high energies must be employed to access the resonance regions, due to the large mass of the beam particles.

An alternative way to excite the nucleon, which has been widely used during the last decades, is the use of reactions induced by the *electromagnetic interaction* such as *photoproduction* and *electroproduction* of *mesons*, an important tool for studying the electromagnetic properties of nucleon resonances which has played a significant role in the tests of *quark models*, such as the ratio of the *electric quadrupole* to the *magnetic dipole* transition amplitudes (*EMR*) in the processes ($\gamma\, N \leftrightarrows \Delta(1232)$).

From the experimental point of view, the database has grown considerably thanks to the progress made in accelerator and detector technology; observables such as the total cross-sections and the electromagnetic multipoles have been measured with higher precision than hadron induced reactions, although the cross-sections corresponding to this type of reactions are three orders of magnitude larger than the electromagnetically induced reactions. All experiments are based at electron accelerators and, in the specific case of photoproduction, two different techniques are employed to produce the photon beams: *bremsstrahlung* and *laser backscattering.* The bremsstrahlung technique is used at ELSA [3] and MAMI [4] (in Germany), CLAS [5] (in United States), and at LNS (in Japan) while laser backscattering is employed at LEGS (in United States), at GRAAL [6] (in France), and at SPring-8 [7] (in Japan).

This work will focus on the particular case of pion photoproduction to evaluate, analytically and numerically through a model that will be described below, physical observables such as the cross-section and the multipole amplitudes which will be compared with the available experimental data to extract the relevant coupling constants of the nucleon resonances. However, from the theoretical point of view, we face the problem that in the *low energy* limit of *quantum chromodynamics* that is, at low momentum transfers Q in the GeV region, where the nucleon and its main resonances live, a *perturbative* analysis is not appropriate [8]. Therefore, we have to adopt an *effective approach* to try to represent in a "simple way" the dynamical content of the theory.

Several models have been proposed for studying the pion photoproduction of nucleon resonances, such as *Breit-Wigner* models [9, 10], *effective Lagrangian approach* models (*ELAs*) [11, 12], *dynamical* models [13], etc. Being phenomenological models, we shall adopt the ELA because it has become an acceptable approach in the energy range from threshold ($\sim$ 0.149 GeV) up to $\sim$ 1.70 GeV in the *center of mass* coordinate system ($\sim$ 1.0 GeV in the *laboratory* coordinate system) for the reaction $\gamma p \ \to \pi N$. Another reason is that the ELA also provides a natural framework to extend the model to other processes such as pion electroproduction, two pion photoproduction, photoproduction of other mesons such as η, etc.

In the ELA all contributions to the reaction are derived from *effective Lagrangian densities* corresponding to the *interaction vertices*, in which each particle is considered as an *effective field* having mass, spin, isospin, strong decay width, etc. [14, 15]. In the specific case of photoproduction of pseudoscalar mesons such pion or η, the two commonly encountered forms of the *meson-nucleon* interaction are through the *pseudoscalar (PS)* or *pseudovector (PV)* couplings, which are equivalent for elementary fields without anomalous magnetic moment. However in the specific case of pion photoproduction, the πNN coupling is chosen to be PV to obtain the right low energy behaviour in accord with *current algebra* results and *chiral symmetry*, due to the small mass of the pion [16]. In the case of the η meson, there is no preference for the coupling to be PS or PV due to the larger mass of the η meson [17].

On the other hand, in the photoproduction of pions off the nucleon, the spin-$\frac{3}{2}$ resonance $\Delta(1232)$ plays the most important role in the first resonance region, however, the treatment of the spin-$\frac{3}{2}$ nucleon resonances namely, vertices and propagator, are not completely consistent in the literature. It deserves special attention because the quantum field theory of particles with spin $\geq \frac{3}{2}$ is an *open* problem since the Lagrangian and the propagator are not unique, there are *arbitrary* parameters in the theory. On one hand, the free-field Lagrangian as well as the propagator for spin-$\frac{3}{2}$ particles contain an arbitrary parameter A which defines the so-called *point transformation**. On the other hand, the interaction Lagrangians are constructed in such a way that they are invariant under the same point transformation as the free Lagrangian, but they depend now on two parameters, A and Z (named *off-shell* parameter), as we will see in the specific case of the coupling of the spin-$\frac{3}{2}$ field to the nucleon and pion fields and the coupling of the spin-$\frac{3}{2}$ field to the electromagnetic and pion fields. Even though the physical amplitudes depend on the Z parameter, it can be set consistently to a fixed value [11, 18].

For the case of a spin-$\frac{3}{2}$ field coupled to a spin-$\frac{1}{2}$ and the derivative of the pion field, the approach that we adopt, the value of the off-shell parameter is fixed to $Z = \frac{1}{2}$, by requiring the interaction to be consistent with the principles of second quantization [12, 19].

*A point transformation is a transformation of the variables, which does not involve time derivatives.

With regard to the A parameter, we make the choice $A = -\frac{1}{3}$, both in the vertices and the propagator [20], in agreement with [21], an election that differs from other works in which the value of A is not fixed consistently in both, vertices and propagator.

In our model, the internal structure of hadrons is taken into account by means of phenomenological form factors which are included, consistently, in the tree level amplitudes by preserving the symmetries of the theory namely, gauge invariance and crossing symmetry, giving rise to additional current contributions beyond the usual Feynman diagrams to *cancel* the resulting gauge-*violating* terms [22, 23, 24].

This work is distributed as follows: in Ch. 2, we fix the notation and list all the basic kinematical formulas for pion photoproduction. In Ch. 3 we present the Lagrangians for all the *free* fields taking part in *pion* photoproduction below ~ 1.7 GeV. In Ch. 4 we present the most general interaction Lagrangians for vertices, compatible with all possible symmetries namely, chiral symmetry, gauge invariance and crossing symmetry. In Ch. 5 we present the general form of the total propagator for the spin-$\frac{3}{2}$ field which is considered first as a stable bare particle that later obtains its empirical mass and width by *dressing* with pions by means of the absorptive *one-loop* self-energy correction to the spin-$\frac{3}{2}$ particle propagator which reproduces the complex-mass prescription for its resonant form. In Ch. 6 we present the analytic expressions for the amplitudes contributing to pion photoproduction off the proton (as well as neutron, for the sake of completeness) at the tree level, without including form factors, which are included later in Ch. 7 for the numerical calculation of the cross-sections corresponding to the specific processes $\gamma p \to \pi^+ n$ and $\gamma p \to \pi^0 p$. Finally, in Ch. 8 we perform the analysis of the relavant electromagnetic multipoles in pion photoproduction namely, $M_{1+}^{\frac{3}{2}}$ and $E_{1+}^{\frac{3}{2}}$.

2. Units and Kinematics

In this chapter we will fix the units, the notation and list all the basic kinematical formulas for photoproduction of *pion* (π) from a *free proton* (p), $\gamma p \to \pi N$, where $N = p$ or n (*neutron*) and the *four-vector momentum* of the incident *photon* (γ) and the outgoing *pion* are denoted by k and q, respectively, while those of the initial and final *nucleon* are $p_{\rm i}$ and $p_{\rm f}$, respectively. For the sake of simplicity we will evaluate the scattering amplitudes in a coordinate system in which $\vec{k}$ and $\vec{p}_{\rm i}$ each lies along a given line, say the $z-$axis of a rectangular coordinate system (that is, $\vec{k} \times \vec{p}_{\rm i} = \vec{0}$). Since the scattering matrix is a *Lorentz invariant*, the general case may then be obtained from a Lorentz transformation. The two most common coordinate systems are the *laboratory* coordinate system, in which $\vec{p}_{\rm i} = \vec{0}$, and the *center of mass* coordinate system, in which $\vec{k} + \vec{p}_{\rm i} = \vec{0}$, as shown in Fig. 2.1, where we indicate the components of each four-vector momentum.

2.1. Units

We will use the system of natural units, where

$$\hbar = c = 1, \tag{2.1}$$

such that

$$[\text{length}] = [\text{time}] = [\text{energy}]^{-1} = [\text{mass}]^{-1}. \tag{2.2}$$

Therefore, in this system, the electric charge of the proton is given by

$$e = \sqrt{4\pi\alpha} = 0.302862. \tag{2.3}$$

2.2. Relativity and Tensors

Our conventions for relativity and tensors follow *Bjorken* and *Drell* [25], *Jackson* [26], and *Peskin* [27]. For example, for the metric tensor, $\eta_{\mu\nu}$, we use

$$\eta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{2.4}$$

such that

$$p^2 = \eta_{\mu\nu} p^\mu p^\nu = E^2 - |\vec{p}|^2, \tag{2.5}$$

where the energy E and the momentum $\vec{p}$ of the particle are represented by the operators

$$E \to i\frac{\partial}{\partial x^0} \equiv i\partial_0, \quad \text{and} \quad \vec{p} \to -i\vec{\nabla}. \tag{2.6}$$

Then, the plane wave $\mathrm{e}^{-ik\cdot x}$ has momentum k_μ, since

$$i\partial_\mu \mathrm{e}^{-ik\cdot x} = k_\mu \mathrm{e}^{-ik\cdot x}. \tag{2.7}$$

2.3. Pauli and Dirac Matrices

We use the Pauli sigma matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \text{and} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2.8}$$

For the Dirac matrices, γ^μ, we use the *Weyl* or *chiral* representation given by

$$\gamma^0 = \begin{pmatrix} \mathbb{0} & \mathbb{1} \\ \mathbb{1} & \mathbb{0} \end{pmatrix}, \quad \text{and} \quad \vec{\gamma} = \begin{pmatrix} \mathbb{0} & \vec{\sigma} \\ -\vec{\sigma} & \mathbb{0} \end{pmatrix}, \tag{2.9}$$

which satisfy the anticommutation relation

$$\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}\mathbb{1}, \tag{2.10}$$

and the additional gamma matrix, γ_5, is defined as

$$\gamma_5 \equiv i\gamma_0\gamma_1\gamma_2\gamma_3 = -\frac{i}{4!}\epsilon^{\mu\nu\alpha\beta}\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\beta = \begin{pmatrix} -\mathbb{1} & \mathbb{0} \\ \mathbb{0} & \mathbb{1} \end{pmatrix}, \tag{2.11}$$

with the properties

$$\gamma_5^\dagger = \gamma_5, \quad \text{and} \quad \{\gamma_5, \gamma_\mu\} = \mathbb{0}. \tag{2.12}$$

2.4. Mandelstam Variables

In the case of 2-body → 2-body processes, it is useful to express the scattering amplitudes in terms of the *Mandelstam variables* that make it easy to apply *crossing relations*. The Mandelstam variables are defined by

$$\begin{aligned} s &\equiv (k + p_\mathrm{i})^2 = (q + p_\mathrm{f})^2, \\ t &\equiv (q - k)^2 = (p_\mathrm{i} - p_\mathrm{f})^2, \\ u &\equiv (k - p_\mathrm{f})^2 = (q - p_\mathrm{i})^2, \end{aligned} \tag{2.13}$$

where,

$$s + t + u = \sum_{i=1}^{4} m_i^2 = 2M_\mathrm{N}^2 + m_\pi^2$$

with $M_\mathrm{N} = 0.938$ GeV and $m_\pi = 0.138$ GeV, the nucleon and pion mass, respectively and for any process, s is the square of the total initial 4-momentum.

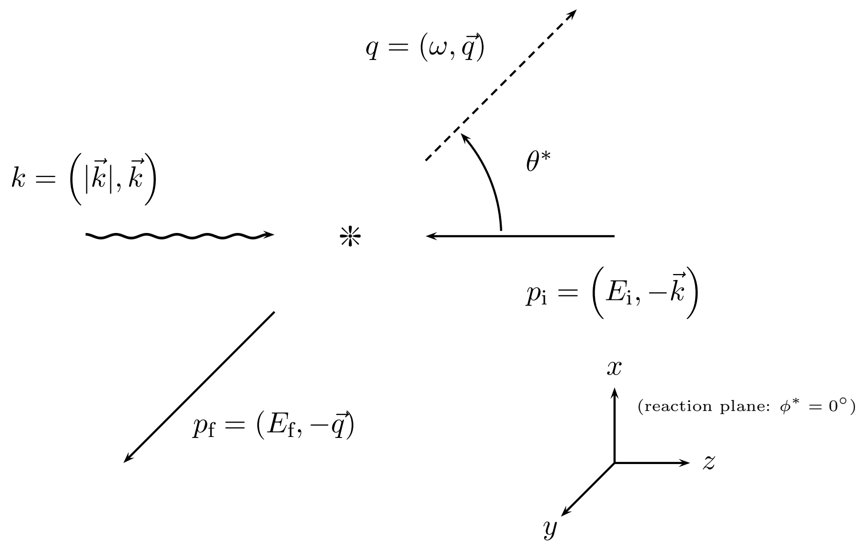


Figure 2.1.: Kinematics of *pion* meson photoproduction in the *c.m.* coordinate system.

2.5. Center of Mass Coordinate System

In the *center of mass* (*c.m.*) coordinate system of the *initial proton* and the *photon*, where the *experimental observables* will be calculated, the Mandelstam variables become

$$\begin{aligned} s &\equiv W^2 = (E_\mathrm{i} + |\vec{k}|)^2, \\ t &= m_\pi^2 - 2\omega|\vec{k}| + 2|\vec{q}||\vec{k}|\cos\theta^*, \\ u &= M_\mathrm{N}^2 - 2E_\mathrm{f}|\vec{k}| - 2|\vec{q}||\vec{k}|\cos\theta^*, \end{aligned} \tag{2.14}$$

where θ^* is the *scattering angle* and $W = \sqrt{s}$, the total energy.
The energies and momenta are determined in terms of W as

$$|\vec{k}| = \frac{W^2 - M_\mathrm{N}^2}{2W}, \quad E_\mathrm{i} = \frac{W^2 + M_\mathrm{N}^2}{2W}, \quad E_\mathrm{f} = \frac{W^2 - m_\pi^2 + M_\mathrm{N}^2}{2W}, \tag{2.15}$$

$$\omega = \frac{W^2 + m_\pi^2 - M_\mathrm{N}^2}{2W}, \quad \text{and} \quad |\vec{q}| = \sqrt{\frac{(W^2 + m_\pi^2 - M_\mathrm{N}^2)^2}{4W^2} - m_\pi^2}. \tag{2.16}$$

Threshold of the Reaction

The threshold for the reaction $\gamma p \to \pi N$ is defined at the pion momentum $|\vec{q}| = 0$, where the *photon lab energy* is

$$E_\gamma = \frac{(m_\pi + M_\mathrm{N})^2 - M_\mathrm{N}^2}{2M_\mathrm{N}} \simeq 0.149\,\mathrm{GeV}, \tag{2.17}$$

corresponding to a c.m. energy of $W \simeq 1.08$ GeV.

3. Free Lagrangians

The relevant degrees of freedom used to describe nuclei depend on the energy resolution by which the nucleus is probed. As discussed in the introduction, the energy range of the more actual experiments is up to ~ 2.0 GeV, thus for energy transfers below ~ 1.7 GeV and momentum transfers below ~ 1.7 GeV/c, the important degrees of freedom are limited to the lowest states of the nucleon and meson spectrum. The main role is played by the *pion*, the *nucleon* and the *nucleon resonances*: $P_{33}(1232)$, $P_{11}(1440)$, $D_{13}(1520)$, $S_{11}(1535)$, $P_{33}(1600)$, $S_{11}(1650)$, and $S_{11}(1710)$; the *vector mesons* ρ and ω also play an important role. Their properties are displayed in Table 3.1. Other mesons such as η, η' and ϕ do not contribute significantly at tree level. In the case of η mesons, first order electromagnetic decays, $\eta \to \pi^0\gamma$ are forbidden by *conservation of angular momentum* [28], while the contribution of the ϕ meson is suppressed by the *OZI rule* [29]. In this section we present the Lagrangians for all the *free* fields taking part in *pion* photoproduction below ~ 1.7 GeV, before to describe the interacting Lagrangians from which we will calculate the *invariant* amplitudes.

3.1. The Pion Field ($\vec{\Phi}_\pi$)

The Lagrangian for the *spin*-0, *isospin*-1 *pion* free field is the *Klein-Gordon* Lagrangian

$$\mathscr{L}_\pi = \frac{1}{2}\left(\partial^\mu \vec{\Phi}_\pi \cdot \partial_\mu \vec{\Phi}_\pi - m_\pi^2 \vec{\Phi}_\pi \cdot \vec{\Phi}_\pi\right), \tag{3.1}$$

where

$$\vec{\Phi}_\pi = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix} \tag{3.2}$$

denotes the three-component pion field in terms of its *cartesian isospin* components, in terms of which the *charge* components $\pi^\pm$ and π^0 of the pion field are defined by [13]

$$\pi^\pm \equiv \frac{\mp\pi_1 - i\pi_2}{\sqrt{2}} \quad \text{and} \quad \pi^0 \equiv \pi_3, \tag{3.3}$$

then the pion field is rewritten as

$$\vec{\Phi}_\pi = \pi^+\hat{\Phi}_+ + \pi^-\hat{\Phi}_- + \pi^0\hat{\Phi}_0, \tag{3.4}$$

with *unit* vectors

$$\hat{\Phi}_+ \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ i \\ 0 \end{pmatrix}, \quad \hat{\Phi}_{\equiv}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix}, \quad \hat{\Phi}_0 \equiv \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \tag{3.5}$$

Hadron	Isospin	Charge states	Spin$^{(\text{parity})}$	Mass (MeV)	Γ_{total} (MeV)
Pion	1	π^+, π^0, π^-	0^-	$m_{\pi^\pm} = 139.6$ $m_{\pi^0} = 135.0$	"stable" 8.02×10^{-6}
ρ-meson	1	ρ^+, ρ^0, ρ^-	1^-	775.3	147.4
ω-meson	0	ω^0	1^-	782.7	8.7
Nucleon	$\frac{1}{2}$	p, n	$\frac{1}{2}^+$	$M_{\text{p}} = 938.3$ $M_{\text{n}} = 939.6$	stable "stable"
$P_{33}(1232)$	$\frac{3}{2}$	Δ^{++}, Δ^+, Δ^0, Δ^-	$\frac{3}{2}^+$	$M_{\Delta^+} = 1206 - 1213$ $M_{\Delta^0} = 1212 - 1214$	$97 - 119$ $103 - 105$
$P_{11}(1440)$	$\frac{1}{2}$	P^+, P^0	$\frac{1}{2}^+$	$1360 - 1380$	$160 - 190$
$D_{13}(1520)$	$\frac{1}{2}$	D^+, D^0	$\frac{3}{2}^-$	$1505 - 1515$	$105 - 120$
$S_{11}(1535)$	$\frac{1}{2}$	S^+, S^0	$\frac{1}{2}^-$	$1500 - 1520$	$110 - 150$
$P_{33}(1600)$	$\frac{3}{2}$	Δ^{++}, Δ^+, Δ^0, Δ^-	$\frac{3}{2}^+$	$1460 - 1560$	$200 - 340$
$S_{11}(1650)$	$\frac{1}{2}$	S^+, S^0	$\frac{1}{2}^-$	$1640 - 1670$	$100 - 170$
$P_{11}(1710)$	$\frac{1}{2}$	P^+, P^0	$\frac{1}{2}^+$	$1680 - 1720$	$80 - 160$

Table 3.1.: Properties of the hadrons considered in this work [1]. The mass and total width of the resonances correspond to the resonance *pole position*, s_{R}, given by $\sqrt{s_{\text{R}}} = M_{\text{R}} - i\frac{\Gamma_{\text{R}}}{2}$.

3.2. Vector Meson Fields

Spin-1 massive particles may be described by the well known *Proca* Lagrangian [30].

1. **The ρ Field ($\vec{\Phi}^\mu_\rho$)**

   The Lagrangian for the *spin*-1, *isospin*-1 ρ free field is

$$\mathscr{L}_\rho = -\frac{1}{4}\vec{W}^{\mu\nu} \cdot \vec{W}_{\mu\nu} + \frac{1}{2}m_\rho^2 \vec{\Phi}^\mu_\rho \cdot \vec{\Phi}_{\rho\mu}, \tag{3.6}$$

   where the tensor $\vec{W}^{\mu\nu}$ is defined by

$$\vec{W}^{\mu\nu} \equiv \partial^\mu \vec{\Phi}^\nu_\rho - \partial^\nu \vec{\Phi}^\mu_\rho. \tag{3.7}$$

   As in the previous case,

$$\vec{\Phi}^\mu_\rho = \begin{pmatrix} \rho_1^\mu \\ \rho_2^\mu \\ \rho_3^\mu \end{pmatrix} \tag{3.8}$$

   denotes the three-component ρ field in terms of its *cartesian isospin* components, in terms of which the *charge* components $\rho^{\mu\pm}$ and $\rho^{\mu 0}$ of the rho field are defined by [13]

$$\rho^{\mu\pm} \equiv \frac{\mp\rho_1^\mu - i\rho_2^\mu}{\sqrt{2}} \quad \text{and} \quad \rho^{\mu 0} \equiv \rho_3^\mu. \tag{3.9}$$

2. **The ω Field (Φ^{μ}_{ω})**
   Similarly, for the *spin*-1, *isospin*-0 ω field

$$\mathscr{L}_{\omega} = -\frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \frac{1}{2} m_{\omega}^{2} \Phi^{\mu}_{\omega} \Phi_{\omega\mu}, \tag{3.10}$$

   where the tensor $B^{\mu\nu}$ is defined by

$$B^{\mu\nu} \equiv \partial^{\mu} \Phi^{\nu}_{\omega} - \partial^{\nu} \Phi^{\mu}_{\omega}. \tag{3.11}$$

3.3. Nucleon (Ψ_N) and Spin-$\frac{1}{2}$ Resonance (Ψ_R) Fields

The Lagrangian for the *spin*-$\frac{1}{2}$, *isospin*-$\frac{1}{2}$ Nucleon (N) and Resonance (R) free fields is given by the well known *Dirac* Lagrangian

$$\mathscr{L}_{X} = \overline{\Psi}_{X} \left(i\gamma^{\mu} \partial_{\mu} - M_{X} \right) \Psi_{X}, \tag{3.12}$$

where M_X is the mass of the spin-$\frac{1}{2}$ baryon ($X = N, R$) and the γ_{μ} matrices satisfy the anticommutation relation

$$\{\gamma_{\mu}, \gamma_{\nu}\} = 2 g_{\mu\nu}. \tag{3.13}$$

In this case, the *nucleon* field Ψ_N, is given by the *isospin doublet*

$$\Psi_{N} = \begin{pmatrix} \psi_{p} \\ \psi_{n} \end{pmatrix}, \tag{3.14}$$

where ψ_p and ψ_n represent the *proton* and *neutron* fields, respectively.
Similarly, for the $P_{11}(1440)$, $S_{11}(1535)$, $S_{11}(1650)$ and $P_{11}(1710)$ resonances

$$\Psi_{R} = \begin{pmatrix} \psi_{R^{+}} \\ \psi_{R^{0}} \end{pmatrix}, \tag{3.15}$$

where the superscripts + and 0 denote the electric charge of the corresponding fields.

3.4. Spin-$\frac{3}{2}$ Resonance Fields (Ψ^{μ}_{Δ} and Ψ^{μ}_{D})

The Lagrangian for the *spin*-$\frac{3}{2}$, *isospin*-$\frac{3}{2}$ (Ψ^{μ}_{Δ}) and the *spin*-$\frac{3}{2}$, *isospin*-$\frac{1}{2}$ (Ψ^{μ}_{D}) resonance free fields is the *Rarita-Schwinger* Lagrangian [19, 31]

$$\mathscr{L}_{X} = \overline{\Psi}^{\mu}_{X} \Lambda_{\mu\alpha} \left[g^{\alpha\beta} \left(i\partial\!\!\!/ - M_{X} \right) + \frac{i}{3} \left(\gamma^{\alpha} \partial\!\!\!/ \gamma^{\beta} - \gamma^{\alpha} \partial^{\beta} - \partial^{\alpha} \gamma^{\beta} \right) + \frac{1}{3} M_{X} \gamma^{\alpha} \gamma^{\beta} \right] \Lambda_{\beta\nu} \Psi^{\nu}_{X}, \tag{3.16}$$

where M_X is the mass of the spin-$\frac{3}{2}$ baryon ($X = \Delta, D$) and the tensor $\Lambda_{\rho\sigma}$ is defined as

$$\Lambda_{\rho\sigma} \equiv g_{\rho\sigma} + \frac{1}{2}(1 + 3A) \gamma_{\rho} \gamma_{\sigma}, \tag{3.17}$$

with A an arbitrary parameter subject to the restriction $A \neq -\frac{1}{2}$.
On the other hand, the spin-$\frac{3}{2}$ field describing the Δ resonances, Ψ^μ_Δ, is a *spinor-vector* field given by the *isospin-$\frac{3}{2}$ quartet*

$$\Psi^\mu_\Delta = \begin{pmatrix} \psi^\mu_{\Delta^{++}} \\ \psi^\mu_{\Delta^+} \\ \psi^\mu_{\Delta^0} \\ \psi^\mu_{\Delta^-} \end{pmatrix}, \tag{3.18}$$

while the spin-$\frac{3}{2}$ field describing the D resonances, Ψ^μ_D, is a *spinor-vector* field given by the *isospin-$\frac{1}{2}$ doublet*

$$\Psi^\mu_D = \begin{pmatrix} \psi^\mu_{D^+} \\ \psi^\mu_{D^0} \end{pmatrix}. \tag{3.19}$$

3.4.1. The Point Transformation

The free Lagrangian given by Eq. (3.16) is *invariant* under the *point transformation* [11, 31]

$$\Psi^\mu_X \to \Psi'^\mu_X = \Psi^\mu_X + a\gamma^\mu\gamma_\nu\Psi^\nu_X, \quad A \to A' = \frac{A - 2a}{1 + 4a}, \tag{3.20}$$

where $a \neq -\frac{1}{4}$, but is otherwise arbitrary.
It implies that physical properties of the *free* field, such as the energy-momentum tensor and the canonical commutation relations are independent of the parameter A [19].

3.4.2. Spin-$\frac{3}{2}$ Resonance Field Propagator

The propagator of the Rarita-Schwinger field deserves special attention and will be considered in Ch. 5 with more detail.

4. Interaction Lagrangians

In the study of photoproduction of *pseudoscalar mesons* such as *pion* off the nucleon, the strong interaction vertex will be treated phenomenologically using the *effective Lagrangian approach* (*ELA*) [11, 17]. The two standard couplings are the *pseudoscalar* (*PS*) and the *pseudovector* (*PV*) which for elementary fields, without anomalous magnetic interactions, are equivalent in the lowest order in strong coupling constant [32]. In the case of our interest, pion photoproduction, the πNN coupling is preferred to be PV according with the *low energy theorem* (*LET*) [16].
The model consists of effective interaction Lagrangians which are splitted into two different types of contributions: the first type consists of the *non-resonant* or *background* term which include the nucleon s- and u-channels, the pion t-channel, the vector meson exchanges (ρ and ω), the *contact* term and the u-channel of the resonance excitations, namely $P_{33}(1232)$, $P_{11}(1440)$, $D_{13}(1520)$, $S_{11}(1535)$, $P_{33}(1600)$, $S_{11}(1650)$, and $P_{11}(1710)$. The second type consists of the s-channel of the above resonance excitations. The corresponding plots of each of these Feynman graphs will be shown later.
In this chapter, we present the most general interaction Lagrangians for vertices, compatible with all possible symmetries: chiral symmetry, gauge invariance and crossing symmetry.

4.1. Strong Interaction

The strong interaction is *invariant* under *time reversal* ($t \to -t$) and *parity* ($\vec{r} \to -\vec{r}$), it is also invariant under *charge conjugation* which transforms particles into antiparticles. *Isospin symmetry* is also an important concept in the physics of the strong interaction, isospin symmetry means that the strong interaction is *invariant* under rotations in *isospin space*. Thus, the total isospin of an interacting system of pions and nucleons is a *conserved* quantity, however, this is broken by *electromagnetic interactions*.

4.1.1. Vertices for Born Terms

The πNN Vertex

The interaction Lagrangian is given by

$$\mathscr{L}_{\pi NN} = -\frac{f_{\pi NN}}{m_\pi}\overline{\Psi}_N \gamma_5 \gamma_\mu \vec{\tau}\, \Psi_N \cdot \partial^\mu \vec{\Phi}_\pi, \tag{4.1}$$

where m_π is the mass of the pion, and $f_{\pi NN}$ is the *pseudovector* coupling constant whose experimental value has been determined accurately from pion-nucleon and nucleon-

nucleon scattering [11, 32]

$$\frac{f_{\pi NN}^2}{4\pi} = 0.0749. \tag{4.2}$$

On the other hand, the scalar product $\vec{\tau} \cdot \vec{\Phi}_\pi$ with the nucleon isospin matrix $\vec{\tau}$ has the form

$$\vec{\tau} \cdot \vec{\Phi}_\pi = \pi^+(\vec{\tau} \cdot \hat{\Phi}_+) + \pi^-(\vec{\tau} \cdot \hat{\Phi}_-) + \pi^0(\vec{\tau} \cdot \hat{\Phi}_0), \tag{4.3}$$

where

$$\vec{\tau} \cdot \hat{\Phi}_+ = -\tau_-, \quad \vec{\tau} \cdot \hat{\Phi}_- = -\tau_+, \quad \vec{\tau} \cdot \hat{\Phi}_0 = \tau_3 \tag{4.4}$$

with the charge "*raising*" and charge "*lowering*" operators τ_+ and τ_- given in the *spherical basis* by [13]

$$\tau_\pm \equiv \frac{\mp\tau_1 - i\tau_2}{\sqrt{2}}. \tag{4.5}$$

4.1.2. Vertices for Vector Meson Terms

In the energy region of our interest, that is from *threshold* up to ~ 1.7 GeV, the main contribution of *vector mesons* to pion photoproduction is given by the ρ and ω exchanges. The role of the ϕ meson is found to be negligible, less than 2% of the $\rho + \omega$ contribution at threshold [17].

The ρNN Vertex

The interaction Lagrangian is given by [11]

$$\mathscr{L}_{\rho NN} = \overline{\Psi}_N \, \vec{\tau} \cdot \left[g_{\rho NN}^v \gamma_\alpha + \frac{g_{\rho NN}^t}{2M_N} \sigma_{\alpha\beta} \partial^\beta \right] \vec{\Phi}_\rho^{\,\alpha} \, \Psi_N, \tag{4.6}$$

where $g_{\rho NN}^v$ and $g_{\rho NN}^t$ are the *vector* and *tensor* couplings of the ρNN vertex, respectively and $\sigma_{\mu\nu}$ is defined by

$$\sigma_{\mu\nu} \equiv \frac{i}{2}[\gamma_\mu, \gamma_\nu]. \tag{4.7}$$

The experimental values of these couplings will be described below.

The ωNN Vertex

Except for the isospin, the interaction Lagrangian in this case is similar to the previous one and is given by [11]

$$\mathscr{L}_{\omega NN} = \overline{\Psi}_N \left[g_{\omega NN}^v \gamma_\alpha + \frac{g_{\omega NN}^t}{2M_N} \sigma_{\alpha\beta} \partial^\beta \right] \Phi_\omega^\alpha \Psi_N, \tag{4.8}$$

where $g_{\omega NN}^v$ and $g_{\omega NN}^t$ are the *vector* and *tensor* couplings of the ωNN vertex, respectively.

The Vector and Tensor Couplings: g^v_{VNN} and g^t_{VNN} ($V = \rho, \omega$)

The experimental values of the vector and tensor couplings of the vector meson-nucleon vertex (ρNN and ωNN) are taken from several sources. For example, analyses of strong interaction processes such as πN and NN scattering using dispersion relations [17] obtain the values

$$g^v_{\rho NN} = 2.63 \pm 0.38, \quad g^v_{\omega NN} = 10.09 \pm 0.93, \tag{4.9}$$

for the vector couplings, and

$$g^t_{\rho NN} = 16.05 \pm 0.82, \quad g^t_{\omega NN} = 1.42 \pm 1.99, \tag{4.10}$$

for the tensor couplings.
On the other hand, analysis of nucleon electromagnetic form factors [11] obtain the values

$$g^v_{\rho NN} = 2.63, \quad g^v_{\omega NN} = 20.86 \pm 0.25, \tag{4.11}$$

and

$$g^t_{\rho NN} = 15.86 \pm 0.52, \quad g^t_{\omega NN} = -3.41 \pm 0.24. \tag{4.12}$$

In Ref. [13], the reported values are

$$g^v_{\rho NN} = 2.66, \quad g^v_{\omega NN} = 7.98, \tag{4.13}$$

for the vector couplings, and

$$g^t_{\rho NN} = 9.84, \quad g^t_{\omega NN} = 0.0, \tag{4.14}$$

for the tensor couplings.
Thus we can see that the values of the couplings $g^v_{\omega NN}$, $g^t_{\rho NN}$, and $g^t_{\omega NN}$ are not well determined experimentally and therefore will be considered as *free* parameters to be varied within the estimated ranges in order to get the best fit.

4.1.3. Vertices for Spin-$\frac{1}{2}$ Resonance Terms

The interaction Lagrangian is given by [17]

$$\mathscr{L}_{\pi NR^\pm} = -\frac{f_{\pi NR^\pm}}{m_\pi} \left(\overline{\Psi}_N \Gamma_\mu \vec{\tau}\, \Psi_R\right) \cdot \partial^\mu \vec{\Phi}_\pi + \text{h.c.}, \tag{4.15}$$

where the coupling $f_{\pi NR^\pm}$ for the $\pi NR^\pm$ vertex is set to [17, 32]

$$\frac{f_{\pi NR^\pm}}{m_\pi} \equiv \pm \frac{g_{\pi NR^\pm}}{M_{R^\pm} \pm M_N} \tag{4.16}$$

with the upper (lower) sign corresponding to *even* (*odd*) parity resonances, and the operator structure for Γ_μ is, respectively, $\Gamma_\mu = \gamma_\mu$ for *odd* parity resonances, and $\Gamma_\mu = \gamma_\mu \gamma_5$ for *even* parity resonances.

Resonance	$\Gamma_{R\,\to\,\pi N}$ (%)	$\lvert f_{\pi NR}\rvert$
$P_{11}(1440)$	55 – 75	0.293 – 0.373
$S_{11}(1535)$	32 – 52	0.121 – 0.180
$S_{11}(1650)$	50 – 70	0.107 – 0.165
$P_{11}(1710)$	5 – 20	0.029 – 0.081

Table 4.1.: Estimated strong coupling constants for the *Spin*-$\frac{1}{2}$ nucleon resonances.

The Strong Coupling: $g_{\pi NR^\pm}$

It will be more convenient to express the couplings in terms of the experimentally observable quantities, namely the partial decay width ($\Gamma_{R^\pm\,\to\,\pi N}$).
The decay width for the process $R^\pm \to \pi N$ is given by [27]

$$\frac{d\Gamma}{d\Omega} = \frac{1}{2M_R}\frac{1}{16\pi^2}\frac{|\vec{q}|}{\sqrt{s}}\overline{|\mathcal{M}_{\rm fi}|^2}, \tag{4.17}$$

where

$$\mathcal{M}_{\rm fi} = -i\frac{f_{\pi NR^\pm}}{m_\pi} I_R\,\overline{u}(p_{\rm f})\not{q}\,[\Gamma]\,u(p_{\rm i}), \tag{4.18}$$

with I_R the corresponding isospin factor (see Table 6.3), $\Gamma = \mathbb{1}\,(\gamma_5)$ for the *even* (*odd*) parity resonances and $\overline{|\mathcal{M}_{\rm fi}|^2}$ denotes the average over the initial spin ($s_{\rm i}$) and sum over the final spins ($s_{\rm f}$), namely

$$\overline{|\mathcal{M}_{\rm fi}|^2} \equiv \frac{1}{2}\sum_{s_{\rm i}}\sum_{s_{\rm f}}|\mathcal{M}_{\rm fi}|^2 \tag{4.19}$$

$$= \frac{1}{2}\frac{f^2_{\pi NR^\pm}}{m_\pi^2} I_R^2 {\rm Tr}\left[(\not{p}_{\rm i} + M_{\rm R})\not{q}\Gamma(\not{p}_{\rm f} + M_{\rm N})\not{q}\Gamma\right] \tag{4.20}$$

$$= 2g^2_{\pi NR^\pm} I_R^2 M_{R^\pm}\left[E_{\rm f}(M_{R^\pm}) \mp M_{\rm N}\right]. \tag{4.21}$$

After integrating over the phase space and summing over all channels ($\Gamma_{R^\pm\,\to\,\pi N} = \Gamma_{R^\pm\,\to\,\pi^+ n} + \Gamma_{R^\pm\,\to\,\pi^0 p}$)

$$\frac{g^2_{\pi NR^\pm}}{4\pi} = \frac{M_{R^\pm}}{3|\vec{q}(M_{R^\pm})|\left[E_{\rm f}(M_{R^\pm}) \mp M_{\rm N}\right]}\Gamma_{R^\pm\,\to\,\pi N}, \tag{4.22}$$

with $E_{\rm f}$ and $|\vec{q}|$ evaluated at $W = M_{R^\pm}$.
The magnitude of the estimated values of the strong coupling constants for the spin-$\frac{1}{2}$ nucleon resonances are displayed in Table 4.1, according with the decay width ranges given in the previous column of the same table [1].

4.1.4. Vertices for Spin-$\frac{3}{2}$ Resonance Terms

The $\pi N \Delta$ Vertex

The interaction of the Δ resonances with the pion and the nucleon has been discussed extensively in the literature [11, 19, 31]. In the present case, we consider the interaction Lagrangian given by [13, 19, 31, 33]

$$\mathscr{L}_{\pi N \Delta} = \frac{f_{\pi N \Delta}}{m_\pi} \left(\overline{\Psi}^\mu_\Delta \vec{T} \, \Theta_{\mu\nu} \Psi_N \right) \cdot \partial^\nu \vec{\Phi}_\pi + \text{h.c.}, \tag{4.23}$$

where $\vec{T}$ is the $N \to \Delta$ *isospin excitation operator* given by [11]

$$T_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & 0 \\ 0 & -1 \end{pmatrix}, \, T_2 = -\frac{i}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & 0 \\ 0 & 1 \end{pmatrix}, \, T_3 = -\sqrt{\frac{2}{3}} \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \tag{4.24}$$

with T_1 ,T_2 and T_3 such that

$$T_i^\dagger T_j = \frac{2}{3}\delta_{ij} - \frac{1}{3} i\epsilon_{ijk}\tau_k, \quad (i,j,k = 1,2,3) \tag{4.25}$$

and the tensor $\Theta_{\mu\nu}$ is defined as [34]

$$\Theta_{\mu\nu} \equiv g_{\mu\nu} + \left[\frac{1}{2}(1+4Z)A + Z \right] \gamma_\mu \gamma_\nu, \tag{4.26}$$

in order to guarantee that the *total* Lagrangian is invariant under the point transformation,

$$\Psi^\mu_X \to \Psi'^\mu_X = \Psi^\mu_X + a\gamma^\mu \gamma_\nu \Psi^\nu_X, \quad A \to A' = \frac{A - 2a}{1 + 4a}, \tag{4.27}$$

and

$$\Psi_N \to \Psi'_N = \Psi_N, \quad \vec{\Phi}_\pi \to \vec{\Phi}'_\pi = \vec{\Phi}_\pi. \tag{4.28}$$

The parameter Z, usually referred as the *off-shell* parameter, is arbitrary and will appear in the physical amplitudes. However, it can be set to a fixed value or just let it run freely to obtain the best possible fit. In this work we shall adopt the former and fix its value to $\frac{1}{2}$ if, in accordance with the principles of second quantization, the *interacting* fields are quantized on a spacelike surface [19].
With this choice, the tensor $\Theta_{\mu\nu}$ becomes

$$\Theta_{\mu\nu} = g_{\mu\nu} + \frac{1}{2}(1 + 3A)\gamma_\mu \gamma_\nu, \tag{4.29}$$

in agreement with [31, 35].
On the other hand, the S-matrix elements for the interaction Lagrangian given by Eq. (4.23) are *independent* of the parameter A according to an *equivalence theorem*

given in Ref. [36].
Therefore we will choose $A = -\frac{1}{3}$ so that

$$\Theta_{\mu\nu} = g_{\mu\nu}, \tag{4.30}$$

and the interaction Lagrangian describing the $\pi N\Delta$ vertex becomes [31, 35]

$$\mathscr{L}_{\pi N\Delta} = \frac{f_{\pi N\Delta}}{m_\pi}\left(\overline{\Psi}^\mu_\Delta \vec{T}\,\Psi_N\right)\cdot \partial_\mu \vec{\Phi}_\pi + \text{h.c.} \tag{4.31}$$

The Strong Coupling: $f_{\pi N\Delta}$

Expressing the coupling in terms of the partial decay width ($\Gamma_{\Delta\,\to\,\pi N}$)

$$\frac{d\Gamma}{d\Omega} = \frac{1}{2M_\Delta}\frac{1}{16\pi^2}\frac{|\vec{q}|}{\sqrt{s}}\overline{|\mathcal{M}_{\text{fi}}|^2}, \tag{4.32}$$

where

$$\mathcal{M}_{\text{fi}} = i\frac{f_{\pi N\Delta}}{m_\pi}I_\Delta\,\overline{u}(p_{\text{f}})u_\alpha(p_{\text{i}})q^\alpha, \tag{4.33}$$

with I_Δ an isospin factor (see Table 6.3), $u_\alpha(p_{\text{i}})$ is the corresponding *Rarita-Schwinger* spinor, and $\overline{|\mathcal{M}_{\text{fi}}|^2}$ denotes the average over the initial spin (s_{i}) and sum over the final spins (s_{f}), namely

$$\overline{|\mathcal{M}_{\text{fi}}|^2} \equiv \frac{1}{4}\sum_{s_{\text{i}}}\sum_{s_{\text{f}}}|\mathcal{M}_{\text{fi}}|^2 \tag{4.34}$$

$$= \frac{1}{4}\frac{f^2_{\pi N\Delta}}{m^2_\pi}I^2_\Delta \text{Tr}\left[q^\alpha(\not{p}_{\text{i}} + M_\Delta)\mathcal{P}^{\frac{3}{2}}_{\alpha\beta}(p_{\text{i}})q^\beta(\not{p}_{\text{f}} + M_{\text{N}})\right], \tag{4.35}$$

where

$$\sum_s u_\alpha(p)\overline{u}_\beta(p) = (\not{p} + M_\Delta)\mathcal{P}^{\frac{3}{2}}_{\alpha\beta}(p), \tag{4.36}$$

with $\mathcal{P}^{\frac{3}{2}}_{\alpha\beta}(p)$, the spin-$\frac{3}{2}$ *projection operator*, given by [32]

$$\mathcal{P}^{\frac{3}{2}}_{\alpha\beta}(p) \equiv g_{\alpha\beta} - \frac{1}{3p^2}p_\alpha p_\beta - \frac{1}{3}\gamma_\alpha\gamma_\beta - \frac{1}{3p^2}\left(p_\alpha\gamma_\beta - p_\beta\gamma_\alpha\right)\not{p}. \tag{4.37}$$

Therefore, in the mass shell of the decaying particle ($p^2 = M^2_\Delta$), we obtain

$$\overline{|\mathcal{M}_{\text{fi}}|^2} = \frac{2}{3}\frac{f^2_{\pi N\Delta}}{m^2_\pi}I^2_\Delta M_\Delta|\vec{q}(M_\Delta)|^2\left[E_{\text{f}}(M_\Delta) + M_{\text{N}}\right]. \tag{4.38}$$

After integrating over the phase space and summing over all channels ($\Gamma_{\Delta\,\to\,\pi N} = \Gamma_{\Delta\,\to\,\pi^+ n} + \Gamma_{\Delta\,\to\,\pi^0 p}$)

$$\frac{f^2_{\pi N\Delta}}{4\pi} = \frac{3m^2_\pi M_\Delta}{|\vec{q}(M_\Delta)|^3\left[E_{\text{f}}(M_\Delta) + M_{\text{N}}\right]}\Gamma_{\Delta\,\to\,\pi N}, \tag{4.39}$$

with E_{f} and $|\vec{q}|$ evaluated at $W = M_\Delta$.
The magnitude of the estimated value of the strong coupling constant for the spin-$\frac{3}{2}$ (isospin-$\frac{3}{2}$) nucleon resonance is displayed in Table 4.2, according with the value of the decay width given in the previous column of the same table [1].

Resonance	$\Gamma_{R\to\pi N}$ (%)	$\lvert f_{\pi NR}\rvert$
$P_{33}(1232)$	99.4	$\begin{cases} 2.214 - 2.452\ (\Delta^+) \\ 2.252 - 2.274\ (\Delta^0) \end{cases}$
$D_{13}(1520)$	55 – 65	1.504 – 1.748
$P_{33}(1600)$	8 – 24	0.311 – 0.703

Table 4.2.: Estimated strong coupling constants for the *Spin*-$\frac{3}{2}$ nucleon resonances.

The πND Vertex

This resonance is similar to the previous one except that it has the opposite parity and isospin-$\frac{1}{2}$. The Lagrangian for the πND vertex is then given by

$$\mathscr{L}_{\pi ND} = -\frac{f_{\pi ND}}{m_\pi}\left(\overline{\Psi}^{\mu}_{D}\gamma_5\vec{\tau}\,\Psi_N\right)\cdot\partial_\mu\vec{\Phi}_\pi + \text{h.c.}, \tag{4.40}$$

where we have made the replacement

$$\overline{\Psi}^{\mu}_{\Delta}\vec{T} \to \overline{\Psi}^{\mu}_{D}\vec{\tau}, \tag{4.41}$$

with

$$\Psi^{\mu}_{D} = \begin{pmatrix}\psi^{\mu}_{D^+}\\ \psi^{\mu}_{D^0}\end{pmatrix}. \tag{4.42}$$

The Strong Coupling: $f_{\pi ND}$

The decay width for the process $D \to \pi N$ is given by

$$\frac{d\Gamma}{d\Omega} = \frac{1}{2M_D}\frac{1}{16\pi^2}\frac{|\vec{q}|}{\sqrt{s}}\overline{|\mathcal{M}_{\text{fi}}|^2}, \tag{4.43}$$

where

$$\mathcal{M}_{\text{fi}} = i\frac{f_{\pi ND}}{m_\pi}I_D\,\overline{u}(p_{\text{f}})\gamma_5 u_\alpha(p_{\text{i}})q^\alpha, \tag{4.44}$$

with $I_D = I_R$, and $\overline{|\mathcal{M}_{\text{fi}}|^2}$ denotes the average over the initial spin (s_{i}) and sum over the final spins (s_{f}), namely

$$\overline{|\mathcal{M}_{\text{fi}}|^2} \equiv \frac{1}{4}\sum_{s_{\text{i}}}\sum_{s_{\text{f}}}|\mathcal{M}_{\text{fi}}|^2 \tag{4.45}$$

$$= -\frac{1}{4}\frac{f^2_{\pi ND}}{m^2_\pi}I^2_D\,\text{Tr}\left[q^\alpha(\not{p}_{\text{i}} + M_D)\mathcal{P}^{\frac{3}{2}}_{\alpha\beta}(p_{\text{i}})\gamma_5 q^\beta(\not{p}_{\text{f}} + M_{\text{N}})\gamma_5\right], \tag{4.46}$$

where $\mathcal{P}^{\frac{3}{2}}_{\alpha\beta}(p_{\text{i}})$ is given by Eq. (4.37). With $M_\Delta \to M_D$, then

$$\overline{|\mathcal{M}_{\text{fi}}|^2} = \frac{2}{3}\frac{f^2_{\pi ND}}{m^2_\pi}I^2_D M_D|\vec{q}(M_D)|^2\left[E_{\text{f}}(M_D) - M_{\text{N}}\right]. \tag{4.47}$$

After integrating over the phase space and summing over all channels ($\Gamma_{D \to \pi N} = \Gamma_{D \to \pi^+ n} + \Gamma_{D \to \pi^0 p}$)

$$\frac{f^2_{\pi N D}}{4\pi} = \frac{m^2_\pi M_D \left[E_{\rm f}(M_D) + M_{\rm N}\right]}{|\vec{q}(M_D)|^5} \Gamma_{D \to \pi N}, \tag{4.48}$$

with $E_{\rm f}$ and $|\vec{q}|$ evaluated at $W = M_D$.
The magnitude of the estimated value of the strong coupling constant for the spin-$\frac{3}{2}$ (isospin-$\frac{1}{2}$) nucleon resonance is displayed in Table 4.2, according with the decay width ranges given in the previous column of the same table [1].

4.2. Electromagnetic Interaction

For the case of the Born terms, the electromagnetic interaction is introduced by means of *minimal coupling*, that is, replacing the differential opertor $\frac{\partial}{\partial x^\mu}$ in the Lagrangian of the system by

$$\frac{\partial}{\partial x^\mu} \to \frac{\partial}{\partial x^\mu} + i\hat{Q}A_\mu, \tag{4.49}$$

where A_μ is the *photon* field and $\hat{Q} \equiv \hat{Q}_N + \hat{Q}_\pi$ is the total charge operator [37].
The *extended structure* of the nucleon and the pion is considered by including the *isoscalar*, the *isovector*, and the pion *form factors*

$$\hat{Q}_N \equiv \frac{e}{2}\left[F^s_1(k^2) + F^v_1(k^2)\tau_3\right], \tag{4.50}$$

and

$$\hat{Q}_\pi \equiv eF_\pi(k^2)\hat{T}_3, \tag{4.51}$$

where

$$F^s_1 \equiv F^p_1 + F^n_1 \quad \text{and} \quad F^v_1 \equiv F^p_1 - F^n_1, \tag{4.52}$$

which at the *photon point* ($k^2 = 0$) take the values

$$F^s_1 = F^v_1 = 1 \quad \text{and} \quad F_\pi = F^v_1 \tag{4.53}$$

to ensure *gauge invariance* [11].
The matrix elements of the isospin operator $\hat{T}_3$ in a cartesian isospin basis are given by

$$\langle \pi_i | \hat{T}_3 | \pi_j \rangle = i\epsilon_{i3j}, \tag{4.54}$$

and the *magnetic moment* of the nucleon is taken *phenomenologically* into account by adding the *magnetic dipole term* [25]

$$i\frac{\mu_{\rm B}}{2}\left[F^s_2(k^2) + F^v_2(k^2)\tau_3\right]\sigma^{\mu\nu}F_{\mu\nu}, \tag{4.55}$$

where

$$F^s_2 \equiv \frac{F^p_2 + F^n_2}{2} \quad \text{and} \quad F^v_2 \equiv \frac{F^p_2 - F^n_2}{2}, \tag{4.56}$$

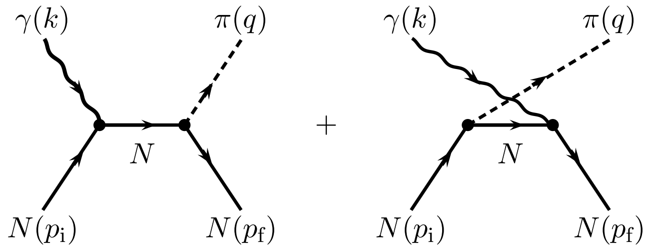


Figure 4.1.: Feynman diagrams for Nucleon Born terms: (a) direct or s-channel, and (b) crossed or u-channel.

which at the *photon point* take the values

$$F_2^p = \kappa^p = 1.79 \quad \text{and} \quad F_2^n = \kappa^n = -1.91 \tag{4.57}$$

in units of $\mu_{\mathrm{B}} \equiv \frac{e}{2M_N}$, $\sigma_{\mu\nu}$ es given by Eq. (4.7) and the electromagnetic field tensor is defined by [27]

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu . \tag{4.58}$$

4.2.1. Vertices for Born Terms

The γNN Vertex

Applying the minimal gauge invariant coupling according to Eqs. (4.50) and (4.55) to the free Dirac Lagrangian given by Eq. (3.12) we obtain the effective γNN interaction Lagrangian

$$\mathscr{L}_{\gamma NN} = -\frac{e}{2} A^\alpha \overline{\Psi}_N \gamma_\alpha \left(F_1^s + F_1^v \tau_3\right) \Psi_N - \frac{e}{4M_N} \overline{\Psi}_N \left(F_2^s + F_2^v \tau_3\right) \sigma_{\alpha\beta} \Psi_N F^{\alpha\beta} . \tag{4.59}$$

From Lagrangians (4.1) and (4.59) we obtain the tree level Feynman diagrams of Fig. 4.1.

The $\gamma\pi NN$ Vertex

In this case the minimal gauge invariant coupling applied to the interaction Lagrangian (4.1) leads to the following effective interaction Lagrangian involving the pion

$$\mathscr{L}_{\gamma\pi NN} = -e \frac{f_{\pi NN}}{m_\pi} \overline{\Psi}_N \gamma_5 \gamma_\mu \left[\vec{\tau} \times \vec{\Phi}_\pi\right]_3 \Psi_N A^\mu . \tag{4.60}$$

From Lagrangians (4.1) and (4.60) we obtain the tree level Feynman diagram of Fig. 4.2a.

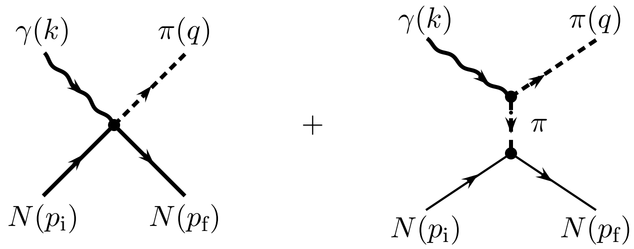


Figure 4.2.: Feynman diagrams for the Born terms: (a) Kroll-Rudermann (contact) term, and (b) pion in flight term.

The $\gamma\pi\pi$ Vertex

Besides the contact term given by Eq. (4.60), the minimal gauge invariant coupling applied to the free Lagrangian (3.1) also leads to the effective $\gamma\pi\pi$ interaction Lagrangian

$$\mathscr{L}_{\gamma\pi\pi} = -e\left[\vec{\Phi}_\pi \times \partial_\mu \vec{\Phi}_\pi\right]_3 A^\mu. \tag{4.61}$$

From Lagrangians (4.1) and (4.61) we obtain the tree level Feynman diagram of Fig. 4.2b.

4.2.2. Vertices for Vector Meson Terms

For the interaction of vector mesons with pion and photon we use the following *standard* Lagrangians [13, 38]:

The $\rho\pi\gamma$ Vertex

For the *isospin*-1 ρ meson,

$$\mathscr{L}_{\rho\pi\gamma} = e\frac{\lambda_{\rho\pi\gamma}}{2m_\pi}\tilde{F}_{\mu\nu}\,\vec{W}^{\mu\nu}\cdot\vec{\Phi}_\pi, \tag{4.62}$$

where the tensor $\vec{W}^{\mu\nu}$ is given by Eq. (3.7) and $\tilde{F}_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$ is the *dual* of $F^{\mu\nu}$. With

$$\epsilon_{\alpha\rho\mu\nu}\,\partial^\mu F^{\alpha\rho} = 0, \tag{4.63}$$

the above Lagrangian may equivalently be written as

$$\mathscr{L}_{\rho\pi\gamma} = -e\frac{\lambda_{\rho\pi\gamma}}{m_\pi}\tilde{F}_{\mu\nu}\,\partial^\mu\vec{\Phi}_\pi\cdot\vec{\Phi}_\rho^{\,\nu}. \tag{4.64}$$

The $\omega\pi\gamma$ Vertex

Similarly, for the *isospin*-0 ω meson,

$$\mathscr{L}_{\omega\pi\gamma} = e\frac{\lambda_{\omega\pi\gamma}}{2m_\pi}\tilde{F}_{\mu\nu}B^{\mu\nu}\left[\vec{\Phi}_\pi\right]_3, \tag{4.65}$$

Vector meson	$\Gamma_{V\to\pi\gamma}$ (keV)	$\|\lambda_{V\pi\gamma}\|$
$\rho(770)$	$\rho^{\pm}$: 67 ± 8	$0.092-0.106$
	ρ^{0}: 89 ± 12	$0.109-0.123$
$\omega(782)$	703 ± 7	$0.310-0.314$

Table 4.3.: Estimated electromagnetic coupling constants for the vector mesons.

where the tensor $B^{\mu\nu}$ is given by Eq. (3.11).
The above Lagrangian is equivalent to

$$\mathscr{L}_{\omega\pi\gamma} = -e\frac{\lambda_{\omega\pi\gamma}}{m_\pi}\tilde{F}_{\mu\nu}\left[\partial^\mu\vec{\Phi}_\pi\right]_3\Phi^\nu_\omega. \tag{4.66}$$

From Lagrangians (4.6), (4.8), (4.64), and (4.66) we obtain the tree level Feynman diagram of Fig. 4.3.

The Electromagnetic Coupling: $\lambda_{V\pi\gamma}$

The electromagnetic $\lambda_{V\pi\gamma}$ couplings can be estimated from the partial decay widths of the vector mesons by using the Lagrangians (4.64) and (4.66).
The decay width for the process $V\to\pi\gamma$ is given by [27]

$$\frac{d\Gamma}{d\Omega} = \frac{1}{2m_V}\frac{1}{16\pi^2}\frac{|\vec{k}|}{\sqrt{s}}\overline{|\mathcal{M}_{\rm fi}|^2}, \tag{4.67}$$

where

$$\mathcal{M}_{\rm fi} = -\frac{e\lambda_{V\pi\gamma}}{m_\pi}\epsilon_{\mu\nu\alpha\beta}\epsilon^{\alpha*}_\lambda k^\beta q^\mu \epsilon^\nu_\sigma(p), \tag{4.68}$$

with ϵ^α_λ and $\epsilon^\nu_\sigma(p)$, the photon and vector meson polarization vectors, respectively.
$\overline{|\mathcal{M}_{\rm fi}|^2}$ denotes the average over the vector meson polarization (σ) and sum over the photon polarization (λ), namely

$$\overline{|\mathcal{M}_{\rm fi}|^2} \equiv \frac{1}{3}\sum_\sigma\sum_\lambda|\mathcal{M}_{\rm fi}|^2 \tag{4.69}$$

$$= \frac{1}{3}\frac{(e\lambda_{V\pi\gamma})^2}{m_\pi^2}\epsilon_{\mu\nu\alpha\beta}\epsilon_{\rho\theta\gamma\delta}k^\beta q^\mu k^\delta q^\rho\left(g^{\theta\nu}-\frac{p^\theta p^\nu}{m_V^2}\right)g^{\alpha\gamma} \tag{4.70}$$

$$= \frac{2}{3}\frac{(e\lambda_{V\pi\gamma})^2}{m_\pi^2}|\vec{k}|^2 m_V^2. \tag{4.71}$$

After integrating over the phase space

$$\frac{(e\lambda_{V\pi\gamma})^2}{4\pi} = \frac{24m_\pi^2 m_V^3}{(m_V^2-m_\pi^2)^3}\Gamma_{V\to\pi\gamma}. \tag{4.72}$$

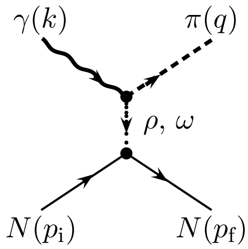


Figure 4.3.: Feynman diagram for vector meson exchanges: ρ and ω.

The magnitude of the estimated values of the electromagnetic coupling constants $\lambda_{\rho\pi\gamma}$ and $\lambda_{\omega\pi\gamma}$ are displayed in Table 4.3, according with the decay width values given in the previous column of the same table [1].

4.2.3. Vertices for Resonance Terms

The $\gamma NR^{\pm}$ Vertex

For a spin-$\frac{1}{2}$ nucleon resonance, the coupling to the photon that preserves gauge invariance is analog to the coupling of the nucleon to the photon given by Eq. (4.59). In this case, the first term of Eq. (4.59) is absent because the difference of the masses of the resonance and the nucleon leads to violation of gauge invariance. Therefore the effective $\gamma NR^{\pm}$ Lagrangian is given by

$$\mathscr{L}_{\gamma NR^{\pm}} = \pm\frac{e}{2(M_N + M_R)}\overline{\Psi}_N\Gamma_{\alpha\beta}\left(\kappa_R^s + \kappa_R^v\tau_3\right)\Psi_R F^{\alpha\beta} + \text{h.c.}, \tag{4.73}$$

where $\kappa_R^p \equiv \kappa_R^s + \kappa_R^v$, and $\kappa_R^n \equiv \kappa_R^s - \kappa_R^v$ are the *transition magnetic couplings* for the *proton* and *neutron* targets, respectively and the operator structure for $\Gamma_{\alpha\beta}$ is $\Gamma_{\alpha\beta} = \gamma_5\sigma_{\alpha\beta}$ for *odd* nucleon resonances (R^-), and $\Gamma_{\alpha\beta} = \sigma_{\alpha\beta}$ for *even* nucleon resonances (R^+).
This vertex is *similar* to the γNN vertex given by Eq. (4.59), except that the first term in the right-hand side of this equation is abscent because its presence violates gauge invariance due to the mass difference of the resonance (R) and the nucleon (N).
From Lagrangians (4.15) and (4.73) we obtain the tree level Feynman diagrams of Fig. 4.4.

The Transition Magnetic Moments: $\kappa_R^{p(n)}$

The transition magnetic moments will be conveniently expressed in terms of the *experimental helicity amplitudes* $A_{\frac{1}{2}}^{p(n)}$ [39].
First, the decay width for the process $R \to \gamma N$ is given by [27]

$$\frac{d\Gamma}{d\Omega} = \frac{1}{2M_R}\frac{1}{16\pi^2}\frac{|\vec{k}|}{\sqrt{s}}\overline{|\mathcal{M}_{\rm fi}|^2}, \tag{4.74}$$

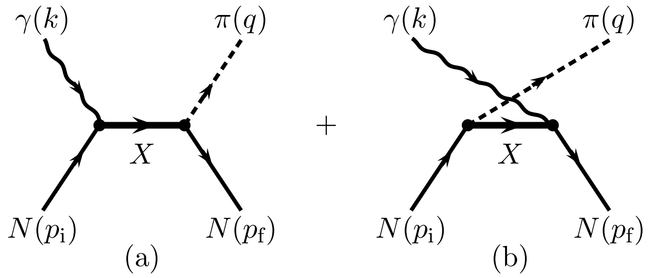


Figure 4.4.: Feynman diagrams for resonance excitations ($X = R, \Delta, D$): (a) direct or s-channel, and (b) crossed or u-channel.

where

$$\mathcal{M}_{\rm fi} = \frac{e\kappa_R}{M_N + M_R} \overline{u}(p_{\rm f}) \, [\not{k}\not{\epsilon}^*] \, u(p_{\rm i}), \tag{4.75}$$

with ϵ_λ^α the photon polarization vector, and $\overline{|\mathcal{M}_{\rm fi}|^2}$ is given by

$$\overline{|\mathcal{M}_{\rm fi}|^2} \equiv \frac{1}{2} \sum_{s_{\rm i}} \sum_{s_{\rm f}} \sum_{\lambda} |\mathcal{M}_{\rm fi}|^2 \tag{4.76}$$

$$= \frac{1}{2} \left(\frac{e\kappa_R}{M_N + M_R} \right)^2 \mathrm{Tr} \left[(\not{p}_{\rm i} + M_{\rm R}) \not{\epsilon} \not{k} (\not{p}_{\rm f} + M_{\rm N}) \not{k} \not{\epsilon}^* \right] \tag{4.77}$$

$$= 8 \left(\frac{e\kappa_R}{M_N + M_R} \right)^2 M_R^2 |\vec{k}|^2. \tag{4.78}$$

Then, integrating over the phase space, one obtains the radiative width

$$\Gamma_{R \to \gamma N} = \left(\frac{e\kappa_R}{M_N + M_R} \right)^2 \frac{|\vec{k}|^3}{\pi} \tag{4.79}$$

$$= \left(\frac{e\kappa_R}{M_N + M_R} \right)^2 \frac{|\vec{k}|^2}{\pi} \frac{M_R^2 - M_N^2}{2M_R}. \tag{4.80}$$

Second, the decay width of a *spin*-$\frac{1}{2}$ resonance can also be determined in terms of the helicity amplitude ($A_{\frac{1}{2}}^{p(n)}$) for the excitation of the nucleon into a resonant state of *helicity*-$\frac{1}{2}$ through [39]

$$\Gamma_{R \to \gamma N} = \frac{|\vec{k}|^2}{\pi} \frac{M_N}{M_R} |A_{\frac{1}{2}}^{p(n)}|^2, \tag{4.81}$$

therefore

$$(e\kappa_R)^2 = 2M_N \left(\frac{M_R + M_N}{M_R - M_N} \right) |A_{\frac{1}{2}}^{p(n)}|^2. \tag{4.82}$$

In Table 4.4 we present their absolute values, according with the ranges of the values of the helicity amplitudes given in the previous column of the same table [1], but their sign will be determined by the best fit.

Resonance	$A_{\frac{1}{2}}^{\binom{p}{n}}$ $(\text{GeV}^{-\frac{1}{2}})$	$\lvert\kappa_R^p\rvert$	$\lvert\kappa_R^n\rvert$
$P_{11}(1440)$	$\{ \begin{matrix} -0.057 - -0.039 \\ 0.035 - 0.055 \end{matrix}$	$0.403 - 0.601$	$0.363 - 0.571$
$S_{11}(1535)$	$\{ \begin{matrix} 0.046 - 0.102 \\ -0.092 - -0.084 \end{matrix}$	$0.429 - 0.957$	$0.789 - 0.862$
$S_{11}(1650)$	$\{ \begin{matrix} 0.015 - 0.038 \\ 0.012 - 0.020 \end{matrix}$	$0.129 - 0.327$	$0.102 - 0.172$
$P_{11}(1710)$	$\{ \begin{matrix} 0.026 - 0.037 \\ -0.060 - 0.006 \end{matrix}$	$0.218 - 0.310$	$0.050 - 0.505$

Table 4.4.: Estimated transition magnetic moments for the *spin*-$\frac{1}{2}$ nucleon resonances.

The $\gamma N\Delta$ Vertex

With respect to the $\gamma N\Delta$ vertex, the so called *normal parity decomposition* (G_1, G_2) given by [12, 40]

$$\mathscr{L}_{\gamma N\Delta} = ie\overline{\Psi}^{\mu}_{\Delta} T_3 \mathbf{\Gamma}^{(\text{NP})}_{\mu\nu} \Psi_{\text{N}} A^{\nu} + \text{h.c.}, \tag{4.83}$$

which will be described below with more detail, has been widely used.
However, another decomposition based upon the same idea as the *Sachs form factors* for the nucleon is also possible [32]. This decomposition, known as the *covariant multipole decomposition* (G_E, G_M), is directly connected to physical quantities, such as the *electric* and *magnetic multipoles* which are of great interest from both experimental and theoretical points of view [13, 40]. This second decomposition is equivalent to the normal parity decomposition when baryons are *on shell* as we will show below [35].

- **The Covariant Multipole Decomposition (MD)**

  The $\gamma N\Delta$ interaction Lagrangian is given by

$$\mathscr{L}_{\gamma N\Delta} = ie\overline{\Psi}^{\mu}_{\Delta} T_3 \mathbf{\Gamma}^{(\text{MD})}_{\mu\nu} \Psi_{N} A^{\nu} + \text{h.c.}, \tag{4.84}$$

  where $\Gamma^{(\text{MD})}_{\mu\nu}$ is written in a covariant multipole decomposition as [13, 40]

$$\mathbf{\Gamma}^{(\text{MD})}_{\mu\nu} \equiv G_M K^M_{\mu\nu} + G_E K^E_{\mu\nu}. \tag{4.85}$$

  G_M and G_E are the *magnetic* and *electric* form factors of the Δ resonance, respectively, and the tensors $K^M_{\mu\nu}$ and $K^E_{\mu\nu}$ are given respectively by

$$K^M_{\mu\nu} \equiv -\frac{3}{2M_N \Sigma M} \epsilon_{\mu\nu\alpha\beta} P^{\alpha} k^{\beta}, \tag{4.86}$$

  and

$$K^E_{\mu\nu} \equiv -K^M_{\mu\nu} - \frac{6}{M_N \Sigma M (\Delta M)^2} \epsilon_{\mu\lambda\alpha\beta} P^{\alpha} k^{\beta} \epsilon_{\nu}{}^{\lambda}{}_{\gamma\delta}\, p^{\gamma}_{\Delta} k^{\delta} i\gamma_5, \tag{4.87}$$

  with $P \equiv \frac{1}{2}(p_i + p_\Delta)$ for the s-channel, $P \equiv \frac{1}{2}(p_f + p_\Delta)$ for the u-channel, $\Sigma M \equiv M_\Delta + M_N$, and $\Delta M \equiv M_\Delta - M_N$.

- **The Normal Parity Decomposition (NP)**

  The most general electromagnetic interaction Lagrangians are given by [41]

$$\mathscr{L}^{(1)}_{\gamma N\Delta} = ie\frac{G_1}{2M_{\rm N}}\overline{\Psi}^{\mu}_{\Delta}T_3\Theta_{\mu\lambda}(X)\gamma_\nu\gamma_5\Psi_{\rm N}F^{\nu\lambda} + \text{h.c.}, \tag{4.88}$$

  and

$$\mathscr{L}^{(2)}_{\gamma N\Delta} = -e\frac{G_2}{2M^2_{\rm N}}\overline{\Psi}^{\mu}_{\Delta}T_3\Theta_{\mu\nu}(Y)\gamma_5(\partial_\lambda\Psi_{\rm N})F^{\nu\lambda} + \text{h.c.}, \tag{4.89}$$

  where the tensor $\Theta_{\mu\nu}$ was defined in Eq. (4.26) and X and Y are off-shell parameters.

  With $X = Y = \frac{1}{2}$ [19] and $A = -\frac{1}{3}$ [35], as it was discussed above, we obtain the so called *normal parity decomposition* [12, 40, 42] for the $\gamma N\Delta$ vertex

$$\mathscr{L}_{\gamma N\Delta} = ie\overline{\Psi}^{\mu}_{\Delta}T_3\mathbf{\Gamma}^{(\rm NP)}_{\mu\nu}\Psi_{\rm N}A^\nu + \text{h.c.}, \tag{4.90}$$

  where

$$\mathbf{\Gamma}^{(\rm NP)}_{\mu\nu} \equiv -i\left[\frac{G_1}{2M_{\rm N}}\mathscr{K}^1_{\mu\nu} - \frac{G_2}{2M^2_{\rm N}}\mathscr{K}^2_{\mu\nu}\right], \tag{4.91}$$

  with the *standard normal parity* set $(\mathscr{K}^1_{\mu\nu}, \mathscr{K}^2_{\mu\nu})$ defined as

$$\mathscr{K}^1_{\mu\nu} \equiv (k_\mu\gamma_\nu - \not{k}g_{\mu\nu})\gamma_5, \tag{4.92}$$

  and

$$\mathscr{K}^2_{\mu\nu} \equiv (k_\mu P_\nu - (P\cdot k)g_{\mu\nu})\gamma_5, \tag{4.93}$$

  in accordance with the notation of [40].

Relation Between the MD and NP Sets

For this purpose we have to make use of the following "non-trivial" relation [35]

$$\begin{aligned}\epsilon_{\alpha\beta\mu\nu}A^\mu B^\nu\gamma_5 = {} & (A\cdot B - \not{A}\not{B})\sigma_{\alpha\beta} + i\not{B}(\gamma_\alpha A_\beta - \gamma_\beta A_\alpha) - i\not{A}(\gamma_\alpha B_\beta - \gamma_\beta B_\alpha) \\ & + i(A_\alpha B_\beta - A_\beta B_\alpha).\end{aligned} \tag{4.94}$$

By taking $A = P$ and $B = k$, we get in general that

$$\begin{aligned}\epsilon_{\mu\nu\alpha\beta}P^\alpha k^\beta\gamma_5 = {} & i\gamma_\mu\left[(P\cdot k - \not{P}\not{k})\gamma_\nu - (\not{k}P_\nu - \not{P}k_\nu)\right] + i(\not{k}P_\mu - \not{P}k_\mu)\gamma_\nu \\ & - i(P\cdot k - \not{P}\not{k})g_{\mu\nu} + i(k_\mu P_\nu - P_\mu k_\nu).\end{aligned} \tag{4.95}$$

In the limit case of Δ *on-shell*,

$$\overline{\Psi}^{\mu}_{\Delta}\gamma_\mu = 0, \quad \overline{\Psi}^{\mu}_{\Delta}p_{\Delta\mu} = 0, \quad \text{and} \quad \overline{\Psi}^{\mu}_{\Delta}\not{p}_{\Delta} = M_\Delta\overline{\Psi}^{\mu}_{\Delta}, \tag{4.96}$$

from which we obtain, in terms of the standard parity set $(\mathscr{K}^1_{\mu\nu}, \mathscr{K}^2_{\mu\nu})$ that

$$\epsilon_{\mu\nu\alpha\beta} P^\alpha k^\beta = i \left[M_\Delta \mathscr{K}^1_{\mu\nu} + \mathscr{K}^2_{\mu\nu} \right] . \tag{4.97}$$

The tensor $K^M_{\mu\nu}$ then becomes

$$K^M_{\mu\nu} = -iK_M \left[M_\Delta \mathscr{K}^1_{\mu\nu} + \mathscr{K}^2_{\mu\nu} \right] , \tag{4.98}$$

where $K_M \equiv \dfrac{3}{2M_N \Sigma M}$.
Next, we make use of the identity

$$-\epsilon_{\mu\lambda\alpha\beta}\, \epsilon_\nu{}^\lambda{}_{\gamma\delta} = \begin{vmatrix} g_{\mu\nu} & g_{\mu\gamma} & g_{\mu\delta} \\ g_{\alpha\nu} & g_{\alpha\gamma} & g_{\alpha\delta} \\ g_{\beta\nu} & g_{\beta\gamma} & g_{\beta\delta} \end{vmatrix} , \tag{4.99}$$

from which we obtain (for Δ on-shell) that

$$\epsilon_{\mu\lambda\alpha\beta} \epsilon_\nu{}^\lambda{}_{\gamma\delta} P^\alpha k^\beta p^\gamma_\Delta k^\delta = -(p_\Delta \cdot k)(k_\mu P_\nu - (P \cdot k) g_{\mu\nu}), \tag{4.100}$$

therefore

$$\epsilon_{\mu\lambda\alpha\beta}\, \epsilon_\nu{}^\lambda{}_{\gamma\delta} P^\alpha k^\beta p^\gamma_\Delta k^\delta (i\gamma_5) = -i(p_\Delta \cdot k) \mathscr{K}^2_{\mu\nu}, \tag{4.101}$$

and

$$K^E_{\mu\nu} = iK_M \left[M_\Delta \mathscr{K}^1_{\mu\nu} + \frac{(3M_\Delta + M_N)}{\Delta M} \mathscr{K}^2_{\mu\nu} \right] , \tag{4.102}$$

where we have made use of

$$2\, p_\Delta \cdot k = M^2_\Delta - M^2_N . \tag{4.103}$$

Finally, from Eq. (4.98) and Eq. (4.102) we get

$$\mathbf{\Gamma}^{(\mathrm{MD})}_{\mu\nu} = -iK_M \left[(G_M - G_E)\, M_\Delta \mathscr{K}^1_{\mu\nu} + \left(G_M - \frac{3M_\Delta + M_N}{\Delta M} G_E \right) \mathscr{K}^2_{\mu\nu} \right] , \tag{4.104}$$

which is the expression of $\mathbf{\Gamma}^{(\mathrm{MD})}_{\mu\nu}$ in terms of the standard parity set $(\mathscr{K}^1_{\mu\nu}, \mathscr{K}^2_{\mu\nu})$.
Comparing Eq. (4.104) to

$$\mathbf{\Gamma}^{(\mathrm{NP})}_{\mu\nu} \equiv -i \left[\frac{G_1}{2M_N} \mathscr{K}^1_{\mu\nu} - \frac{G_2}{2M^2_N} \mathscr{K}^2_{\mu\nu} \right] , \tag{4.105}$$

we find that

$$\frac{G_1}{2M_N} = (G_M - G_E)\, M_\Delta K_M , \tag{4.106}$$

and

$$\frac{G_2}{2M^2_N} = -\left(G_M - \frac{3M_\Delta + M_N}{\Delta M} G_E \right) K_M . \tag{4.107}$$

By using the effective values of G_M and G_E given by [35]

$$G_M = 2.97 \pm 0.08, \quad \text{and} \quad G_E = 0.055 \pm 0.010, \tag{4.108}$$

we obtain that

$$G_1 = 4.93, \quad \text{and} \quad G_2 = -2.68. \tag{4.109}$$

From Eqs. (4.106) and (4.107) we obtain that

$$G_M = \frac{1}{6}\left[\frac{3M_\Delta + M_N}{M_\Delta}G_1 + \frac{\Delta M}{M_N}G_2\right], \tag{4.110}$$

and

$$G_E = \frac{1}{6}\left[\frac{\Delta M}{M_\Delta}G_1 + \frac{\Delta M}{M_N}G_2\right], \tag{4.111}$$

which agree with Eq. (54) of Jones-Scadron's paper [40] if we identify

$$G_1 \to G_1^{\text{JS}} \equiv \frac{G_1}{2M_N}, \quad \text{and} \quad G_2 \to G_2^{\text{JS}} \equiv \frac{G_2}{2M_N^2}. \tag{4.112}$$

With $G_1 = 4.93$ and $G_2 = -2.68$, then

$$G_1^{\text{JS}} = 2.62\,\text{GeV}^{-1}, \quad \text{and} \quad G_2^{\text{JS}} = -1.51\,\text{GeV}^{-2}. \tag{4.113}$$

The Ratio of Electric Quadrupole to Magnetic Dipole Amplitudes for $\Delta(1232)$

The ratio of electric quadrupole to magnetic dipole transition amplitudes R_{EM} in the process $\gamma N \rightleftarrows \Delta$ is an important quantity by means of which theories for effective forces between quarks are tested in order to understand the structure of hadrons. However, from the experimental point of view, the determination of the R_{EM} is not precise, current measured values of electromagnetic helicity amplitudes lead to different values for the R_{EM}, which range from -0.034 to -0.010 [1].
The R_{EM} is defined by [13, 42]

$$R_{EM} \equiv \frac{f_{E_2}}{f_{M_1}}, \tag{4.114}$$

where the M_1 and E_2 multipole amplitudes of the resonance production $\gamma N \to \Delta$, are given, respectively by

$$f_{M_1} \equiv \frac{e}{6}\sqrt{\frac{|\vec{k}|}{M_\Delta M_N}}\left[(3M_\Delta + M_N)\frac{G_1}{2M_N} + M_\Delta \Delta M \frac{G_2}{2M_N^2}\right], \tag{4.115}$$

and

$$f_{E_2} \equiv -\frac{e}{3}\sqrt{\frac{M_\Delta}{M_N}|\vec{k}|}\frac{|\vec{k}|}{\Sigma M}\left[\frac{G_1}{2M_N} + M_\Delta\frac{G_2}{2M_N^2}\right], \tag{4.116}$$

which may be written, by means of Eq. (4.106) and Eq. (4.107), in terms of the *Sachs*-type form factors G_M and G_E as

$$f_{M_1} = \frac{e}{2M_N}\sqrt{\frac{M_\Delta}{M_N}|\vec{k}|}\,G_M, \tag{4.117}$$

and

$$f_{E_2} = -\frac{e}{2M_N}\sqrt{\frac{M_\Delta}{M_N}|\vec{k}|}\frac{2M_\Delta}{M_\Delta^2 - M_N^2}|\vec{k}|\, G_E. \tag{4.118}$$

Finally, in the Δ-rest frame, the photon momentum is

$$|\vec{k}| = \frac{M_\Delta^2 - M_N^2}{2M_\Delta}, \tag{4.119}$$

therefore the R_{EM} is expressed as

$$R_{EM} \equiv \frac{f_{E_2}}{f_{M_1}} = -\frac{G_E}{G_M}. \tag{4.120}$$

The values $G_M = 2.97$, and $G_E = 0.055$ give the ratio

$$R_{EM} = -0.0185 \pm 0.0039. \tag{4.121}$$

The γND Vertex

The difference of this case with respect to the previous one is that this resonance has opposite parity and, both the $I = 0$ and the $I = 1$ components of the photon contribute. Thus the γND interaction Lagrangian is similar to the $\gamma N\Delta$ interaction Lagrangian given by Eq. (4.90) if we make the replacements

$$G_i \to \frac{1}{2}(G_i^s + G_i^v \tau_3), \quad i = 1, 2. \tag{4.122}$$

and

$$\overline{\Psi}_\Delta^\mu T_3 \to \overline{\Psi}_D^\mu. \tag{4.123}$$

Therefore

$$\mathscr{L}_{\gamma ND} = e\overline{\Psi}_D^\mu \left[\frac{1}{4M_N}(G_1^s + G_1^v\tau_3)\mathscr{K}_{\mu\nu}^1 - \frac{1}{4M_N^2}(G_2^s + G_2^v\tau_3)\mathscr{K}_{\mu\nu}^2\right]\gamma_5\Psi_N A^\nu + \text{h.c.}, \tag{4.124}$$

where the isoscalar and isovector electromagnetic couplings G_i^s and G_i^v are defined by $G_i^p + G_i^n$ and $G_i^p - G_i^n$, respectively, with $G_1^p = -5.570$, $G_2^p = 0.624$, $G_1^n = 0.853$, and $G_2^n = 0.100$ [12].

5. The Spin-$\frac{3}{2}$ Propagator

In this chapter we present the general form of the total *nonrenormalized* propagator for the massive Rarita-Schwinger field with all spin components. In addition to the leading component of spin-$\frac{3}{2}$, the massive off-shell spin-$\frac{3}{2}$ field incorporates two spin-$\frac{1}{2}$ components, which cannot be eliminated from the amplitudes. In general, for the massive off-shell fields with spin $J \geq 1$, there are contributions involving the spin-$(J-1)$ sector in the effective amplitudes [43]. The case of the *renormalized* propagator will be considered at the end of the chapter.

5.1. Free (Bare) Propagator

Applying the *Euler-Lagrange* equations to the Lagrangian for the free spin-$\frac{3}{2}$ field given by Eq. (3.16), we obtain the *wave equation* for the spin-$\frac{3}{2}$ particle

$$\Xi_{\mu\nu}\Psi_X^\nu = 0, \tag{5.1}$$

where

$$\begin{aligned}\Xi_{\mu\nu} &\equiv \Lambda_{\mu\alpha}\left[g^{\alpha\beta}\left(i\partial\!\!\!/ - M_X\right) + \frac{i}{3}\left(\gamma^\alpha\partial\!\!\!/\gamma^\beta - \gamma^\alpha\partial^\beta - \partial^\alpha\gamma^\beta\right) + \frac{1}{3}M_X\gamma^\alpha\gamma^\beta\right]\Lambda_{\beta\nu} \\ &= \left(i\partial\!\!\!/ - M_X\right)g_{\mu\nu} + iA\left(\gamma_\mu\partial_\nu + \gamma_\nu\partial_\mu\right) + \frac{i}{2}\left(3A^2 + 2A + 1\right)\gamma_\mu\partial\!\!\!/\gamma_\nu \\ &+ \left(3A^2 + 3A + 1\right)M_X\gamma_\mu\gamma_\nu. \end{aligned}\tag{5.2}$$

Eq. (5.1) leads to the *constraint* equations

$$\gamma_\mu\Psi_X^\mu = 0, \quad \text{and} \quad \partial_\mu\Psi_X^\mu = 0, \tag{5.3}$$

which are necessary to eliminate the redundant components of the free spin-$\frac{3}{2}$ field Ψ_X^μ from sixteen to eight (four spin projections for the particle and the other four for the anti-particle). However, in the presence of interactions, these constraints do not hold in general, but it is possible to derive the necessary number of constraints for a certain type of interactions [19].
The propagator for the free spin-$\frac{3}{2}$ field is [34]

$$\langle 0|\mathscr{T}\Psi_X^\mu(x)\bar{\Psi}_X^\nu(y)|0\rangle = d^{\mu\nu}(\partial)\int\frac{d^4p}{(2\pi)^4}\frac{i}{p^2 - M_X^2 + i\epsilon}e^{-ip\cdot(x-y)}, \tag{5.4}$$

where the operator $d_{\mu\nu}(\partial)$ is given by

$$d_{\mu\nu}(\partial) \equiv (i\not{\partial} + M_X)\left[g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{i}{3M_X}(\gamma_\mu\partial_\nu - \gamma_\nu\partial_\mu) + \frac{2}{3M_X^2}\partial_\mu\partial_\nu\right] - \frac{1}{3M_X^2}\frac{A+1}{2A+1}$$
$$\times\left[\left(\frac{i}{2}\frac{A+1}{2A+1}\not{\partial} - \frac{A}{2A+1}M_X\right)\gamma_\mu\gamma_\nu + i\gamma_\mu\partial_\nu + i\frac{A}{2A+1}\gamma_\nu\partial_\mu\right](\Box + M_X^2). \quad (5.5)$$

In *momentum space*, the free propagator becomes [17, 19]

$$G_{\mu\nu}(p) = \frac{i(\not{p} + M_X)}{p^2 - M_X^2 + i\epsilon}\left[g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3M_X}(\gamma_\mu p_\nu - \gamma_\nu p_\mu) - \frac{2}{3M_X^2}p_\mu p_\nu\right]$$
$$+ \frac{i}{3M_X^2}\frac{A+1}{2A+1}\left[\left(\frac{A+1}{2(2A+1)}\not{p} - \frac{A}{2A+1}M_X\right)\gamma_\mu\gamma_\nu + \gamma_\mu p_\nu + \frac{A}{2A+1}\gamma_\nu p_\mu\right]. \quad (5.6)$$

On the other hand, as it was stated in Sec. 3.4, the physical properties of the free field are independent of the parameter A, which we have taken equal to $A = -\frac{1}{3}$ [21, 31]. This choice yields the expression for the *bare* (*unperturbed*) spin-$\frac{3}{2}$ propagator [31, 33]

$$G_{\mu\nu}(p) = \frac{i(\not{p} + M_X)}{p^2 - M_X^2 + i\epsilon}\left[g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3M_X}(\gamma_\mu p_\nu - \gamma_\nu p_\mu) - \frac{2}{3M_X^2}p_\mu p_\nu\right]$$
$$+ i\frac{2}{3M_X^2}\left[(\not{p} + M_X)\gamma_\mu\gamma_\nu + \gamma_\mu p_\nu - \gamma_\nu p_\mu\right], \quad (5.7)$$

differing from the traditional choice $A = -1$ which leads to the well-known *Rarita-Schwinger* propagator [11, 12]

$$G^{RS}_{\mu\nu}(p) = \frac{i(\not{p} + M_X)}{p^2 - M_X^2 + i\epsilon}\left[g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3M_X}(\gamma_\mu p_\nu - \gamma_\nu p_\mu) - \frac{2}{3M_X^2}p_\mu p_\nu\right]. \quad (5.8)$$

Spin Operators

It will be convenient to consider the set of *spin operators* [33, 43]

$$(\mathcal{P}^{\frac{3}{2}})_{\mu\nu} \equiv g_{\mu\nu} - \frac{2}{3p^2}p_\mu p_\nu - \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{1}{3p^2}(\gamma_\mu p_\nu - \gamma_\nu p_\mu)\not{p}, \quad (5.9)$$

$$(\mathcal{P}^{\frac{1}{2}}_{11})_{\mu\nu} \equiv \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3p^2}p_\mu p_\nu - \frac{1}{3p^2}(\gamma_\mu p_\nu - \gamma_\nu p_\mu)\not{p}, \quad (5.10)$$

$$(\mathcal{P}^{\frac{1}{2}}_{22})_{\mu\nu} \equiv \frac{1}{p^2}p_\mu p_\nu, \quad (5.11)$$

$$(\mathcal{P}^{\frac{1}{2}}_{21})_{\mu\nu} \equiv \sqrt{\frac{3}{p^2}}\frac{1}{3p^2}\left(-i\sigma_{\mu\alpha}p^\alpha\right)\not{p}p_\nu, \quad (5.12)$$

$$(\mathcal{P}^{\frac{1}{2}}_{12})_{\mu\nu} \equiv \sqrt{\frac{3}{p^2}}\frac{1}{3p^2}\left(-i\sigma_{\nu\alpha}p^\alpha\right)\not{p}p_\mu, \quad (5.13)$$

which satisfy the orthonormality condition

$$(\mathcal{P}^I_{ij})^{\mu\lambda}(\mathcal{P}^J_{kl})_{\lambda\nu} = (\mathcal{P}^I_{il})^\mu{}_\nu\, \delta^{IJ}\delta_{jk}, \tag{5.14}$$

and the commutation and anti-commutation relations

$$[\not p, (\mathcal{P}^{\frac{3}{2}})_{\mu\nu}] = 0, \quad \text{and} \quad \begin{cases} [\not p, (\mathcal{P}^{\frac{1}{2}}_{ij})_{\mu\nu}] = 0, & \text{if } i = j, \\ \{\not p, (\mathcal{P}^{\frac{1}{2}}_{ij})_{\mu\nu}\} = 0, & \text{if } i \neq j. \end{cases} \tag{5.15}$$

From these, $P^{\frac{3}{2}}$, $\mathcal{P}^{\frac{1}{2}}_{11}$, and $\mathcal{P}^{\frac{1}{2}}_{22}$, are *projection* operators

$$(\mathcal{P}^{\frac{3}{2}})_{\mu\nu} + (\mathcal{P}^{\frac{1}{2}}_{11})_{\mu\nu} + (\mathcal{P}^{\frac{1}{2}}_{22})_{\mu\nu} = g_{\mu\nu}, \tag{5.16}$$

while $\mathcal{P}^{\frac{1}{2}}_{21}$ and $\mathcal{P}^{\frac{1}{2}}_{12}$ are *nilpotent* operators [44].
In terms of the projection operators, the bare propagator given in Eq. (5.7) becomes

$$G_{\mu\nu}(p) = \frac{i(\not p + M_X)}{p^2 - M_X^2 + i\epsilon}(\mathcal{P}^{\frac{3}{2}})_{\mu\nu} + i\frac{2}{M_X^2}(\not p + M_X)(\mathcal{P}^{\frac{1}{2}}_{11})_{\mu\nu} + i\frac{\sqrt{3}}{M_X\sqrt{p^2}}\not p\left[(\mathcal{P}^{\frac{1}{2}}_{12})_{\mu\nu} - (\mathcal{P}^{\frac{1}{2}}_{21})_{\mu\nu}\right]. \tag{5.17}$$

This expression for the bare propagator in terms of the projection operators will be useful in next section.

5.2. Total (Dressed) Propagator

The bare spin-$\frac{3}{2}$ propagator is singular at $p^2 = M_X^2$ and should be *dressed* by including a *self-energy* (Σ) which gives to it a width corresponding to an *unstable* particle [45]. This self-energy includes the lowest order πN one-loop contribution of Fig. 5.1 as well as other higher order contributions which will not be considered here.
The expression for the corresponding dressed propagator ($\tilde{G}^{\mu\nu}$) is more difficult and has not been solved conclusively yet. In this work we will make use of the analytic expression for the propagator given in Refs. [33, 44] which takes into account all spin components. The dressed propagator is obtained by solving the *Dyson-Schwinger* equation [30]

$$\tilde{G}_{\mu\nu}(p) = G_{\mu\nu}(p) + \tilde{G}_{\mu\alpha}(p)\,\Sigma^{\alpha\beta}(p)\,G_{\beta\nu}(p), \tag{5.18}$$

or equivalently for the inverse propagators

$$\tilde{G}_{\mu\nu}(p)^{-1} = G_{\mu\nu}(p)^{-1} - \Sigma_{\mu\nu}(p), \tag{5.19}$$

where the one-loop self-energy correction is given by

$$\Sigma^{\mu\nu}(p) = -i\left(\frac{f_{\pi NX}}{m_\pi}\right)^2 \int \frac{d^4v}{(2\pi)^4}\frac{\not p + \not v + M_N}{(p+v)^2 - M_N^2}\frac{v^\mu v^\nu}{v^2 - m_\pi^2}. \tag{5.20}$$

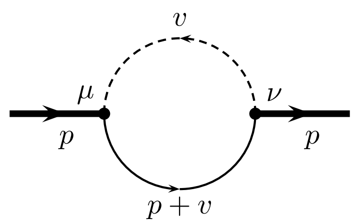


Figure 5.1.: One-loop πN self-energy correction to the spin-$\frac{3}{2}$ propagator.

We evaluate the discontinuity of the loop according to the *Cutkosky rule* [27] by replacing

$$\frac{1}{p^2 - m^2} \to -2\pi i \delta(p^2 - m^2) \tag{5.21}$$

in each cut propagator, from which $\Sigma^{\mu\nu}(p)$ becomes [33]

$$\Sigma^{\mu\nu}(p) = i \left(\frac{f_{\pi NX}}{m_\pi} \right)^2 \int \frac{d^4 v}{(2\pi)^2} \left(\not{p} + \not{v} + M_N \right) v^\mu v^\nu \, \delta \left((p+v)^2 - M_N^2 \right) \delta \left(v^2 - m_\pi^2 \right). \tag{5.22}$$

Integrating over v^0 we obtain

$$\Sigma^{\mu\nu}(p) = i \left(\frac{f_{\pi NX}}{2\pi m_\pi} \right)^2 \int \frac{d^3 \vec{v}}{2\omega_\pi} \left(\not{p} + \not{v} + M_N \right) v^\mu v^\nu \frac{1}{2\sqrt{p^2}} \delta \left(\omega_\pi + \frac{p^2 + m_\pi^2 - M_N^2}{2\sqrt{p^2}} \right) \times \tag{5.23}$$

$$\times \, \Theta \left(p^2 - (M_N + m_\pi)^2 \right),$$

where $\omega_\pi^2 \equiv |\vec{v}|^2 + m_\pi^2$.
Evaluating the volume integral, taking into account that $\int d^3\vec{v} = 4\pi \int |\vec{v}|^2 \, d|\vec{v}|$, we obtain

$$\Sigma^{\mu\nu}(p) = \sum_{i=1}^{10} \bar{J}_i (\mathcal{P}_i)^{\mu\nu}, \tag{5.24}$$

where the projection operators $(\mathcal{P}_i)^{\mu\nu}$ are defined in terms of the spin projection operators given above by

$$\begin{aligned}
&(\mathcal{P}_1)^{\mu\nu} \equiv \Lambda^+ (\mathcal{P}^{\frac{3}{2}})^{\mu\nu}, \quad (\mathcal{P}_2)^{\mu\nu} \equiv \Lambda^- (\mathcal{P}^{\frac{3}{2}})^{\mu\nu}, \quad (\mathcal{P}_3)^{\mu\nu} \equiv \Lambda^+ (\mathcal{P}^{\frac{1}{2}}_{11})^{\mu\nu}, \\
&(\mathcal{P}_4)^{\mu\nu} \equiv \Lambda^- (\mathcal{P}^{\frac{1}{2}}_{11})^{\mu\nu}, \quad (\mathcal{P}_5)^{\mu\nu} \equiv \Lambda^+ (\mathcal{P}^{\frac{1}{2}}_{22})^{\mu\nu}, \quad (\mathcal{P}_6)^{\mu\nu} \equiv \Lambda^- (\mathcal{P}^{\frac{1}{2}}_{22})^{\mu\nu}, \\
&(\mathcal{P}_7)^{\mu\nu} \equiv \Lambda^+ (\mathcal{P}^{\frac{1}{2}}_{21})^{\mu\nu}, \quad (\mathcal{P}_8)^{\mu\nu} \equiv \Lambda^- (\mathcal{P}^{\frac{1}{2}}_{21})^{\mu\nu}, \quad (\mathcal{P}_9)^{\mu\nu} \equiv \Lambda^+ (\mathcal{P}^{\frac{1}{2}}_{12})^{\mu\nu}, \\
&(\mathcal{P}_{10})^{\mu\nu} \equiv \Lambda^- (\mathcal{P}^{\frac{1}{2}}_{12})^{\mu\nu},
\end{aligned} \tag{5.25}$$

with $\Lambda^{\pm} \equiv \dfrac{\sqrt{p^2} \pm \not{p}}{2\sqrt{p^2}}$, and the coefficients $\bar{J}_i$ are given by [33]

$$\bar{J}_1 = \bar{J}_3 \equiv -i\left(\frac{f_{\pi NX}}{2\pi m_\pi}\right)^2 \frac{I_0}{12p^2}\left[\frac{(\sqrt{p^2}+M_N)^2 - m_\pi^2}{4\sqrt{p^2}}\right]\lambda(p^2, M_N^2, m_\pi^2), \tag{5.26}$$

$$\bar{J}_2 = \bar{J}_4 \equiv i\left(\frac{f_{\pi NX}}{2\pi m_\pi}\right)^2 \frac{I_0}{12p^2}\left[\frac{(\sqrt{p^2}-M_N)^2 - m_\pi^2}{4\sqrt{p^2}}\right]\lambda(p^2, M_N^2, m_\pi^2), \tag{5.27}$$

$$\bar{J}_5 \equiv i\left(\frac{f_{\pi NX}}{2\pi m_\pi}\right)^2 \frac{I_0}{4p^2}\left[\frac{(\sqrt{p^2}+M_N)^2 - m_\pi^2}{4\sqrt{p^2}}\right]\left(p^2 - M_N^2 + m_\pi^2\right)^2, \tag{5.28}$$

$$\bar{J}_6 \equiv -i\left(\frac{f_{\pi NX}}{2\pi m_\pi}\right)^2 \frac{I_0}{4p^2}\left[\frac{(\sqrt{p^2}-M_N)^2 - m_\pi^2}{4\sqrt{p^2}}\right]\left(p^2 - M_N^2 + m_\pi^2\right)^2, \tag{5.29}$$

$$\bar{J}_7 = \bar{J}_8 = \bar{J}_9 = \bar{J}_{10} \equiv i\left(\frac{f_{\pi NX}}{2\pi m_\pi}\right)^2 \frac{I_0}{48p^2}\sqrt{\frac{3}{p^2}}\left(p^2 - M_N^2 + m_\pi^2\right)\lambda(p^2, M_N^2, m_\pi^2), \tag{5.30}$$

where

$$\lambda(x,y,z) \equiv (x-y)^2 + (x-z)^2 + (y-z)^2 - x^2 - y^2 - z^2, \tag{5.31}$$

and

$$I_0 \equiv \frac{\pi}{2p^2}\lambda^{\frac{1}{2}}(p^2, M_N^2, m_\pi^2)\,\Theta\left(p^2 - (M_N + m_\pi)^2\right). \tag{5.32}$$

By replacing these results into Eq. (5.18) we obtain the following expression for the dressed propagator

$$\begin{aligned}\tilde{G}_{\mu\nu}(p) = \frac{i}{1-J_2}&\left\{\frac{(\not{p}+\tilde{M}_X)}{p^2-\tilde{M}_X^2}\left[g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{\tilde{M}_X}{3p^2}(\gamma_\mu p_\nu - \gamma_\nu p_\mu) - \frac{2}{3p^2}p_\mu p_\nu\right]\right.\\ &- \left[\frac{\Sigma E/3 - \Sigma G}{2p^2}\not{p} - \frac{\Delta E/3 - \Delta G}{2\sqrt{p^2}}\right]\gamma_\mu p_\nu + \left[\frac{\Sigma E/3 - \Sigma G}{2p^2}\not{p} - \frac{\Delta E/3 + \Delta G}{2\sqrt{p^2}}\right]\gamma_\nu p_\mu\\ &\left. + \frac{1}{3}\left[\frac{\Delta E}{2\sqrt{p^2}}\not{p} - \frac{\Sigma E}{2}\right]\gamma_\mu\gamma_\nu - \frac{1}{p^2}\left[\frac{\Delta E/3 + \Delta F - 2\Delta G}{2\sqrt{p^2}}\not{p} - \frac{\Sigma E/3 - \Sigma F}{2}\right]p_\mu p_\nu\right\},\end{aligned} \tag{5.33}$$

where $\Delta E \equiv E_+ - E_-$, $\Sigma E \equiv E_+ + E_-$, etc. with

$$E_\pm \equiv \frac{2\tilde{M}_X \mp 2\sqrt{p^2} + A_\pm}{-\tilde{M}_X^2 + B_\pm}, \quad F_\pm \equiv \frac{3\frac{J_3 \mp \sqrt{p^2}J_4}{1-J_2}}{-\tilde{M}_X^2 + B_\pm}, \quad G_\pm \equiv \frac{\tilde{M}_X - \frac{J_1 \pm \sqrt{3}J_7}{1-J_2}}{-\tilde{M}_X^2 + B_\pm}, \tag{5.34}$$

$$A_\pm \equiv \frac{3(J_5 \pm \sqrt{p^2}J_6) - 2(J_1 \pm \sqrt{p^2}J_2)}{1-J_2}, \tag{5.35}$$

$$B_\pm \equiv \frac{2M_X(J_1 + J_3 \pm \sqrt{3}J_7 \mp \sqrt{p^2}J_4) + 2\sqrt{p^2}(\mp J_3 + \sqrt{p^2}J_4) + J_1^2}{(1-J_2^2)^2}, \tag{5.36}$$

and the *effective* mass term, $\tilde{M}_X$, is defined by

$$\tilde{M}_X \equiv \frac{M_X + J_1}{1 - J_2}. \tag{5.37}$$

The J_i coefficients are defined in terms of the $\bar{J}_i$ coefficientes given in Eqs. (5.26) - (5.30) by means of

$$J_{2n-1} \equiv \frac{\bar{J}_{2n-1} + \bar{J}_{2n}}{2}, \quad \text{and} \quad J_{2n} \equiv \frac{\bar{J}_{2n-1} - \bar{J}_{2n}}{2\sqrt{p^2}}, \quad n = 1, \cdots, 5. \tag{5.38}$$

Finally, in terms of the spin projection operators, the dressed propagator becomes

$$\begin{aligned}\tilde{G}_{\mu\nu}(p) =& \frac{i}{1 - J_2}\left\{\frac{\not{p} + M_X}{p^2 - M_X^2}(\mathcal{P}^{\frac{3}{2}})_{\mu\nu} - \frac{1}{2}\Sigma E\,(\mathcal{P}^{\frac{1}{2}}_{11})_{\mu\nu} + \frac{1}{2\sqrt{p^2}}\Delta E\,\not{p}(\mathcal{P}^{\frac{1}{2}}_{11})_{\mu\nu}\right. \\ & - \frac{1}{2}\Sigma F\,(\mathcal{P}^{\frac{1}{2}}_{22})_{\mu\nu} - \frac{1}{2\sqrt{p^2}}\Delta F\,\not{p}(\mathcal{P}^{\frac{1}{2}}_{22})_{\mu\nu} - \frac{\sqrt{3}}{2}\Delta G\left[(\mathcal{P}^{\frac{1}{2}}_{12})_{\mu\nu} + (\mathcal{P}^{\frac{1}{2}}_{21})_{\mu\nu}\right] \\ & \left. - \frac{\sqrt{3}}{2\sqrt{p^2}}\Sigma G\,\not{p}\left[(\mathcal{P}^{\frac{1}{2}}_{12})_{\mu\nu} - (\mathcal{P}^{\frac{1}{2}}_{21})_{\mu\nu}\right]\right\}. \end{aligned} \tag{5.39}$$

5.2.1. The Complex Mass Scheme

The effective mass term defined above is given explicitly by

$$\begin{aligned}\tilde{M}_X &\equiv \frac{M_X + J_1}{1 - J_2} \\ &= (M_X + J_1)(1 + J_2 + J_2^2 + \cdots) \\ &= M_X + J_1 + M_X J_2 + \cdots + (\sqrt{p^2}J_2 - \sqrt{p^2}J_2) \\ &= M_X + (J_1 + \sqrt{p^2}J_2) + (M_X - \sqrt{p^2})J_2 + \cdots \\ &= M_X + \bar{J}_1 + (M_X - \sqrt{p^2})J_2 + \mathcal{O}(g^4), \end{aligned} \tag{5.40}$$

where we have made use of $\bar{J}_1 = J_1 + \sqrt{p^2}J_2$ and $g \equiv \frac{f_{\pi NX}}{m_\pi}$.
By neglecting terms of the order $\mathcal{O}(g^4)$ and $\mathcal{O}((M_X - \sqrt{p^2})g^2)$, which are expected to be small in the *resonance region* ($\sqrt{p^2} \simeq M_X$), the effective mass term is then given approximately by

$$\tilde{M}_X \simeq M_X - i\left(\frac{f_{\pi NX}}{2\pi m_\pi}\right)^2 \frac{I_0}{12p^2}\left[\frac{(\sqrt{p^2} + M_N)^2 - m_\pi^2}{4\sqrt{p^2}}\right]\lambda(p^2, M_N^2, m_\pi^2). \tag{5.41}$$

On the other hand, according to the *complex-mass scheme* (CMS), which is the most straightforward method to describe unstable particles in perturbation theory [46], the effective mass is given by

$$\tilde{M}_X \simeq M_X - \frac{i}{2}\Gamma_X(s), \tag{5.42}$$

where $\Gamma_X(s)$ is the *energy-dependent* decay width of the resonance, with $s = p^2$. Comparing Eq. (5.41) and Eq. (5.42) we find that the decay width, $\Gamma_X(s)$, becomes

$$\Gamma_X(s) = \frac{f_{\pi N X}^2}{4\pi m_\pi^2} \frac{1}{12 s^2} \left[\frac{(\sqrt{s} + M_N)^2 - m_\pi^2}{4\sqrt{s}} \right] \lambda^{\frac{3}{2}}(s, M_N^2, m_\pi^2), \tag{5.43}$$

which agrees with the expression for $\Gamma_{\Delta \to \pi N}$ given in Eq. (4.39) when $\sqrt{s} = M_\Delta$.

5.2.2. The Renormalized Propagator

The *renormalized* propagator, $G^R_{\mu\nu}(p)$, is defined by [33]

$$\tilde{G}_{\mu\nu}(p) \equiv (1 - J_2)^{-1} G^R_{\mu\nu}(p), \tag{5.44}$$

where the factor $(1 - J_2)^{-1}$ is absorbed as a component of the X wavefunction renormalization constant.
By keeping terms of order g^2 in the coefficients of the projection operators in Eq. (5.39), the renormalized propagator becomes

$$G^R_{\mu\nu}(p) \simeq \frac{i(\not{p} + \tilde{M}_X)}{p^2 - \tilde{M}_X^2} (\mathcal{P}^{\frac{3}{2}})_{\mu\nu} + i\frac{2}{\tilde{M}_X^2}(\not{p} + \tilde{M}_X)(\mathcal{P}^{\frac{1}{2}}_{11})_{\mu\nu} + i\frac{\sqrt{3}}{\tilde{M}_X\sqrt{p^2}}\not{p}\left[(\mathcal{P}^{\frac{1}{2}}_{12})_{\mu\nu} - (\mathcal{P}^{\frac{1}{2}}_{21})_{\mu\nu}\right]. \tag{5.45}$$

Then, by comparing Eq. (5.17) and Eq. (5.45), we conclude that the form of the renormailized propagator is, up to order g^2, identical to that of the bare propagator under the replacement $M_X \to \tilde{M}_X = M_X - \frac{i}{2}\Gamma_X(s)$.

6. Scattering Amplitudes

The central problem in the study of scattering processes is the calculation of $\mathbb{S}$-matrix elements between *on-shell* states. Given the interaction Lagrangians, $\mathscr{L}_{\text{int}}$, which describe the interactions involved in pion photoproduction, the $\mathbb{S}$-*matrix* is [27, 32]

$$\mathbb{S} \equiv \mathscr{T} e^{i \int d^4x \, \mathscr{L}_{\text{int}}(x)}, \tag{6.1}$$

where $\mathscr{T}$ denotes the *time-ordered product* of the meson, nucleon and photon field operators.
The $\mathbb{S}$-matrix has the following structure: if the particles involved do not interact at all, then $\mathbb{S}$ is simply the *identity operator* ($\mathbb{1}$), but if the theory contains interactions, we define the $\mathbb{T}$-matrix by

$$\mathbb{S} \equiv \mathbb{1} + i\,\mathbb{T}, \tag{6.2}$$

from which we define the *invariant* matrix element $\mathcal{M}$ by [27, 32]

$$\langle \vec{p}_{\text{f}}, \vec{q} | \mathbb{S} - \mathbb{1} | \vec{p}_{\text{i}}, \vec{k} \rangle \equiv (2\pi)^4 \delta^4(p_{\text{i}} + k - p_{\text{f}} - q)\, i\mathcal{M}(p_{\text{i}}, k \to p_{\text{f}}, q), \tag{6.3}$$

which is *useful* because it allows us to separate all the physics that depends on the details of the interaction Lagrangian (*dynamics*) from all the physics that does not (*kinematics*).
In the following sections we present each of the analytic expressions for the amplitudes contributing to pion photoproduction off the proton (as well as neutron, for the sake of completeness) at the tree level, without including form factors. It is worth to mention that at low energies, the use of a *pseudovector* coupling scheme in pion photoproduction is favorable in the energy region near threshold but starts to *diverge* above the delta resonance region in comparison with the current available experimental data. Later we will calculate the same amplitudes by including form factors which account for the structure of the interacting particles not included in the model, or to *regularize* those quantities which would otherwise be *divergent.*

6.1. Born Terms

1. **Nucleon**

   To *first order* in e and $f_{\pi NN}$, the Lagrangians (4.1) and (4.59), yield the invariant amplitudes for the nucleon term

$$i\mathcal{M}_N^s = -ie\frac{f_{\pi NN}}{m_\pi}\mathbf{I}_N\,\overline{u}(p_{\text{f}})\left[\gamma_5 \not{q}\, i\frac{\not{p}_{\text{i}} + \not{k} + M_N}{s - M_N^2}\left(\not{\epsilon} - \frac{\kappa_p}{2M_{\text{N}}}\not{\epsilon}\not{k}\right)\right]u(p_{\text{i}}), \tag{6.4}$$

	$\mathbf{I}_N$				$\mathbf{I}_c$				$\mathbf{I}_\pi$			
Channel	$p\pi^-$	$p\pi^0$	$n\pi^+$	$n\pi^0$	$p\pi^-$	$p\pi^0$	$n\pi^+$	$n\pi^0$	$p\pi^-$	$p\pi^0$	$n\pi^+$	$n\pi^0$
γp	0	1	$-\sqrt{2}$	0	0	0	$\sqrt{2}$	0	0	0	$-\sqrt{2}$	0
γn	$\sqrt{2}$	0	0	-1	$-\sqrt{2}$	0	0	0	$\sqrt{2}$	0	0	0

Table 6.1.: Isospin factors for nucleon Born terms.

for the s-channel in pion photoproduction on proton, where $\mathbf{I}_N$ is an *isospin* factor given in Tab. 6.1, and

$$i\mathcal{M}_N^s = ie\frac{f_{\pi NN}}{m_\pi}\frac{\kappa_n}{2M_N}\mathbf{I}_N\,\overline{u}(p_\mathrm{f})\left[\gamma_5 \not{q}\, i\frac{\not{p}_\mathrm{i} + \not{k} + M_N}{s - M_N^2}\not{\epsilon}\not{k}\right] u(p_\mathrm{i}), \tag{6.5}$$

for the s-channel in pion photoproduction on neutron.

For the u-channel,

$$i\mathcal{M}_N^u = -ie\frac{f_{\pi NN}}{m_\pi}\mathbf{I}_N\,\overline{u}(p_\mathrm{f})\left[\left(\not{\epsilon} - \frac{\kappa_p}{2M_N}\not{\epsilon}\not{k}\right) i\frac{\not{p}_\mathrm{f} - \not{k} + M_N}{u - M_N^2}\gamma_5\not{q}\right] u(p_\mathrm{i}), \tag{6.6}$$

for the processes $\gamma p \to \pi^0 p$ and $\gamma n \to \pi^- p$, and

$$i\mathcal{M}_N^u = ie\frac{f_{\pi NN}}{m_\pi}\frac{\kappa_n}{2M_N}\mathbf{I}_N\,\overline{u}(p_\mathrm{f})\left[\not{\epsilon}\not{k}\, i\frac{\not{p}_\mathrm{f} - \not{k} + M_N}{u - M_N^2}\gamma_5\not{q}\right] u(p_\mathrm{i}), \tag{6.7}$$

for the processes $\gamma p \to \pi^+ n$ and $\gamma n \to \pi^0 n$.

2. **Kroll-Rudermann (Contact)**

   The Lagrangian (4.60) yields the invariant amplitude for the Kroll-Rudermann term of Fig. 4.2a

$$i\mathcal{M}_\mathrm{c} = \pm e\frac{f_{\pi NN}}{m_\pi}\mathbf{I}_\mathrm{c}\,\overline{u}(p_\mathrm{f})\left[\gamma_5\not{\epsilon}\right] u(p_\mathrm{i}), \tag{6.8}$$

   where the (+) sign corresponds to π^+ photoproduction and the (−) sign corresponds to π^- photoproduction. $\mathbf{I}_\mathrm{c}$ is an *isospin* factor given in Table 6.1.

3. **Pion in Flight or t-channel**

   To *first order* in e and $f_{\pi NN}$, the Lagrangians (4.1) and (4.61), yield the invariant amplitude for the pion *in flight* term

$$i\mathcal{M}_\pi^t = ie\frac{f_{\pi NN}}{m_\pi}\mathbf{I}_\pi\, i\frac{q\cdot\epsilon}{t - m_\pi^2}\overline{u}(p_\mathrm{f})\left[\gamma_5\left(\not{q} - \not{k}\right)\right] u(p_\mathrm{i}), \tag{6.9}$$

   where $\mathbf{I}_\pi$ is an *isospin* factor given in Table 6.1.

	I_ρ				I_ω			
Channel	$p\pi^-$	$p\pi^0$	$n\pi^+$	$n\pi^0$	$p\pi^-$	$p\pi^0$	$n\pi^+$	$n\pi^0$
γp	0	1	$-\sqrt{2}$	0	0	1	0	0
γn	$\sqrt{2}$	0	0	-1	0	0	0	1

Table 6.2.: Isospin factors for ρ and ω mesons.

6.2. Vector Meson Terms

To *first order* in e and $g_{\rho NN}$, the Lagrangians (4.6) and (4.64), yield the invariant amplitude for the ρ meson term

$$i\mathcal{M}^t_\rho = \pm e\frac{\lambda_{\rho\pi\gamma}}{m_\pi}\mathbf{I}_\rho\, i\frac{\epsilon_{\sigma\mu\alpha\nu}\epsilon^\sigma q^\mu k^\alpha}{t-m_\rho^2}\overline{u}(p_\mathrm{f})\left[g^v_{\rho NN}\gamma^\nu + i\frac{g^t_{\rho NN}}{2M_\mathrm{N}}\sigma^{\nu\beta}(q-k)_\beta\right]u(p_\mathrm{i}), \tag{6.10}$$

where the $(+)$ sign corresponds to $\pi^\pm$ photoproduction and the $(-)$ sign corresponds to π^0 photoproduction. $\mathbf{I}_\rho$ is an *isospin* factor given in Tab. 6.2.
Similarly, for the ω meson term, the Lagrangians (4.8) and (4.66), yield the invariant amplitude

$$i\mathcal{M}^t_\omega = e\frac{\lambda_{\omega\pi\gamma}}{m_\pi}\mathbf{I}_\omega\, i\frac{\epsilon_{\sigma\mu\alpha\nu}\epsilon^\sigma q^\mu k^\alpha}{t-m_\omega^2}\overline{u}(p_\mathrm{f})\left[g^v_{\omega NN}\gamma^\nu + i\frac{g^t_{\omega NN}}{2M_\mathrm{N}}\sigma^{\nu\beta}(q-k)_\beta\right]u(p_\mathrm{i}), \tag{6.11}$$

where $\mathbf{I}_\omega$ is an *isospin* factor given in Table 6.2.

6.3. Resonance Terms

1. **Spin-$\frac{1}{2}$ Resonances of Negative Parity:** $S_{11}(1535)$ **and** $S_{11}(1650)$

   To *first order* in e and $f_{\pi NR^-}$, the Lagrangians (4.15) and (4.73), yield the invariant amplitudes for the negative parity resonances of spin-$\frac{1}{2}$ (R^-)

   $$i\mathcal{M}^s_{R^-} = -ie\frac{f_{\pi NR^-}}{m_\pi}\frac{\kappa^j_{R^-}}{\Sigma M}\mathbf{I}_R\,\overline{u}(p_\mathrm{f})\left[\gamma_5\not{q}\, i\frac{\not{p}_\mathrm{i}+\not{k}-M_{R^-}}{s-M^2_{R^-}}\not{\epsilon}\not{k}\right]u(p_\mathrm{i}) \tag{6.12}$$

   for the s-channel, where $\kappa^j_{R^-} = \kappa^p_{R^-}(\kappa^n_{R^-})$ for pion photoproduction on proton (neutron), and $\mathbf{I}_R$ is an *isospin* factor given in Table 6.3.

   For the u-channel,

   $$i\mathcal{M}^u_{R^-} = -ie\frac{f_{\pi NR^-}}{m_\pi}\frac{\kappa^j_{R^-}}{\Sigma M}\mathbf{I}_R\,\overline{u}(p_\mathrm{f})\left[\not{\epsilon}\not{k}\, i\frac{\not{p}_\mathrm{f}-\not{k}-M_{R^-}}{u-M^2_{R^-}}\gamma_5\not{q}\right]u(p_\mathrm{i}), \tag{6.13}$$

   where $\kappa^j_{R^-} = \kappa^p_{R^-}$ for the processes $\gamma p \to \pi^0 p$ and $\gamma n \to \pi^- p$, and $\kappa^j_{R^-} = \kappa^n_{R^-}$ for the processes $\gamma p \to \pi^+ n$ and $\gamma n \to \pi^0 n$.

2. **Spin-$\frac{1}{2}$ Resonances of Positive Parity:** $P_{11}(1440)$ **and** $P_{11}(1710)$

   Similarly, to *first order* in e and $f_{\pi N R^+}$, the Lagrangians (4.15) and (4.73), yield the invariant amplitudes for the positive parity resonances of spin-$\frac{1}{2}$ (R^+)

$$i\mathcal{M}^{s}_{R^+} = ie\frac{f_{\pi N R^+}}{m_\pi}\frac{\kappa^j_{R^+}}{\Sigma M}\mathbf{I}_R\,\overline{u}(p_\mathrm{f})\left[\gamma_5 \not{q}\, i\frac{\not{p}_\mathrm{i} + \not{k} + M_{R^+}}{s - M^2_{R^+}}\not{\epsilon}\not{k}\right]u(p_\mathrm{i}) \tag{6.14}$$

   for the s-channel, where $\kappa^j_{R^+} = \kappa^p_{R^+}$ ($\kappa^n_{R^+}$) for pion photoproduction on proton (neutron).

   For the u-channel,

$$i\mathcal{M}^{u}_{R^+} = ie\frac{f_{\pi N R^+}}{m_\pi}\frac{\kappa^j_{R^+}}{\Sigma M}\mathbf{I}_R\,\overline{u}(p_\mathrm{f})\left[\not{\epsilon}\not{k}\, i\frac{\not{p}_\mathrm{f} - \not{k} + M_{R^+}}{u - M^2_{R^+}}\gamma_5\not{q}\right]u(p_\mathrm{i}), \tag{6.15}$$

   where $\kappa^j_{R^+} = \kappa^p_{R^+}$ for the processes $\gamma p \to \pi^0 p$ and $\gamma n \to \pi^- p$, and $\kappa^j_{R^+} = \kappa^n_{R^+}$ for the processes $\gamma p \to \pi^+ n$ and $\gamma n \to \pi^0 n$.

3. **Spin-$\frac{3}{2}$ Resonances of Isospin-$\frac{3}{2}$:** $P_{33}(1232)$ **and** $P_{33}(1600)$

   To *first order* in e and $f_{\pi N\Delta}$, and following the covariant multipole decomposition (MD) or the normal parity set (NP), the Lagrangians (4.23) and (4.84), yield the invariant amplitudes

$$i\mathcal{M}^{s}_{\Delta} = e\frac{f_{\pi N\Delta}}{m_\pi}\mathbf{I}_\Delta\,\overline{u}(p_\mathrm{f})\left[q_\mu\, iG^{\mu\alpha}(p_\Delta)\Gamma_{\alpha\beta}\epsilon^\beta\right]u(p_\mathrm{i}), \tag{6.16}$$

   for the s-channel, where $\mathbf{I}_\Delta$ is an *isospin* factor given in Table 6.3, $G^{\mu\nu}(p_\Delta)$ is the spin-$\frac{3}{2}$ propagator discussed in the previous chapter and $\Gamma_{\alpha\beta} = \Gamma^{(\mathrm{MD})}_{\alpha\beta}$ ($\Gamma^{(\mathrm{NP})}_{\alpha\beta}$).

   For the u-channel,

$$i\mathcal{M}^{u}_{\Delta} = \mp e\frac{f_{\pi N\Delta}}{m_\pi}\mathbf{I}_\Delta\,\overline{u}(p_\mathrm{f})\left[\tilde{\Gamma}_{\mu\nu}\epsilon^\nu\, iG^{\mu\alpha}(p_\Delta)q_\alpha\right]u(p_\mathrm{i}), \tag{6.17}$$

   where $\tilde{\Gamma}_{\mu\nu} = \tilde{\Gamma}^{(\mathrm{MD})}_{\mu\nu}$ ($\tilde{\Gamma}^{(\mathrm{NP})}_{\mu\nu}$), with

$$\tilde{\Gamma}_{\mu\nu} \equiv \gamma_0\Gamma^\dagger_{\mu\nu}\gamma_0, \tag{6.18}$$

   and $\Gamma_{\mu\nu} = \Gamma^{(\mathrm{MD})}_{\mu\nu}(\Gamma^{(\mathrm{NP})}_{\mu\nu})$. The negative (positive) sign corresponds to π^0 (π^+) photoproduction on proton.

4. **Spin-$\frac{3}{2}$ Resonances of Isospin-$\frac{1}{2}$:** $D_{13}(1520)$

   To *first order* in e and $f_{\pi N D}$, the Lagrangians (4.40) and (4.124), and following the normal parity set, yield the invariant amplitudes

$$i\mathcal{M}^{s}_{D} = -ie\frac{f_{\pi N D}}{m_\pi}\mathbf{I}_D\,\overline{u}(p_\mathrm{f})\left[\gamma_5 q_\mu\, iG^{\mu\alpha}(p_D)K^{p-}_{\alpha\beta}\epsilon^\beta\gamma_5\right]u(p_\mathrm{i}), \tag{6.19}$$

	$I_{R(D)}$				I_Δ			
Channel	$p\pi^-$	$p\pi^0$	$n\pi^+$	$n\pi^0$	$p\pi^-$	$p\pi^0$	$n\pi^+$	$n\pi^0$
γp	0	1	$-\sqrt{2}$	0	0	2/3	$\sqrt{2}/3$	0
γn	$\sqrt{2}$	0	0	-1	$\sqrt{2}/3$	0	0	2/3

Table 6.3.: Isospin factors for isospin-$\frac{1}{2}$ (R, D) and isospin-$\frac{3}{2}$ (Δ) resonance terms.

for the s-channel in pion photoproduction on proton (neutron), where $\mathbf{I}_D$ is an *isospin* factor given in Table 6.3, and

$$K^{p\pm}_{\alpha\beta} \equiv \frac{G^p_1}{2M_N}\mathscr{K}^1_{\alpha\beta} \pm \frac{G^p_2}{2M^2_N}\mathscr{K}^2_{\alpha\beta}. \tag{6.20}$$

In the covariant multipole decomposition, it may be written as

$$i\mathcal{M}^s_D = e\frac{f_{\pi ND}}{m_\pi}\,\mathbf{I}_D\,\overline{u}(p_\mathrm{f})\left[\gamma_5 q_\mu\, iG^{\mu\alpha}(p_D)\Gamma^{(\mathrm{MD})}_{\alpha\beta}\epsilon^\beta\gamma_5\right]u(p_\mathrm{i}), \tag{6.21}$$

where $\Gamma^{(\mathrm{MD})}_{\mu\nu}$ is given by Eq. (4.85).
In this case, the *magnetic* and *electric* form factors of the D resonance, G^p_M and G^p_E, are given, respectively by

$$G^p_M = \frac{1}{6}\left[\frac{3M_D + M_N}{M_D}G^p_1 + \frac{\Delta M}{M_N}G^p_2\right], \tag{6.22}$$

and

$$G^p_E = \frac{1}{6}\left[\frac{\Delta M}{M_D}G^p_1 + \frac{\Delta M}{M_N}G^p_2\right]. \tag{6.23}$$

With $G^p_1 = -5.570$ and $G^p_2 = 0.624$, we obtain the values

$$G^p_M = -3.298 \quad \text{and} \quad G^p_E = -0.288. \tag{6.24}$$

For the u-channel,

$$i\mathcal{M}^u_D = -ie\frac{f_{\pi ND}}{m_\pi}\,\mathbf{I}_D\,\overline{u}(p_\mathrm{f})\left[\gamma_5 K_{\mu\nu}\epsilon^\nu G^{\mu\alpha}(p_D)q_\alpha\gamma_5\right]u(p_\mathrm{i}), \tag{6.25}$$

where $K_{\alpha\beta} = K^{p+}_{\alpha\beta}$ for the processes $\gamma p \to \pi^0 p$ and $\gamma n \to \pi^- p$, while

$$K_{\alpha\beta} = K^{n+}_{\alpha\beta} \equiv \frac{G^n_1}{2M_N}\mathscr{K}^1_{\alpha\beta} + \frac{G^n_2}{2M^2_N}\mathscr{K}^2_{\alpha\beta} \tag{6.26}$$

for the processes $\gamma p \to \pi^+ n$ and $\gamma n \to \pi^0 n$.

Similarly, with G^n_1 =0.853 and $G^n_2 = 0.100$, we obtain the value of the *magnetic* and *electric* form factors, G^n_M and G^n_E, respectively

$$G^n_M = 1.575 \quad \text{and} \quad G^n_E = 0.064. \tag{6.27}$$

7. Gauge Invariance and Form Factors

Gauge invariance is one of the central issues in the description of the interaction of photons with hadronic systems. In the case of pion photoproduction off the nucleon at the tree level Feynman diagrams, this condition is guaranteed with *bare*, *point like* particles. However, the tree-level (total) amplitude is no longer gauge invariant if one makes use of (*off-shell*) *hadronic form-factors* to account for the internal structure of extended particles such as *mesons* and *baryons* which are not point-like.
In order to *preserve* gauge invariance, we will need to construct additional current contributions beyond the usual Feynman diagrams to *cancel* the resulting gauge-*violating* terms.
For *bare* nucleons, the tree-level amplitudes may be written as [22, 23, 24]

$$i\mathcal{M}_{\rm fi} = e\frac{f_{\pi NN}}{m_\pi}\mathbf{I}_N \sum_{j=1}^{4} A_j\,\overline{u}(p_{\rm f})\left[\epsilon_\alpha \mathcal{M}_j^\alpha\right]u(p_{\rm i}), \tag{7.1}$$

which represents an expansion based on the operators

$$\mathcal{M}_1^\alpha \equiv -\gamma_5\gamma^\alpha \not{k}, \tag{7.2}$$
$$\mathcal{M}_2^\alpha \equiv 2\gamma_5\left(p_{\rm f}\cdot k\,p_{\rm i}^\alpha - p_{\rm i}\cdot k\,p_{\rm f}^\alpha\right), \tag{7.3}$$
$$\mathcal{M}_3^\alpha \equiv \gamma_5\left(p_{\rm i}\cdot k\,\gamma^\alpha - p_{\rm i}^\alpha \not{k}\right), \tag{7.4}$$
$$\mathcal{M}_4^\alpha \equiv \gamma_5\left(p_{\rm f}\cdot k\,\gamma^\alpha - p_{\rm f}^\alpha \not{k}\right), \tag{7.5}$$

where each of the operators $\mathcal{M}_1^\alpha, \cdots, \mathcal{M}_4^\alpha$ is *gauge invariant* by itself, that is $k_\alpha \mathcal{M}_i^\alpha = 0$. The coefficient functions $A_1, \cdots, A_4$ will be calculated below for each of the processes $\gamma\, p \to n\, \pi^+$ and $\gamma\, p \to p\, \pi^0$, respectively.

7.1. Coefficient Functions

1. $\gamma\, p \to n\, \pi^+$

   The terms inside the brackets of the amplitudes given by Eq. (6.4), Eq. (6.7), Eq. (6.8), and Eq. (6.9), factoring out the polarization vector ϵ_α, become respectively

$$\frac{(1+\kappa_p)}{s-M_{\rm N}^2}\,2p_{\rm i}\cdot k\,\gamma_5\gamma^\alpha - \frac{C_p}{s-M_{\rm N}^2}\,2p_{\rm i}\cdot k\,\gamma_5\gamma^\alpha\not{k} - \frac{(1+\kappa_p)}{s-M_{\rm N}^2}\,2M_{\rm N}\,\gamma_5\gamma^\alpha\not{k}$$
$$+\frac{4M_{\rm N}}{s-M_{\rm N}^2}\,p_{\rm i}^\alpha\,\gamma_5 - \frac{2\kappa_p}{s-M_{\rm N}^2}\,p_{\rm i}^\alpha\gamma_5\not{k}, \tag{7.6}$$

$$\frac{\kappa_n}{u-M_{\rm N}^2}\,2p_{\rm f}\cdot k\,\gamma_5\gamma^\alpha+\frac{C_n}{u-M_{\rm N}^2}\,2p_{\rm f}\cdot k\,\gamma_5\gamma^\alpha\not{k}-\frac{\kappa_n}{u-M_{\rm N}^2}\,2M_{\rm N}\,\gamma_5\gamma^\alpha\not{k}$$
$$-\frac{2\kappa_n}{u-M_{\rm N}^2}\,p_{\rm f}^\alpha\gamma_5\not{k}, \tag{7.7}$$

$$-\gamma_5\gamma^\alpha, \tag{7.8}$$

and

$$\frac{4M_{\rm N}}{t-m_\pi^2}\,p_{\rm i}^\alpha\,\gamma_5-\frac{4M_{\rm N}}{t-m_\pi^2}\,p_{\rm f}^\alpha\,\gamma_5, \tag{7.9}$$

where $C_p \equiv \frac{\kappa_p}{M_{\rm N}^2}$, and $C_n \equiv \frac{\kappa_n}{M_{\rm N}^2}$.

Then, the contribution to the total π^+ photoproduction amplitude given by the *nucleon* s- and u-channels, the *contact* term, and the *pion* t-channel to the expansion given by Eq. (7.1) leads to the coefficient functions

$$A_1 \equiv 2M_{\rm N}\left(\frac{1+\kappa_p}{s-M_{\rm N}^2}+\frac{\kappa_n}{u-M_{\rm N}^2}\right)+\frac{\kappa_p+\kappa_n}{2M_{\rm N}}, \tag{7.10}$$

$$A_2 \equiv \frac{4M_{\rm N}}{(s-M_{\rm N}^2)(t-m_\pi^2)}, \tag{7.11}$$

$$A_3 \equiv \frac{2\kappa_p}{s-M_{\rm N}^2}, \tag{7.12}$$

$$A_4 \equiv \frac{2\kappa_n}{u-M_{\rm N}^2}. \tag{7.13}$$

From these we can see that the terms proportional to $\mathcal{M}_1^\alpha$, $\mathcal{M}_3^\alpha$ and $\mathcal{M}_4^\alpha$ arise from *purely magnetic* contributions (κ_p and κ_n) and therefore are always *gauge invariant* by themselves, regardless of whether one uses form factors or not. The problem is with the term A_2 which arises from the sum of the *electric* contributions of the *nucleon* s-channel and the *pion* t-channel, this is known as the A_2 *problem* [22, 23, 24].

2. $\gamma\, p \to p\, \pi^0$

Similarly, for the process $\gamma\, p \to p\, \pi^0$, the terms inside the brackets of the amplitudes given by Eq. (6.4), and Eq. (6.6), factoring out the polarization vector ϵ_α, become respectively

$$\frac{(1+\kappa_p)}{s-M_{\rm N}^2}\,2p_{\rm i}\cdot k\,\gamma_5\gamma^\alpha-\frac{C_p}{s-M_{\rm N}^2}\,2p_{\rm i}\cdot k\,\gamma_5\gamma^\alpha\not{k}-\frac{(1+\kappa_p)}{s-M_{\rm N}^2}\,2M_{\rm N}\,\gamma_5\gamma^\alpha\not{k}$$
$$+\frac{4M_{\rm N}}{s-M_{\rm N}^2}\,p_{\rm i}^\alpha\,\gamma_5-\frac{2\kappa_p}{s-M_{\rm N}^2}\,p_{\rm i}^\alpha\gamma_5\not{k}, \tag{7.14}$$

and

$$\frac{(1+\kappa_p)}{u-M_{\rm N}^2}\,2p_{\rm f}\cdot k\,\gamma_5\gamma^\alpha+\frac{C_p}{u-M_{\rm N}^2}\,2p_{\rm f}\cdot k\,\gamma_5\gamma^\alpha\not{k}-\frac{(1+\kappa_p)}{u-M_{\rm N}^2}\,2M_{\rm N}\,\gamma_5\gamma^\alpha\not{k}$$
$$+\frac{4M_{\rm N}}{u-M_{\rm N}^2}\,p_{\rm f}^\alpha\,\gamma_5-\frac{2\kappa_p}{u-M_{\rm N}^2}\,p_{\rm f}^\alpha\gamma_5\not{k}. \tag{7.15}$$

In this case, the contribution to the total π^0 photoproduction amplitude given by the *nucleon* s- and u-channels to the expansion given by Eq. (7.1) leads to the coefficient functions

$$A_1 \equiv 2M_{\rm N}(1+\kappa_p)\left(\frac{1}{s-M_{\rm N}^2}+\frac{1}{u-M_{\rm N}^2}\right)+\frac{\kappa_p}{M_{\rm N}}, \tag{7.16}$$

$$A_2 \equiv -\frac{4M_{\rm N}}{(s-M_{\rm N}^2)(u-M_{\rm N}^2)}, \tag{7.17}$$

$$A_3 \equiv \frac{2\kappa_p}{s-M_{\rm N}^2}, \tag{7.18}$$

$$A_4 \equiv \frac{2\kappa_p}{u-M_{\rm N}^2}, \tag{7.19}$$

which differ a bit from the previous case because the *contact* and *pion* t-channel terms are abscent in π^0 photoproduction (see isospin factors, Table 6.1).

7.2. Form Factors

We now consider the *nucleons* as composite objects by introducing a momentum dependent *strong form factor* at the πNN vertex of each *Born term*

$$F_1 \equiv F_1(s) = f[(p_{\rm i}+k)^2, M_{\rm N}^2, m_\pi^2], \tag{7.20}$$

$$F_2 \equiv F_2(u) = f[M_{\rm N}^2, (p_{\rm f}-k)^2, m_\pi^2], \tag{7.21}$$

$$F_3 \equiv F_3(t) = f[M_{\rm N}^2, M_{\rm N}^2, (p_{\rm i}-p_{\rm f})^2], \tag{7.22}$$

which are chosen as a function of the squares of the four momenta of its three legs [22, 24, 23].

The *total* amplitude given by Eq. (7.1) then becomes

$$i\mathcal{M}'_{\rm fi} = e\frac{f_{\pi NN}}{m_\pi}\mathbf{I}_N\,\epsilon_\alpha\overline{u}(p_{\rm f})\left[\sum_{j=1}^{4}A'_j\mathcal{M}_j^\alpha+\mathcal{M}_{\rm vio}^\alpha\right]u(p_{\rm i}), \tag{7.23}$$

where the coefficient functions for the process $\gamma\, p \to n\, \pi^+$ are given by

$$A_1 \to A'_1 \equiv 2M_{\rm N}\left(\frac{F_1(1+\kappa_p)}{s-M_{\rm N}^2}+\frac{F_2\kappa_n}{u-M_{\rm N}^2}\right)+\frac{F_1\kappa_p+F_2\kappa_n}{2M_{\rm N}}, \tag{7.24}$$

$$A_2 \to A'_2 \equiv \frac{4\mathcal{F}M_{\rm N}}{(s-M_{\rm N}^2)(t-m_\pi^2)}, \tag{7.25}$$

$$A_3 \to A'_3 \equiv \frac{2F_1\kappa_p}{s-M_{\rm N}^2}, \tag{7.26}$$

$$A_4 \to A'_4 \equiv \frac{2F_2\kappa_n}{u-M_{\rm N}^2}, \tag{7.27}$$

and the additional *gauge-invariance-violating* term, M^{α}_{vio}, is given by

$$\mathcal{M}^{\alpha}_{\text{vio}} \equiv 4M_{\text{N}}\gamma_5 \left(\frac{(F_1 - \mathcal{F})p^{\alpha}_{\text{i}}}{s - M^2_{\text{N}}} + \frac{(F_3 - \mathcal{F})q^{\alpha}}{t - m^2_{\pi}} \right) + (F_1 - 1)\gamma_5\gamma^{\alpha}. \tag{7.28}$$

It is important to mention that, after including the form factors, the additional *form factor* $\mathcal{F}$ in the coefficient A_2 is undefined and has been included "strategically" in the following way

$$\begin{aligned} F_1(t - m^2_{\pi})p^{\alpha}_{\text{i}} + F_3(s - M^2_{\text{N}})(p^{\alpha}_{\text{i}} - p^{\alpha}_{\text{f}}) \longrightarrow F_1(t - m^2_{\pi})p^{\alpha}_{\text{i}} + F_3(s - M^2_{\text{N}})(p^{\alpha}_{\text{i}} - p^{\alpha}_{\text{f}}) \\ + \mathcal{F}(u - M^2_{\text{N}})p^{\alpha}_{\text{i}} - \mathcal{F}(u - M^2_{\text{N}})p^{\alpha}_{\text{i}} \\ + \mathcal{F}(s - M^2_{\text{N}})p^{\alpha}_{\text{f}} - \mathcal{F}(s - M^2_{\text{N}})p^{\alpha}_{\text{f}}, \end{aligned} \tag{7.29}$$

from which, with $s - M^2_{\text{N}} = 2p_{\text{i}} \cdot k$ and $u - M^2_{\text{N}} = -2p_{\text{f}} \cdot k$, we obtain that

$$\begin{aligned} F_1(t - m^2_{\pi})p^{\alpha}_{\text{i}} + F_3(s - M^2_{\text{N}})(p^{\alpha}_{\text{i}} - p^{\alpha}_{\text{f}}) = 2\mathcal{F}(p_{\text{f}} \cdot k\, p^{\alpha}_{\text{i}} - p_{\text{i}} \cdot k\, p^{\alpha}_{\text{f}}) + (F_1 - \mathcal{F})(t - m^2_{\pi})p^{\alpha}_{\text{i}} \\ + (F_3 - \mathcal{F})(s - M^2_{\text{N}})q^{\alpha}, \end{aligned} \tag{7.30}$$

where $q = p_{\text{i}} - p_{\text{f}}$.
In this way we have isolated the *gauge-invariance-violating* term given by Eq. (7.28) in a form that makes the comparison with Eq. (7.1) easier and the full amplitude $i\mathcal{M}'_{\text{fi}}$ *does not depend* on it since the sum of the $\mathcal{F}$ contributions from Eq. (7.28) exactly cancels the A'_2 term.
Notice that the *pointlike* Born terms are recovered by setting all form factors equal to unity.
In order to *restore gauge-invariance* we have to introduce an *additional contact current* (that is, a term *free* of poles), $\mathcal{M}^{\alpha}_{\text{c}}$, with *on-shell* matrix elements cancelling exactly the gauge-violating term given by Eq. (7.28), that is

$$\epsilon_{\alpha}\overline{u}(p_{\text{f}})\,[\mathcal{M}^{\alpha}_{\text{c}}]\,u(p_{\text{i}}) \equiv -\epsilon_{\alpha}\overline{u}(p_{\text{f}})\,[\mathcal{M}^{\alpha}_{\text{vio}}]\,u(p_{\text{i}}). \tag{7.31}$$

Then by adding this *contact* term to Eq. (7.23), we obtain the *gauge-invariant* amplitude

$$i\mathcal{M}'_{\text{fi}} = e\frac{f_{\pi NN}}{m_{\pi}}\mathbf{I}_N\,\epsilon_{\alpha}\overline{u}(p_{\text{f}}) \left[\sum_{j=1}^{4} A'_j \mathcal{M}^{\alpha}_j \right] u(p_{\text{i}}), \tag{7.32}$$

which *depends* on the *undefined* form factor $\mathcal{F}$. However, the functional form of $\mathcal{F}$ is not arbitrary, it is *constrained* because the resulting amplitudes should obey the constrains imposed by *gauge invariance* and *crossing symmetry.* In addition the contact term given by Eq. (7.31) must be free of poles, therefore $F_1(s)$, $F_2(u)$ and $F_3(t)$ must be such that

$$F_1(M^2_{\text{N}}) = F_2(M^2_{\text{N}}) = F_3(m^2_{\pi}) = 1. \tag{7.33}$$

Then, for example, one possible choice for the form factor $\mathcal{F}$ which satisfies the above constrains is [24]

$$\begin{aligned} \mathcal{F}(s,u,t) = F_1(s) + F_2(u) + F_3(t) - F_1(s)F_2(u) - F_1(s)F_3(t) - F_2(u)F_3(t) + \\ + F_1(s)F_2(u)F_3(t). \end{aligned} \tag{7.34}$$

Similarly, the coefficient functions for the process $\gamma\, p \to p\, \pi^0$ are given by

$$A_1 \to A_1' \equiv 2M_{\rm N}(1+\kappa_p)\left(\frac{F_1}{s-M_{\rm N}^2}+\frac{F_2}{u-M_{\rm N}^2}\right)+\frac{\kappa_p}{2M_{\rm N}}\,(F_1+F_2)\,, \tag{7.35}$$

$$A_2 \to A_2' \equiv -\frac{4\mathcal{F}M_{\rm N}}{(s-M_{\rm N}^2)(u-M_{\rm N}^2)}, \tag{7.36}$$

$$A_3 \to A_3' \equiv \frac{2F_1\kappa_p}{s-M_{\rm N}^2}, \tag{7.37}$$

$$A_4 \to A_4' \equiv \frac{2F_2\kappa_p}{u-M_{\rm N}^2}, \tag{7.38}$$

and the additional *gauge-invariance-violating* term in this case is given by

$$\mathcal{M}^{\alpha}_{\rm vio} \equiv 4M_{\rm N}\gamma_5\left(\frac{(F_1-\mathcal{F})p_{\rm i}^{\alpha}}{s-M_{\rm N}^2}+\frac{(F_2-\mathcal{F})p_{\rm f}^{\alpha}}{u-M_{\rm N}^2}\right)+(F_1-F_2)\gamma_5\gamma^{\alpha}. \tag{7.39}$$

7.3. Scattering Amplitudes

By means of the above analysis, we obtain the gauge invariant scattering amplitudes including form factors.

7.3.1. Born Terms

1. **Nucleon**

   For the nucleon term

$$i\mathcal{M}'^{s}_{N} = -iF_1 e\frac{f_{\pi NN}}{m_\pi}\mathbf{I}_N\,\overline{u}(p_{\rm f})\left[\gamma_5\not{q}\,i\frac{\not{p}_{\rm i}+\not{k}+M_N}{s-M_N^2}\left(\not{\epsilon}-\frac{\kappa_p}{2M_{\rm N}}\not{\epsilon}\not{k}\right)\right]u(p_{\rm i}) \tag{7.40}$$

$$+\,2iM_N(F_1-\mathcal{F})e\frac{f_{\pi NN}}{m_\pi}\mathbf{I}_N\,\overline{u}(p_{\rm f})\left[\gamma_5\,i\frac{\not{p}_{\rm i}+M_N}{s-M_N^2}\not{\epsilon}\right]u(p_{\rm i}), \tag{7.41}$$

   for the s-channel.
   For the u-channel,

$$i\mathcal{M}'^{u}_{N} = -iF_2 e\frac{f_{\pi NN}}{m_\pi}\frac{\kappa_n}{2M_N}\sqrt{2}\,\overline{u}(p_{\rm f})\left[\not{\epsilon}\not{k}\,i\frac{\not{p}_{\rm f}-\not{k}+M_N}{u-M_N^2}\gamma_5\not{q}\right]u(p_{\rm i}), \tag{7.42}$$

   for π^+ photoproduction on proton, and

$$i\mathcal{M}'^{u}_{N} = -iF_2 e\frac{f_{\pi NN}}{m_\pi}\,\overline{u}(p_{\rm f})\left[\left(\not{\epsilon}-\frac{\kappa_p}{2M_N}\not{\epsilon}\not{k}\right)\,i\frac{\not{p}_{\rm f}-\not{k}+M_N}{u-M_N^2}\gamma_5\not{q}\right]u(p_{\rm i}) \tag{7.43}$$

$$+\,2iM_N(F_2-\mathcal{F})e\frac{f_{\pi NN}}{m_\pi}\,\overline{u}(p_{\rm f})\left[\not{\epsilon}\,i\frac{\not{p}_{\rm f}+M_N}{u-M_N^2}\gamma_5\right]u(p_{\rm i}), \tag{7.44}$$

   for π^0 photoproduction on proton.

2. **Kroll-Rudermann (Contact)**

$$i\mathcal{M}'_{\rm c} = -F_1 e \frac{f_{\pi NN}}{m_\pi} \sqrt{2}\, \overline{u}(p_{\rm f}) \left[\gamma_5 \not{\epsilon}\right] u(p_{\rm i}), \tag{7.45}$$

for π^+ photoproduction on proton, and for π^0 photoproduction on proton, there appears a contact (non-physical) term given by

$$i\mathcal{M}'_{\rm c} = -(F_1 - F_2) e \frac{f_{\pi NN}}{m_\pi} \overline{u}(p_{\rm f}) \left[\gamma_5 \not{\epsilon}\right] u(p_{\rm i}). \tag{7.46}$$

3. **Pion in Flight or t-channel**

$$i\mathcal{M}'^{t}_{\pi} = -iF_3 e \frac{f_{\pi NN}}{m_\pi} \sqrt{2}\, i \frac{q \cdot \epsilon}{t - m_\pi^2} \overline{u}(p_{\rm f}) \left[\gamma_5 \left(\not{q} - \not{k}\right)\right] u(p_{\rm i}) \tag{7.47}$$

$$- 2iM_N (F_3 - \mathcal{F}) e \frac{f_{\pi NN}}{m_\pi} \sqrt{2}\, i \frac{q \cdot \epsilon}{t - m_\pi^2} \overline{u}(p_{\rm f}) \left[\gamma_5\right] u(p_{\rm i}), \tag{7.48}$$

for π^+ photoproduction on proton.

For the *numerical evaluation* of the scattering amplitudes, we will choose *covariant* vertex parametrizations without any *singularities* on the real axis. One common vertex parametrization used is of the form [11]

$$F_1(s) = \frac{\Lambda^4}{\Lambda^4 + (s - M_{\rm N}^2)^2}, \tag{7.49}$$

$$F_2(u) = \frac{\Lambda^4}{\Lambda^4 + (u - M_{\rm N}^2)^2}, \tag{7.50}$$

$$F_3(t) = \frac{\Lambda^4}{\Lambda^4 + (t - m_\pi^2)^2}, \tag{7.51}$$

where Λ is some *cutoff* parameter to be determined from the fitting.

7.3.2. Vector Meson and Resonance Terms

The terms corresponding to *vector mesons* and *resonances* are all gauge invariant *independently*, therefore do not depent on other prescriptions for restoring gauge invariance. It is important to mention, that in the case of the spin-$\frac{3}{2}$ resonances a form factor must be included to regularize the behaviour of the propagator at high energies.

8. Electromagnetic Multipoles

In the study of pion photoproduction via the intermediate excitation of resonances it is convenient to decompose the initial and final state into multipole components since the intermediate resonance has definite *parity* and *angular momentum*.
In the initial state the photon with orbital angular momentum ($\vec{L}_\gamma$) relative to the target nucleon

$$L_\gamma = 1,\ 2,\ \cdots, \tag{8.1}$$

spin ($\vec{S}_\gamma$)

$$S_\gamma = 1, \tag{8.2}$$

total angular momentum ($\vec{J}_\gamma$)

$$J_\gamma = L_\gamma + 1,\ L_\gamma,\ L_\gamma - 1 \tag{8.3}$$

and parity (P_γ)

$$P_\gamma = \begin{cases} (-1)^{L_\gamma} & \text{for the } \textit{electric } (EL_\gamma) - \text{multipoles,} \\ (-1)^{L_\gamma+1} & \text{for the } \textit{magnetic } (ML_\gamma) - \text{multipoles} \end{cases} \tag{8.4}$$

couples *electromagnetically* [47] to the target nucleon with spin ($\vec{J}_\mathrm{N}$)

$$J_\mathrm{N} = \frac{1}{2} \tag{8.5}$$

and parity (P_N)

$$P_\mathrm{N} = 1 \tag{8.6}$$

to produce a resonance with *spin* ($\vec{J}_\mathrm{R}$)

$$J_\mathrm{R} = J_\gamma + \frac{1}{2},\ J_\gamma - \frac{1}{2} \tag{8.7}$$

and parity (P_R)

$$P_\mathrm{R} = P_\mathrm{N} \cdot P_\gamma = P_\gamma. \tag{8.8}$$

The resonance subsequently decays by the *strong interaction* to the nucleon ground state via the emission of the pion with spin 0, parity $P_\pi = -1$ and orbital angular momentum ($\vec{L}_\pi$) relative to the *recoiling* nucleon, such that

$$J_\mathrm{R} = L_\pi + \frac{1}{2},\ L_\pi - \frac{1}{2} \tag{8.9}$$

photon M-pole	initial state (L^P, j_p^P)	intermediate state $J_{N^*}^P$	final state (j_p^P, l^P)	multipole
$E1$	$(1^-, \frac{1}{2}^+)$	$\frac{1}{2}^-$	$(\frac{1}{2}^+, 0^-)$	E_{0+}
		$\frac{3}{2}^-$	$(\frac{1}{2}^+, 2^-)$	E_{2-}
$M1$	$(1^+, \frac{1}{2}^+)$	$\frac{1}{2}^+$	$(\frac{1}{2}^+, 1^+)$	M_{1-}
		$\frac{3}{2}^+$	$(\frac{1}{2}^+, 1^+)$	M_{1+}
$E2$	$(2^+, \frac{1}{2}^+)$	$\frac{3}{2}^+$	$(\frac{1}{2}^+, 1^+)$	E_{1+}
		$\frac{5}{2}^+$	$(\frac{1}{2}^+, 3^+)$	E_{3-}
$M2$	$(2^-, \frac{1}{2}^+)$	$\frac{3}{2}^-$	$(\frac{1}{2}^+, 2^-)$	M_{2-}
		$\frac{5}{2}^-$	$(\frac{1}{2}^+, 2^-)$	M_{2+}

Table 8.1.: Lowest order *multipoles* for photoproduction of *pion* meson [2].

and

$$P_{\mathrm{R}} = P_{\mathrm{N}} \cdot P_\pi \cdot (-1)^{L_\pi} = (-1)^{L_\pi + 1}. \tag{8.10}$$

Parity and angular momentum *conservation* lead to the following *selection rules*

$$P_{\mathrm{R}} = P_\gamma = (-1)^{L_\pi + 1}, \tag{8.11}$$

$$J_{\mathrm{R}} = J_\gamma + \frac{1}{2},\ J_\gamma - \frac{1}{2} = L_\pi + \frac{1}{2},\ L_\pi - \frac{1}{2}, \tag{8.12}$$

allowing the two possibilities for L_γ

$$L_\gamma = \begin{cases} L_\pi \pm 1, & \text{for } EL_\gamma \\ L_\pi & \text{for } ML_\gamma. \end{cases} \tag{8.13}$$

The corresponding photoproduction *multipoles* will be denoted by $E_{l\pm}$ and $M_{l\pm}$, where E and M stand for the *electric* and *magnetic* photon multipoles, respectively, l denotes the relative angular momentum of the *final meson* (L_π), and '+' or '−' indicate whether the *spin* (1/2) of the nucleon must be *added* to or *substracted* from l to form the total angular momentum J_{R} of the intermediate state.
The lowest *electromagnetic excitation modes* and the corresponding states of the *pion-proton* system with the relevant quantum numbers are summarized in Table 8.1. From this we can see that each resonance can be excited by one electric and one magnetic multipole, with the exception of *spin*-1/2 resonances, which can only be excited by one multipole (E_{0+} for *negative parity* states and M_{1-} for *positive parity* states).

8.1. Isospin Amplitudes

For the calculation of the electromagnetic multipoles we will use the following isospin decomposition of the *invariant* amplitude for a pion with *isospin* j [2, 11, 32]

$$\mathcal{M} = \chi_{\mathrm{f}}^{\dagger} \left(\mathcal{M}^{0}\, \tau_j + \mathcal{M}^{-}\, \frac{1}{2} \left[\tau_j, \tau_3 \right] + \mathcal{M}^{+}\, \delta_{j3} \right) \pi_j\, \chi_{\mathrm{i}}, \tag{8.14}$$

where the isospin decomposition amplitudes $\mathcal{M}^0$, $\mathcal{M}^+$ and $\mathcal{M}^-$ are related to the *physical* amplitudes by

$$\mathcal{M}(\gamma p \to \pi^0 p) \equiv \mathcal{M}^{(\pi^0 p)} = \mathcal{M}^{+} + \mathcal{M}^{0}, \tag{8.15}$$

$$\mathcal{M}(\gamma p \to \pi^+ n) \equiv \mathcal{M}^{(\pi^+ n)} = \sqrt{2} \left(\mathcal{M}^{-} + \mathcal{M}^{0} \right), \tag{8.16}$$

for the case of pion photoproduction on proton.
To build up the multipoles it is convenient to change the *isospin basis* from $(\mathcal{M}^0, \mathcal{M}^-, \mathcal{M}^+)$ to $(\mathcal{M}^{\frac{3}{2}},\ {}_p\mathcal{M}^{\frac{1}{2}},\ {}_n\mathcal{M}^{\frac{1}{2}})$. Both bases are related by means of [2]

$$\mathcal{M}^{\frac{3}{2}} = \mathcal{M}^{+} - \mathcal{M}^{-}, \tag{8.17}$$

$${}_p\mathcal{M}^{\frac{1}{2}} = \frac{1}{3}\mathcal{M}^{+} + \frac{2}{3}\mathcal{M}^{-} + \mathcal{M}^{0}, \tag{8.18}$$

$${}_n\mathcal{M}^{\frac{1}{2}} = -\frac{1}{3}\mathcal{M}^{+} - \frac{2}{3}\mathcal{M}^{-} + \mathcal{M}^{0}, \tag{8.19}$$

in terms of which the physical amplitudes become

$$\mathcal{M}(\gamma p \to \pi^0 p) \equiv \mathcal{M}^{(\pi^0 p)} = {}_p\mathcal{M}^{\frac{1}{2}} + \frac{2}{3}\mathcal{M}^{\frac{3}{2}}, \tag{8.20}$$

$$\mathcal{M}(\gamma p \to \pi^+ n) \equiv \mathcal{M}^{(\pi^+ n)} = \sqrt{2} \left({}_p\mathcal{M}^{\frac{1}{2}} - \frac{1}{3}\mathcal{M}^{\frac{3}{2}} \right). \tag{8.21}$$

Then the invariant amplitudes in the isospin decomposition that shall be needed for the calculation of the electromagnetic multipoles are given below.

8.1.1. Born Terms

1. **Nucleon**

$$i\mathcal{M}_{\mathrm{N}}^{s,+} = i\mathcal{M}_{\mathrm{N}}^{s,-} = i\frac{e}{2}\frac{f_{\pi NN}}{m_\pi}\, \overline{u}(p_{\mathrm{f}}) \left[\not{q}\gamma_5 i \frac{\not{p}_{\mathrm{i}} + \not{k} + M_{\mathrm{N}}}{s - M_{\mathrm{N}}^2} \left(F_1^{\mathrm{v}} \not{\epsilon} - \frac{F_2^{\mathrm{v}}}{2M_{\mathrm{N}}} \not{\epsilon}\not{k} \right) \right] u(p_{\mathrm{i}}), \tag{8.22}$$

where $F_1^{\mathrm{v}} = 1$ and $F_2^{\mathrm{v}} = 1.85$, according to Eq. (4.53), Eq. (4.56), and the values given in Eq. (4.57).

$$i\mathcal{M}_{\mathrm{N}}^{s,0} = i\frac{e}{2}\frac{f_{\pi NN}}{m_\pi}\, \overline{u}(p_{\mathrm{f}}) \left[\not{q}\gamma_5 i \frac{\not{p}_{\mathrm{i}} + \not{k} + M_{\mathrm{N}}}{s - M_{\mathrm{N}}^2} \left(F_1^{\mathrm{s}} \not{\epsilon} - \frac{F_2^{\mathrm{s}}}{2M_{\mathrm{N}}} \not{\epsilon}\not{k} \right) \right] u(p_{\mathrm{i}}), \tag{8.23}$$

where $F_1^{\rm s} = 1$ and $F_2^{\rm s} = -0.12$.

From these we obtain the isospin amplitudes for the nucleon s-channel

$$i\,{}_p\mathcal{M}_{\rm N}^{s,\,\frac{1}{2}} = i\frac{e}{2}\frac{f_{\pi NN}}{m_\pi}\,\overline{u}(p_{\rm f})\left[\not{q}\gamma_5 i\frac{\not{p}_{\rm i}+\not{k}+M_{\rm N}}{s-M_{\rm N}^2}\left(F_1^{\rm v}\not{\epsilon}-\frac{F_2^{\rm v}}{2M_{\rm N}}\not{\epsilon}\not{k}\right)\right]u(p_{\rm i}), \tag{8.24}$$

and

$$i\mathcal{M}_{\rm N}^{s,\,\frac{3}{2}} = 0. \tag{8.25}$$

Similarly, for the nucleon u-channel we obtain

$$i\mathcal{M}_{\rm N}^{u,\,+} = -i\mathcal{M}_{\rm N}^{u,\,-} = i\frac{e}{2}\frac{f_{\pi NN}}{m_\pi}\,\overline{u}(p_{\rm f})\left[\left(F_1^{\rm v}\not{\epsilon}-\frac{F_2^{\rm v}}{2M_N}\not{\epsilon}\not{k}\right)i\frac{\not{p}_{\rm f}-\not{k}+M_N}{u-M_N^2}\not{q}\gamma_5\right]u(p_{\rm i}), \tag{8.26}$$

and

$$i\mathcal{M}_{\rm N}^{u,\,0} = i\frac{e}{2}\frac{f_{\pi NN}}{m_\pi}\,\overline{u}(p_{\rm f})\left[\left(F_1^{\rm s}\not{\epsilon}-\frac{F_2^{\rm s}}{2M_N}\not{\epsilon}\not{k}\right)i\frac{\not{p}_{\rm f}-\not{k}+M_N}{u-M_N^2}\not{q}\gamma_5\right]u(p_{\rm i}). \tag{8.27}$$

From these we obtain the isospin amplitudes for the nucleon u-channel

$$i\,{}_p\mathcal{M}_{\rm N}^{u,\,\frac{1}{2}} = -\,i\frac{e}{6}\frac{f_{\pi NN}}{m_\pi}\,\overline{u}(p_{\rm f})\left[\left(F_1^{\rm v}\not{\epsilon}-\frac{F_2^{\rm v}}{2M_N}\not{\epsilon}\not{k}\right)i\frac{\not{p}_{\rm f}-\not{k}+M_N}{u-M_N^2}\not{q}\gamma_5\right]u(p_{\rm i}) \tag{8.28}$$

$$+\,i\frac{e}{2}\frac{f_{\pi NN}}{m_\pi}\,\overline{u}(p_{\rm f})\left[\left(F_1^{\rm s}\not{\epsilon}-\frac{F_2^{\rm s}}{2M_N}\not{\epsilon}\not{k}\right)i\frac{\not{p}_{\rm f}-\not{k}+M_N}{u-M_N^2}\not{q}\gamma_5\right]u(p_{\rm i}), \tag{8.29}$$

and

$$i\mathcal{M}_{\rm N}^{u,\,\frac{3}{2}} = ie\frac{f_{\pi NN}}{m_\pi}\,\overline{u}(p_{\rm f})\left[\left(F_1^{\rm v}\not{\epsilon}-\frac{F_2^{\rm v}}{2M_N}\not{\epsilon}\not{k}\right)i\frac{\not{p}_{\rm f}-\not{k}+M_N}{u-M_N^2}\not{q}\gamma_5\right]u(p_{\rm i}). \tag{8.30}$$

2. **Kroll-Rudermann (Contact)**

$$i\mathcal{M}_{\rm c}^{-} = ie\frac{f_{\pi NN}}{m_\pi}\,\overline{u}(p_{\rm f})\,[i\gamma_5\not{\epsilon}]\,u(p_{\rm i}), \tag{8.31}$$

and

$$i\mathcal{M}_{\rm c}^{+} = i\mathcal{M}_{\rm c}^{0} = 0. \tag{8.32}$$

From these we obtain the isospin amplitudes for the contact term

$$i\,{}_p\mathcal{M}_{\rm c}^{\frac{1}{2}} = i\frac{2}{3}e\frac{f_{\pi NN}}{m_\pi}\,\overline{u}(p_{\rm f})\,[i\gamma_5\not{\epsilon}]\,u(p_{\rm i}), \tag{8.33}$$

and

$$i\mathcal{M}_{\rm c}^{\frac{3}{2}} = -ie\frac{f_{\pi NN}}{m_\pi}\,\overline{u}(p_{\rm f})\,[i\gamma_5\not{\epsilon}]\,u(p_{\rm i}). \tag{8.34}$$

3. **Pion in Flight or t-channel**

$$i\mathcal{M}_{\pi}^{t,-} = ieF_1^{\mathrm{v}} \frac{f_{\pi NN}}{m_\pi} \frac{i}{t-m_\pi^2} (q\cdot\epsilon)\, \overline{u}(p_{\mathrm{f}}) \left[\left(\not{q} - \not{k}\right)\gamma_5 \right] u(p_{\mathrm{i}}), \tag{8.35}$$

and

$$i\mathcal{M}_{\pi}^{t,+} = i\mathcal{M}_{\pi}^{t,0} = 0. \tag{8.36}$$

From these we obtain the isospin amplitudes for the t-channel

$$i\,{}_{p}\mathcal{M}_{\pi}^{t,\frac{1}{2}} = i\frac{2}{3} eF_1^{\mathrm{v}} \frac{f_{\pi NN}}{m_\pi} \frac{i}{t-m_\pi^2} (q\cdot\epsilon)\, \overline{u}(p_{\mathrm{f}}) \left[\left(\not{q} - \not{k}\right)\gamma_5 \right] u(p_{\mathrm{i}}), \tag{8.37}$$

and

$$i\mathcal{M}_{\pi}^{t,\frac{3}{2}} = -ieF_1^{\mathrm{v}} \frac{f_{\pi NN}}{m_\pi} \frac{i}{t-m_\pi^2} (q\cdot\epsilon)\, \overline{u}(p_{\mathrm{f}}) \left[\left(\not{q} - \not{k}\right)\gamma_5 \right] u(p_{\mathrm{i}}). \tag{8.38}$$

8.1.2. Vector Meson Terms

1. ρ **Meson**

$$i\mathcal{M}_{\rho}^{t,0} = ie\frac{\lambda_{\rho\pi\gamma}}{m_\pi} \frac{\epsilon_{\lambda\sigma\nu\mu} k^\sigma q^\nu \epsilon^\lambda}{t-m_\rho^2} \overline{u}(p_{\mathrm{f}}) \left[g_\rho^{\mathrm{v}}\gamma^\mu - \frac{g_\rho^{\mathrm{t}}}{2M_{\mathrm{N}}} i\sigma^{\mu\beta} (q-k)_\beta \right] u(p_{\mathrm{i}}), \tag{8.39}$$

and

$$i\mathcal{M}_{\rho}^{t,+} = i\mathcal{M}_{\rho}^{t,-} = 0. \tag{8.40}$$

From these we obtain the isospin amplitudes for the ρ meson

$$i\,{}_{p}\mathcal{M}_{\rho}^{t,\frac{1}{2}} = ie\frac{\lambda_{\rho\pi\gamma}}{m_\pi} \frac{\epsilon_{\lambda\sigma\nu\mu} k^\sigma q^\nu \epsilon^\lambda}{t-m_\rho^2} \overline{u}(p_{\mathrm{f}}) \left[g_\rho^{\mathrm{v}}\gamma^\mu - \frac{g_\rho^{\mathrm{t}}}{2M_{\mathrm{N}}} i\sigma^{\mu\beta} (q-k)_\beta \right] u(p_{\mathrm{i}}), \tag{8.41}$$

and

$$i\mathcal{M}_{\rho}^{t,\frac{3}{2}} = 0. \tag{8.42}$$

2. ω **Meson**

$$i\mathcal{M}_{\omega}^{t,+} = ie\frac{\lambda_{\omega\pi\gamma}}{m_\pi} \frac{\epsilon_{\lambda\sigma\nu\mu} k^\sigma q^\nu \epsilon^\lambda}{t-m_\omega^2} \overline{u}(p_{\mathrm{f}}) \left[g_\omega^{\mathrm{v}}\gamma^\mu - \frac{g_\omega^{\mathrm{t}}}{2M_{\mathrm{N}}} i\sigma^{\mu\beta} (q-k)_\beta \right] u(p_{\mathrm{i}}), \tag{8.43}$$

and

$$i\mathcal{M}_{\omega}^{t,-} = i\mathcal{M}_{\omega}^{t,0} = 0. \tag{8.44}$$

From these we obtain the isospin amplitudes for the ω meson

$$i\,{}_{p}\mathcal{M}_{\omega}^{t,\frac{1}{2}} = i\frac{e}{3}\frac{\lambda_{\omega\pi\gamma}}{m_\pi} \frac{\epsilon_{\lambda\sigma\nu\mu} k^\sigma q^\nu \epsilon^\lambda}{t-m_\omega^2} \overline{u}(p_{\mathrm{f}}) \left[g_\omega^{\mathrm{v}}\gamma^\mu - \frac{g_\omega^{\mathrm{t}}}{2M_{\mathrm{N}}} i\sigma^{\mu\beta} (q-k)_\beta \right] u(p_{\mathrm{i}}), \tag{8.45}$$

and

$$i\mathcal{M}_{\omega}^{t,\frac{3}{2}} = ie\frac{\lambda_{\omega\pi\gamma}}{m_\pi} \frac{\epsilon_{\lambda\sigma\nu\mu} k^\sigma q^\nu \epsilon^\lambda}{t-m_\omega^2} \overline{u}(p_{\mathrm{f}}) \left[g_\omega^{\mathrm{v}}\gamma^\mu - \frac{g_\omega^{\mathrm{t}}}{2M_{\mathrm{N}}} i\sigma^{\mu\beta} (q-k)_\beta \right] u(p_{\mathrm{i}}). \tag{8.46}$$

8.1.3. Resonance Terms

1. **Spin-$\frac{1}{2}$ Nucleon Resonances of Negative Parity:** $S_{11}(1535)$ **and** $S_{11}(1650)$

$$i\mathcal{M}^{s,+}_{R^-} = i\mathcal{M}^{s,-}_{R^-} = i\frac{e}{2}\frac{f_{\pi NR^-}}{m_\pi}\frac{\kappa^v_{R^-}}{\Sigma M}\,\overline{u}(p_\mathrm{f})\left[\not{\epsilon}\gamma_5 i\frac{\not{p}_\mathrm{i}+\not{k}-M_{R^-}}{s-M^2_{R^-}}\not{\epsilon}\not{k}\right]u(p_\mathrm{i}), \tag{8.47}$$

and

$$i\mathcal{M}^{s,0}_{R^-} = i\frac{e}{2}\frac{f_{\pi NR^-}}{m_\pi}\frac{\kappa^s_{R^-}}{\Sigma M}\,\overline{u}(p_\mathrm{f})\left[\not{\epsilon}\gamma_5 i\frac{\not{p}_\mathrm{i}+\not{k}-M_{R^-}}{s-M^2_{R^-}}\not{\epsilon}\not{k}\right]u(p_\mathrm{i}), \tag{8.48}$$

From these we obtain the isospin amplitudes for the negative parity resonances s-channel

$$i\,{}_p\mathcal{M}^{s,\frac{1}{2}}_{R^-} = i\frac{e}{2}\frac{f_{\pi NR^-}}{m_\pi}\frac{\kappa^p_{R^-}}{\Sigma M}\,\overline{u}(p_\mathrm{f})\left[\not{\epsilon}\gamma_5 i\frac{\not{p}_\mathrm{i}+\not{k}-M_{R^-}}{s-M^2_{R^-}}\not{\epsilon}\not{k}\right]u(p_\mathrm{i}), \tag{8.49}$$

and

$$i\mathcal{M}^{s,\frac{3}{2}}_{R^-} = 0. \tag{8.50}$$

Similarly, for the u-channel we obtain

$$i\mathcal{M}^{u,+}_{R^-} = -i\mathcal{M}^{u,-}_{R^-} = i\frac{e}{2}\frac{f_{\pi NR^-}}{m_\pi}\frac{\kappa^v_{R^-}}{\Sigma M}\,\overline{u}(p_\mathrm{f})\left[\not{\epsilon}\not{k} i\frac{\not{p}_\mathrm{f}-\not{k}-M_{R^-}}{u-M^2_{R^-}}\not{\epsilon}\gamma_5\right]u(p_\mathrm{i}), \tag{8.51}$$

and

$$i\mathcal{M}^{u,0}_{R^-} = i\frac{e}{2}\frac{f_{\pi NR^-}}{m_\pi}\frac{\kappa^s_{R^-}}{\Sigma M}\,\overline{u}(p_\mathrm{f})\left[\not{\epsilon}\not{k} i\frac{\not{p}_\mathrm{f}-\not{k}-M_{R^-}}{u-M^2_{R^-}}\not{\epsilon}\gamma_5\right]u(p_\mathrm{i}). \tag{8.52}$$

From these we obtain the isospin amplitudes for the negative parity resonances u-channel

$$i\,{}_p\mathcal{M}^{u,\frac{1}{2}}_{R^-} = -\,i\frac{e}{6}\frac{f_{\pi NR^-}}{m_\pi}\frac{\kappa^v_{R^-}}{\Sigma M}\,\overline{u}(p_\mathrm{f})\left[\not{\epsilon}\not{k} i\frac{\not{p}_\mathrm{f}-\not{k}-M_{R^-}}{u-M^2_{R^-}}\not{\epsilon}\gamma_5\right]u(p_\mathrm{i}) \tag{8.53}$$

$$+\,i\frac{e}{2}\frac{f_{\pi NR^-}}{m_\pi}\frac{\kappa^s_{R^-}}{\Sigma M}\,\overline{u}(p_\mathrm{f})\left[\not{\epsilon}\not{k} i\frac{\not{p}_\mathrm{f}-\not{k}-M_{R^-}}{u-M^2_{R^-}}\not{\epsilon}\gamma_5\right]u(p_\mathrm{i}), \tag{8.54}$$

and

$$i\mathcal{M}^{u,\frac{3}{2}}_{R^-} = ie\frac{f_{\pi NR^-}}{m_\pi}\frac{\kappa^v_{R^-}}{\Sigma M}\,\overline{u}(p_\mathrm{f})\left[\not{\epsilon}\not{k} i\frac{\not{p}_\mathrm{f}-\not{k}-M_{R^-}}{u-M^2_{R^-}}\not{\epsilon}\gamma_5\right]u(p_\mathrm{i}). \tag{8.55}$$

2. **Spin-$\frac{1}{2}$ Nucleon Resonances of Positive Parity:** $P_{11}(1440)$ **and** $P_{11}(1710)$

$$i\mathcal{M}^{s,+}_{R^+} = i\mathcal{M}^{s,-}_{R^+} = -i\frac{e}{2}\frac{f_{\pi NR^+}}{m_\pi}\frac{\kappa^v_{R^+}}{\Sigma M}\,\overline{u}(p_\mathrm{f})\left[\not{\epsilon}\gamma_5 i\frac{\not{p}_\mathrm{i}+\not{k}+M_{R^+}}{s-M^2_{R^+}}\not{\epsilon}\not{k}\right]u(p_\mathrm{i}), \tag{8.56}$$

and

$$i\mathcal{M}^{s,\,0}_{R^+} = -i\frac{e}{2}\frac{f_{\pi N R^+}}{m_\pi}\frac{\kappa^s_{R^+}}{\Sigma M}\,\overline{u}(p_\mathrm{f})\left[\not{\epsilon}\gamma_5 i\frac{\not{p}_\mathrm{i}+\not{k}+M_{R^+}}{s-M^2_{R^+}}\not{\epsilon}\not{k}\right]u(p_\mathrm{i}). \tag{8.57}$$

From these we obtain the isospin amplitudes for the positive parity resonances s-channel

$$i\,{}_p\mathcal{M}^{s,\,\frac{1}{2}}_{R^+} = -i\frac{e}{2}\frac{f_{\pi N R^+}}{m_\pi}\frac{\kappa^p_{R^+}}{\Sigma M}\,\overline{u}(p_\mathrm{f})\left[\not{\epsilon}\gamma_5 i\frac{\not{p}_\mathrm{i}+\not{k}+M_{R^+}}{s-M^2_{R^+}}\not{\epsilon}\not{k}\right]u(p_\mathrm{i}), \tag{8.58}$$

and

$$i\mathcal{M}^{s,\,\frac{3}{2}}_{R^+} = 0. \tag{8.59}$$

Similarly, for the u-channel we obtain

$$i\mathcal{M}^{u,\,+}_{R^+} = -i\mathcal{M}^{u,\,-}_{R^+} = -i\frac{e}{2}\frac{f_{\pi N R^+}}{m_\pi}\frac{\kappa^v_{R^+}}{\Sigma M}\,\overline{u}(p_\mathrm{f})\left[\not{\epsilon}\not{k} i\frac{\not{p}_\mathrm{f}-\not{k}+M_{R^+}}{u-M^2_{R^+}}\not{q}\gamma_5\right]u(p_\mathrm{i}), \tag{8.60}$$

and

$$i\mathcal{M}^{u,\,0}_{R^+} = -i\frac{e}{2}\frac{f_{\pi N R^+}}{m_\pi}\frac{\kappa^s_{R^+}}{\Sigma M}\,\overline{u}(p_\mathrm{f})\left[\not{\epsilon}\not{k} i\frac{\not{p}_\mathrm{f}-\not{k}+M_{R^+}}{u-M^2_{R^+}}\not{q}\gamma_5\right]u(p_\mathrm{i}). \tag{8.61}$$

From these we obtain the isospin amplitudes for the positive parity resonances u-channel

$$i\,{}_p\mathcal{M}^{u,\,\frac{1}{2}}_{R^+} = i\frac{e}{6}\frac{f_{\pi N R^+}}{m_\pi}\frac{\kappa^v_{R^+}}{\Sigma M}\,\overline{u}(p_\mathrm{f})\left[\not{\epsilon}\not{k} i\frac{\not{p}_\mathrm{f}-\not{k}+M_{R^+}}{u-M^2_{R^+}}\not{q}\gamma_5\right]u(p_\mathrm{i}) \tag{8.62}$$

$$-\,i\frac{e}{2}\frac{f_{\pi N R^+}}{m_\pi}\frac{\kappa^s_{R^+}}{\Sigma M}\,\overline{u}(p_\mathrm{f})\left[\not{\epsilon}\not{k} i\frac{\not{p}_\mathrm{f}-\not{k}+M_{R^+}}{u-M^2_{R^+}}\not{q}\gamma_5\right]u(p_\mathrm{i}), \tag{8.63}$$

and

$$i\mathcal{M}^{u,\,\frac{3}{2}}_{R^+} = -ie\frac{f_{\pi N R^+}}{m_\pi}\frac{\kappa^v_{R^+}}{\Sigma M}\,\overline{u}(p_\mathrm{f})\left[\not{\epsilon}\not{k} i\frac{\not{p}_\mathrm{f}-\not{k}+M_{R^+}}{u-M^2_{R^+}}\not{q}\gamma_5\right]u(p_\mathrm{i}). \tag{8.64}$$

3. **Spin-$\frac{3}{2}$ Nucleon Resonances of Isospin-$\frac{3}{2}$:** $P_{33}(1232)$

$$i\mathcal{M}^{s,\,+}_{\Delta} = -2i\mathcal{M}^{s,\,-}_{\Delta} = i\frac{e}{3}\frac{f_{\pi N\Delta}}{m_\pi}\,\overline{u}(p_\mathrm{f})\left[q_\mu G^{\mu\alpha}(p_\Delta)\left(G_M K^M_{\alpha\beta}+G_E K^E_{\alpha\beta}\right)\epsilon^\beta\right]u(p_\mathrm{i}), \tag{8.65}$$

and

$$i\mathcal{M}^{s,\,0}_{\Delta} = 0. \tag{8.66}$$

From these we obtain the isospin amplitudes for the Δ resonance s-channel

$$i\,{}_p\mathcal{M}^{s,\,\frac{1}{2}}_{\Delta} = 0, \tag{8.67}$$

and

$$i\mathcal{M}_{\Delta}^{s,\frac{3}{2}} = i\frac{e}{2}\frac{f_{\pi N\Delta}}{m_\pi}\,\overline{u}(p_{\mathrm{f}})\left[q_\mu G^{\mu\alpha}(p_\Delta)\left(G_M K^M_{\alpha\beta} + G_E K^E_{\alpha\beta}\right)\epsilon^\beta\right]u(p_{\mathrm{i}}). \tag{8.68}$$

Similarly, for the Δ resonance u-channel we obtain

$$i\mathcal{M}_{\Delta}^{u,+} = 2i\mathcal{M}_{\Delta}^{u,-} = -i\frac{e}{3}\frac{f_{\pi N\Delta}}{m_\pi}\,\overline{u}(p_{\mathrm{f}})\left[\epsilon^\nu\left(G_M K^M_{\mu\nu} + G_E K^E_{\mu\nu}\right)G^{\mu\alpha}(p_\Delta)q_\alpha\right]u(p_{\mathrm{i}}), \tag{8.69}$$

and

$$i\mathcal{M}_{\Delta}^{u,0} = 0. \tag{8.70}$$

From these we obtain the isospin amplitudes for the Δ resonance u-channel

$$i\,{}_p\mathcal{M}_{\Delta}^{u,\frac{1}{2}} = -i\frac{2}{9}e\frac{f_{\pi N\Delta}}{m_\pi}\,\overline{u}(p_{\mathrm{f}})\left[\epsilon^\nu\left(G_M K^M_{\mu\nu} + G_E K^E_{\mu\nu}\right)G^{\mu\alpha}(p_\Delta)q_\alpha\right]u(p_{\mathrm{i}}), \tag{8.71}$$

and

$$i\mathcal{M}_{\Delta}^{u,\frac{3}{2}} = -i\frac{e}{6}\frac{f_{\pi N\Delta}}{m_\pi}\,\overline{u}(p_{\mathrm{f}})\left[\epsilon^\nu\left(G_M K^M_{\mu\nu} + G_E K^E_{\mu\nu}\right)G^{\mu\alpha}(p_\Delta)q_\alpha\right]u(p_{\mathrm{i}}). \tag{8.72}$$

4. **Spin-$\frac{3}{2}$ Nucleon Resonances of Isospin-$\frac{1}{2}$:** $D_{13}(1520)$

$$i\mathcal{M}_{D}^{s,+} = i\mathcal{M}_{D}^{s,-} = -ie\frac{f_{\pi ND}}{m_\pi}\,\overline{u}(p_{\mathrm{f}})\left[q_\mu\gamma_5 G^{\mu\alpha}(p_D)K^{v-}_{\alpha\beta}\gamma_5\epsilon^\beta\right]u(p_{\mathrm{i}}), \tag{8.73}$$

and

$$i\mathcal{M}_{D}^{s,0} = -ie\frac{f_{\pi ND}}{m_\pi}\,\overline{u}(p_{\mathrm{f}})\left[q_\mu\gamma_5 G^{\mu\alpha}(p_D)K^{s-}_{\alpha\beta}\gamma_5\epsilon^\beta\right]u(p_{\mathrm{i}}), \tag{8.74}$$

where

$$K^{s(v)\pm}_{\alpha\beta} \equiv \frac{G_1^{s(v)}}{4M_N}\mathscr{K}^1_{\alpha\beta} \pm \frac{G_2^{s(v)}}{4M_N^2}\mathscr{K}^2_{\alpha\beta}. \tag{8.75}$$

From these we obtain the isospin amplitudes for the D resonance s-channel

$$i\,{}_p\mathcal{M}_{D}^{s,\frac{1}{2}} = -ie\frac{f_{\pi ND}}{m_\pi}\,\overline{u}(p_{\mathrm{f}})\left[q_\mu\gamma_5 G^{\mu\alpha}(p_D)K^{p-}_{\alpha\beta}\gamma_5\epsilon^\beta\right]u(p_{\mathrm{i}}), \tag{8.76}$$

and

$$i\mathcal{M}_{D}^{s,\frac{3}{2}} = 0. \tag{8.77}$$

Similarly, for the D resonance u-channel we obtain

$$i\mathcal{M}_{D}^{u,+} = -i\mathcal{M}_{D}^{u,-} = ie\frac{f_{\pi ND}}{m_\pi}\,\overline{u}(p_{\mathrm{f}})\left[\epsilon^\nu\gamma_5 K^{v+}_{\mu\nu}G^{\mu\alpha}(p_D)\gamma_5 q_\alpha\right]u(p_{\mathrm{i}}), \tag{8.78}$$

and

$$i\mathcal{M}_{D}^{u,0} = ie\frac{f_{\pi ND}}{m_\pi}\,\overline{u}(p_{\mathrm{f}})\left[\epsilon^\nu\gamma_5 K^{s+}_{\mu\nu}G^{\mu\alpha}(p_D)\gamma_5 q_\alpha\right]u(p_{\mathrm{i}}). \tag{8.79}$$

From these we obtain the isospin amplitudes for the D resonance u-channel

$$i\,{}_p\mathcal{M}_D^{u,\,\frac{1}{2}} = -\,i\frac{e}{3}\frac{f_{\pi ND}}{m_\pi}\,\overline{u}(p_{\rm f})\left[\epsilon^\nu\gamma_5 K^{v+}_{\mu\nu}G^{\mu\alpha}(p_D)\gamma_5 q_\alpha\right]u(p_{\rm i}) \tag{8.80}$$

$$+\,ie\frac{f_{\pi ND}}{m_\pi}\,\overline{u}(p_{\rm f})\left[\epsilon^\nu\gamma_5 K^{s+}_{\mu\nu}G^{\mu\alpha}(p_D)\gamma_5 q_\alpha\right]u(p_{\rm i}), \tag{8.81}$$

and

$$i\mathcal{M}_D^{u,\,\frac{3}{2}} = 2ie\frac{f_{\pi ND}}{m_\pi}\,\overline{u}(p_{\rm f})\left[\epsilon^\nu\gamma_5 K^{v+}_{\mu\nu}G^{\mu\alpha}(p_D)\gamma_5 q_\alpha\right]u(p_{\rm i}). \tag{8.82}$$

Notice that for a given isospin resonance the *direct* term contributes only to a single isospin channel, $\frac{1}{2}$, while the *crossed* term contributes to both channels, $\frac{1}{2}$ and $\frac{3}{2}$.

8.2. Helicity Amplitudes

In the *c.m.* coordinate system, we quantize the initial and final spins along the directions of $\hat{k}$ and $\hat{q}$ so that *spin up* corresponds to a *negative helicity*

$$\chi^{\uparrow}_{\rm i,f} \to \lambda_{\rm i,f} = -\frac{1}{2} \tag{8.83}$$

and *viceversa*

$$\chi^{\downarrow}_{\rm i,f} \to \lambda_{\rm i,f} = +\frac{1}{2}. \tag{8.84}$$

Then the amplitudes $\mathcal{M}_{\rm fi}$ become the *helicity amplitudes* $\mathcal{M}_{\mu\lambda}$, where

$$\lambda \equiv \lambda_\gamma - \lambda_{\rm i} \tag{8.85}$$

is the initial helicity state along the photon and

$$\mu = -\lambda_{\rm f} \tag{8.86}$$

is the final helicity state along the pion.

For pion photoproduction the eight possible helicity amplitudes $\mathcal{M}_{\mu\lambda}$ are not independent because for *real*, *transverse* photons, $\lambda_\gamma = \pm 1$ and the four amplitudes with $\lambda_\gamma = -1$ are related to the four with $\lambda_\gamma = +1$ by *parity symmetry* [9, 48]

$$\mathcal{M}_{-\mu,-\lambda}(\theta,\phi,\sqrt{s}) = -{\rm e}^{i(\lambda-\mu)(\pi-2\phi)}\mathcal{M}_{\mu\lambda}(\theta,\phi,\sqrt{s}). \tag{8.87}$$

8.2.1. Partial Wave Analysis

The *angular momentum decomposition* of the helicity amplitudes $\mathcal{M}_{\mu\lambda}(\theta,\phi,\sqrt{s})$ is written as [9]

$$\mathcal{M}_{\mu\lambda}(\theta,\phi,\sqrt{s}) = \sum_j \mathcal{M}^j_{\mu\lambda}(\sqrt{s})(2j+1)\,d^j_{\lambda\mu}(\theta)\,{\rm e}^{i(\lambda-\mu)\phi}, \tag{8.88}$$

where the $d^j_{\lambda\mu}(\theta)$ are *Wigner* d-functions given by [1]

$$d^j_{\frac{1}{2}\frac{1}{2}}(\theta) = \frac{1}{l+1}\cos\frac{\theta}{2}\left(P'_{l+1} - P'_l\right),\ d^j_{\frac{1}{2}\frac{3}{2}}(\theta) = \frac{1}{l+1}\sin\frac{\theta}{2}\left(\sqrt{\frac{l}{l+2}}P'_{l+1} + \sqrt{\frac{l+2}{l}}P'_l\right), \tag{8.89}$$

$$d^j_{-\frac{1}{2}\frac{1}{2}}(\theta) = \frac{1}{l+1}\sin\frac{\theta}{2}\left(P'_{l+1} + P'_l\right),\ d^j_{-\frac{1}{2}\frac{3}{2}}(\theta) = \frac{1}{l+1}\sin\frac{\theta}{2}\left(\sqrt{\frac{l}{l+2}}P'_{l+1} + \sqrt{\frac{l+2}{l}}P'_l\right), \tag{8.90}$$

with $j = l + \frac{1}{2}$ and $P'_l \equiv dP_l/d\cos\theta$.
On the other hand, since the functions

$$\sqrt{(2j+1)}\, d^j_{\lambda\mu}(\theta)\, \mathrm{e}^{i(\lambda-\mu)\phi}, \tag{8.91}$$

for different values of j, are mutually *orthogonal* and *normalized* to 4π, when integrated over $d\Omega$, the *helicity coefficients* $\mathcal{M}^j_{\mu\lambda}(\sqrt{s})$ are given by

$$\mathcal{M}^j_{\mu\lambda}(\sqrt{s}) = \frac{1}{4\pi}\int d\Omega\, \mathcal{M}_{\mu\lambda}(\theta,\phi,\sqrt{s})\, d^j_{\lambda\mu}(\theta)\, \mathrm{e}^{-i(\lambda-\mu)\phi}. \tag{8.92}$$

These coefficients depend only on $\sqrt{s}$ and refer to states of *definite* j but *mixed* parity. By separating the ϕ phase factor, the following four standard *helicity amplitudes* are defined [17]

$$H_1(\theta,\sqrt{s}) \equiv \mathrm{e}^{-i\phi}\mathcal{M}_{\frac{1}{2}\frac{3}{2}}(\theta,\phi,\sqrt{s}), \tag{8.93}$$

$$H_2(\theta,\sqrt{s}) \equiv \mathcal{M}_{\frac{1}{2}\frac{1}{2}}(\theta,\phi,\sqrt{s}), \tag{8.94}$$

$$H_3(\theta,\sqrt{s}) \equiv \mathrm{e}^{-2i\phi}\mathcal{M}_{-\frac{1}{2}\frac{3}{2}}(\theta,\phi,\sqrt{s}), \tag{8.95}$$

$$H_4(\theta,\sqrt{s}) \equiv \mathrm{e}^{-i\phi}\mathcal{M}_{-\frac{1}{2}\frac{1}{2}}(\theta,\phi,\sqrt{s}), \tag{8.96}$$

from which we obtain, for example, the four *helicity coefficients*

$$\mathcal{M}^{\frac{3}{2}}_{\frac{1}{2}\frac{3}{2}}(\sqrt{s}) = \frac{1}{2}\int d\cos\theta\, H_1(\theta,\sqrt{s})\, d^{\frac{3}{2}}_{\frac{3}{2}\frac{1}{2}}(\theta), \tag{8.97}$$

$$\mathcal{M}^{\frac{3}{2}}_{\frac{1}{2}\frac{1}{2}}(\sqrt{s}) = \frac{1}{2}\int d\cos\theta\, H_2(\theta,\sqrt{s})\, d^{\frac{3}{2}}_{\frac{1}{2}\frac{1}{2}}(\theta), \tag{8.98}$$

$$\mathcal{M}^{\frac{3}{2}}_{-\frac{1}{2}\frac{3}{2}}(\sqrt{s}) = \frac{1}{2}\int d\cos\theta\, H_3(\theta,\sqrt{s})\, d^{\frac{3}{2}}_{\frac{3}{2},-\frac{1}{2}}(\theta), \tag{8.99}$$

$$\mathcal{M}^{\frac{3}{2}}_{\frac{1}{2}\frac{3}{2}}(\sqrt{s}) = \frac{1}{2}\int d\cos\theta\, H_4(\theta,\sqrt{s})\, d^{\frac{3}{2}}_{\frac{1}{2},-\frac{1}{2}}(\theta), \tag{8.100}$$

which shall be relevant in the calculation of the multipoles.

8.2.2. Helicity Elements

Final states of *definite parity* are formed by the sum and difference of final states having *opposite* helicity, μ and $-\mu$. Thus the sum and difference

$$\mathcal{M}^j_{\frac{1}{2}\lambda} \pm \mathcal{M}^j_{-\frac{1}{2}\lambda} \tag{8.101}$$

of the two final helicity states for given initial helicity do correspond to *definite parity.* These combinations are called *helicity elements* and are defined by [17]

$$A_{l+} \equiv -\frac{1}{\sqrt{2}}\left(\mathcal{M}^j_{\frac{1}{2}\frac{1}{2}} + \mathcal{M}^j_{-\frac{1}{2}\frac{1}{2}}\right), \tag{8.102}$$

$$A_{(l+1)-} \equiv \frac{1}{\sqrt{2}}\left(\mathcal{M}^j_{\frac{1}{2}\frac{1}{2}} - \mathcal{M}^j_{-\frac{1}{2}\frac{1}{2}}\right), \tag{8.103}$$

$$B_{l+} \equiv \sqrt{\frac{2}{l(l+2)}}\left(\mathcal{M}^j_{\frac{1}{2}\frac{3}{2}} + \mathcal{M}^j_{-\frac{1}{2}\frac{3}{2}}\right), \tag{8.104}$$

$$B_{(l+1)-} \equiv -\sqrt{\frac{2}{l(l+2)}}\left(\mathcal{M}^j_{\frac{1}{2}\frac{3}{2}} - \mathcal{M}^j_{-\frac{1}{2}\frac{3}{2}}\right), \tag{8.105}$$

where $l\pm$ refer to the two states with pion *orbital angular momentum* l and *total angular momentum* $j = l \pm \frac{1}{2}$.

8.3. Multipole Amplitudes

The relations between the multipoles and the helicity elements are given by [17]

$$A_{l+} = \frac{1}{2}\left[lM_{l+} + (l+2)E_{l+}\right], \tag{8.106}$$

$$A_{(l+1)-} = \frac{1}{2}\left[(l+2)M_{(l+1)-} - lE_{(l+1)-}\right], \tag{8.107}$$

$$B_{l+} = E_{l+} - M_{l+}, \tag{8.108}$$

$$B_{(l+1)-} = E_{(l+1)-} + M_{(l+1)-}. \tag{8.109}$$

Then the first multipoles are

$$E^I_{1+}(\sqrt{s}) = -\frac{\sqrt{2}}{4}\left[\left(\mathcal{M}^{I\frac{3}{2}}_{\frac{1}{2}\frac{1}{2}} + \mathcal{M}^{I\frac{3}{2}}_{-\frac{1}{2}\frac{1}{2}}\right) - \frac{1}{\sqrt{3}}\left(\mathcal{M}^{I\frac{3}{2}}_{\frac{1}{2}\frac{3}{2}} + \mathcal{M}^{I\frac{3}{2}}_{-\frac{1}{2}\frac{3}{2}}\right)\right], \tag{8.110}$$

$$M^I_{1+}(\sqrt{s}) = -\frac{\sqrt{2}}{4}\left[\left(\mathcal{M}^{I\frac{3}{2}}_{\frac{1}{2}\frac{1}{2}} + \mathcal{M}^{I\frac{3}{2}}_{-\frac{1}{2}\frac{1}{2}}\right) + \sqrt{3}\left(\mathcal{M}^{I\frac{3}{2}}_{\frac{1}{2}\frac{3}{2}} + \mathcal{M}^{I\frac{3}{2}}_{-\frac{1}{2}\frac{3}{2}}\right)\right], \tag{8.111}$$

where I indicates the *isospin* in the final state (Eqs. (8.17) - (8.19)).
For example, with $I = \frac{3}{2}$

$$M^{\frac{3}{2}}_{1+}(\sqrt{s}) = \frac{\sqrt{2}M_N}{64\pi\sqrt{s}}\int_{-1}^{1} d\cos\theta \left[\cos\frac{\theta}{2}(3\cos\theta - 1)H^{\frac{3}{2}}_{2}(\theta,\sqrt{s}) - \sin\frac{\theta}{2}(3\cos\theta + 1)H^{\frac{3}{2}}_{4}(\theta,\sqrt{s})\right]$$
$$- 3\frac{\sqrt{2}M_N}{64\pi\sqrt{s}}\int_{-1}^{1} d\cos\theta \left[\sin\frac{\theta}{2}(\cos\theta + 1)H^{\frac{3}{2}}_{1}(\theta,\sqrt{s}) + \cos\frac{\theta}{2}(\cos\theta - 1)H^{\frac{3}{2}}_{3}(\theta,\sqrt{s})\right], \quad (8.112)$$

and

$$E^{\frac{3}{2}}_{1+}(\sqrt{s}) = \frac{\sqrt{2}M_N}{64\pi\sqrt{s}}\int_{-1}^{1} d\cos\theta \left[\cos\frac{\theta}{2}(3\cos\theta - 1)H^{\frac{3}{2}}_{2}(\theta,\sqrt{s}) - \sin\frac{\theta}{2}(3\cos\theta + 1)H^{\frac{3}{2}}_{4}(\theta,\sqrt{s})\right]$$
$$+ \frac{\sqrt{2}M_N}{64\pi\sqrt{s}}\int_{-1}^{1} d\cos\theta \left[\sin\frac{\theta}{2}(\cos\theta + 1)H^{\frac{3}{2}}_{1}(\theta,\sqrt{s}) + \cos\frac{\theta}{2}(\cos\theta - 1)H^{\frac{3}{2}}_{3}(\theta,\sqrt{s})\right], \quad (8.113)$$

where

$$H^{\frac{3}{2}}_{1}(\theta,\sqrt{s}) \equiv \mathrm{e}^{-i\phi}\mathcal{M}^{\frac{3}{2}}_{\frac{1}{2}\frac{3}{2}}(\theta,\phi,\sqrt{s})$$
$$= \mathrm{e}^{-i\phi}\,\overline{u}(p_{\mathrm{f}},\uparrow)\left[\mathcal{M}^{\frac{3}{2}}(\lambda_\gamma = 1,\theta,\phi,\sqrt{s})\right]u(p_{\mathrm{i}},\downarrow), \quad (8.114)$$

etc.

These multipoles, for example, are of interest because they provide valuable information about the $P_{33}(1232)$ resonance, as it was discussed in Ch. 4.

9. Results and Conclusions

9.1. Results

In this section we present the results obtained for the parameters of the nucleon resonances namely, mass, width, strong coupling constants and magnetic moments, by fitting the total cross-section given by Eq. (9.11), with the tree-level amplitudes obtained in Sec. 7.3 for the reactions, $\gamma\, p \to \pi^+ n$ and $\gamma\, p \to \pi^0 p$.
For the calculation of the cross-section and other observables such as the *electromagnetic multipoles*, which will described with more detail in next chapter, we use pion-nucleon center-of-mass system (c.m.) with the photon direction pointing along the positive z-axis and the pion momentum in the zx plane, that is, with polar angle θ and azimuthal angle $\phi = 0$, as shown in Fig. 2.1. In this system the Dirac spinors $u(p_\mathrm{i})$ and $\overline{u}(p_\mathrm{f})$, used in evaluating the amplitudes become

$$u(p_\mathrm{i}, \uparrow) = \sqrt{\frac{E_\mathrm{i} + M_\mathrm{N}}{2M_\mathrm{N}}} \begin{pmatrix} \chi_\mathrm{i}^\uparrow \\ -\frac{\vec{\sigma}\cdot\vec{k}}{E_\mathrm{i}+M_\mathrm{N}} \chi_\mathrm{i}^\uparrow \end{pmatrix}, \tag{9.1}$$

$$u(p_\mathrm{i}, \downarrow) = \sqrt{\frac{E_\mathrm{i} + M_\mathrm{N}}{2M_\mathrm{N}}} \begin{pmatrix} \chi_\mathrm{i}^\downarrow \\ -\frac{\vec{\sigma}\cdot\vec{k}}{E_\mathrm{i}+M_\mathrm{N}} \chi_\mathrm{i}^\downarrow \end{pmatrix}, \tag{9.2}$$

and

$$\overline{u}(p_\mathrm{f}, \uparrow) = \sqrt{\frac{E_\mathrm{f} + M_\mathrm{N}}{2M_\mathrm{N}}} \begin{pmatrix} \chi_\mathrm{f}^{\uparrow\dagger} & \chi_\mathrm{f}^{\uparrow\dagger} \frac{\vec{\sigma}\cdot\vec{q}}{E_\mathrm{f}+M_\mathrm{N}} \end{pmatrix}, \tag{9.3}$$

$$\overline{u}(p_\mathrm{f}, \downarrow) = \sqrt{\frac{E_\mathrm{f} + M_\mathrm{N}}{2M_\mathrm{N}}} \begin{pmatrix} \chi_\mathrm{f}^{\downarrow\dagger} & \chi_\mathrm{f}^{\downarrow\dagger} \frac{\vec{\sigma}\cdot\vec{q}}{E_\mathrm{f}+M_\mathrm{N}} \end{pmatrix}, \tag{9.4}$$

where the spinors of the initial and final nucleon are, respectively

$$\chi_\mathrm{i}^\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi_\mathrm{i}^\downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{9.5}$$

and

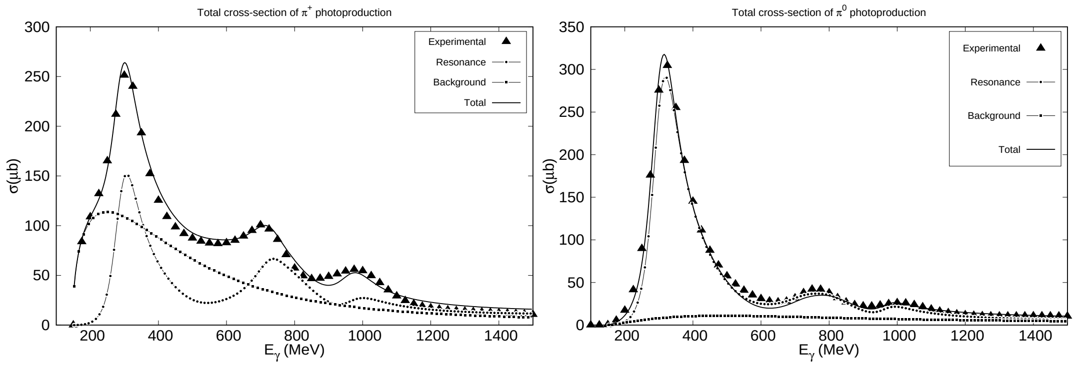
$$\chi_\mathrm{f}^\uparrow = \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} \end{pmatrix}, \quad \chi_\mathrm{f}^\downarrow = \begin{pmatrix} -\sin\frac{\theta}{2} \\ \cos\frac{\theta}{2} \end{pmatrix}, \tag{9.6}$$

with

$$\vec{\sigma}\cdot\hat{k}\,\chi_\mathrm{i}^{\uparrow(\downarrow)} = \pm\chi_\mathrm{i}^{\uparrow(\downarrow)} \tag{9.7}$$

and

$$\vec{\sigma}\cdot\hat{q}\,\chi_\mathrm{f}^{\uparrow(\downarrow)} = \pm\chi_\mathrm{f}^{\uparrow(\downarrow)}, \tag{9.8}$$



(a) (b)

Figure 9.1.: Calculated total cross-sections in μb of pion photoproduction off proton for different photon energies up to $\sim$ 1.7 GeV in the laboratory frame: (a) π^+ and (b) π^0. The experimental data are taken from the Data Analysis Center of the George Washington University <`http://gwdac.phys.gwu.edu`>.

so that *spin up* would correspond, in the c.m. system, to a *negative helicity* and *viceversa.* For *real* photons, the *photon polarization* vector has two independent components which we have taken to be

$$\epsilon^{\mu}_{\lambda} = \frac{1}{\sqrt{2}}(0; -\lambda, -i, 0), \tag{9.9}$$

with $\lambda = \pm 1$.
On the other hand, the averaged differential cross-section for pion photoproduction is given by [32]

$$\frac{d\sigma}{d\Omega^*} = \frac{|\vec{q}|}{2|\vec{k}|} \frac{M_{\rm N}^2}{16\pi^2 s} \frac{1}{2} \sum_{s_{\rm i}} \sum_{s_{\rm f}} \sum_{\lambda} |\overline{u}(p_{\rm f}) \mathcal{M} u(p_{\rm i})|^2, \tag{9.10}$$

from which, integrating over $d\Omega^*$, the total cross-section is calculated according to

$$\sigma(\sqrt{s}) = \int \frac{d\sigma}{d\Omega^*}\, d\Omega^* = 2\pi \int_0^{\pi} \frac{d\sigma}{d\Omega^*}\, \sin\theta^*\, d\theta^*. \tag{9.11}$$

Our results for the total cross sections are shown in Fig. 9.1a and Fig. 9.1b for the two reactions of interest: $\gamma\, p \to \pi^+ n$ and $\gamma\, p \to \pi^0 p$, and for the whole energy region from threshold up to $\sim$ 1.7 GeV.

9.1.1. First Resonance Region

The so called first resonance region consist of the $\Delta(1232)$ resonance only, corresponding to an energy range from 150 MeV (threshold) to $\sim$ 630 MeV in the laboratory frame system. In this case we use the magnetic (G_M) and electric (G_E) form factors given in Eq. (4.108), as input parameters [35], for which the ratio R_{EM} given by Eq. (4.121) is in good agreement with the value given by Ref [1],

$$R_{EM} = -0.025 \pm 0.005. \tag{9.12}$$

The parameters that give the best fit to the experimental data, corresponding to this region, are displayed in table 9.1.

9.1.2. Second Resonance Region

This region consists of the spin-$\frac{1}{2}$ resonances $P_{11}(1440)$ and $S_{11}(1535)$, and the spin-$\frac{3}{2}$ nucleon resonances $D_{13}(1520)$ and $P_{33}(1600)$, corresponding to an energy range from $\sim$ 630 MeV to $\sim$ 930 MeV in the laboratory frame system. The parameters that give the best fit to the experimental data in this region are displayed in table 9.1.
The behaviour of the propagator for the case of spin-$\frac{1}{2}$ resonances at high energies does not requiere the inclusion of a form factor.
According to the analysis performed for the $\Delta(1232)$ resonance electromagnetic vertex, we can estimate the magnetic (G_M) and electric (G_E) form factors of the $\Delta(1600)$ resonance, obtaining

$$G_M = 0.260, \quad \text{and} \quad G_E = 0.030, \tag{9.13}$$

from which we determine the helicity amplitudes $A_{\frac{1}{2}}$ and $A_{\frac{3}{2}}$ for this resonance, obtaining

$$A_{\frac{1}{2}} = -0.012\,\text{GeV}^{-\frac{1}{2}}, \quad \text{and} \quad A_{\frac{3}{2}} = -0.035\,\text{GeV}^{-\frac{1}{2}}. \tag{9.14}$$

We observe that these *estimated* values are in close agreement with the measured experimetal values given in Ref. [1] for two different experiments, namely

$$A_{\frac{1}{2}} = \begin{cases} -0.051 \pm 0.010 \ \text{GeV}^{-\frac{1}{2}} \\ -0.018 \pm 0.015 \ \text{GeV}^{-\frac{1}{2}} \end{cases}, \quad A_{\frac{3}{2}} = \begin{cases} -0.055 \pm 0.010 \ \text{GeV}^{-\frac{1}{2}} \\ -0.025 \pm 0.015 \ \text{GeV}^{-\frac{1}{2}} \end{cases}. \tag{9.15}$$

In the model proposed in Ref. [49], for example, they obtain the values

$$G_M = 0.202 \pm 0.148, \quad \text{and} \quad G_E = 0.000, \tag{9.16}$$

for the magnetic and the electric form factors, respectively and

$$A_{\frac{1}{2}} = -0.0154 \pm 0.0113\,\text{GeV}^{-\frac{1}{2}}, \quad \text{and} \quad A_{\frac{3}{2}} = -0.0266 \pm 0.0196\,\text{GeV}^{-\frac{1}{2}}. \tag{9.17}$$

for the helicity amplitudes $A_{\frac{1}{2}}$ and $A_{\frac{3}{2}}$.
Finally, by means of Eq. (4.120), we estimate the ratio of electric quadrupole to magnetic dipole transition amplitudes R_{EM} for this resonance as

$$R_{EM} = -\frac{G_E}{G_M} = -0.115, \tag{9.18}$$

which is not yet reported in Ref. [1].

Spin-$\frac{1}{2}$ Resonances	$f_{\pi NR}$	M_R (GeV)	Γ_R (GeV)	κ_R^p	κ_R^n	Λ (GeV)
$P_{11}(1440)$	0.373	1.380	0.180	−0.601	0.400	–
$S_{11}(1535)$	−0.153	1.510	0.110	0.920	−0.690	–
$S_{11}(1650)$	−0.96	1.640	0.100	0.47	−0.430	–
$P_{11}(1710)$	0.055	1.680	0.090	−0.335	0.335	–
Spin-$\frac{3}{2}$ Resonances	$f_{\pi NR}$	M_R (GeV)	Γ_R (GeV)	G_M	G_E	Λ (GeV)
$P_{33}(1232)$	2.202	1.213	0.108	2.970	0.055	0.70
$D_{13}(1520)$	−1.509	1.505	0.105	−3.298	−0.192	0.50
$P_{33}(1600)$	−0.671	1.510	0.200	−0.260	−0.030	0.50

Table 9.1.: Best fit parameters for the first, second and third resonance regions.

9.1.3. Third Resonance Region

This region consists of the spin-$\frac{1}{2}$ resonances $S_{11}(1650)$ and $P_{11}(1710)$, corresponding to an energy range from ~ 930 MeV to ~ 1100 MeV in the laboratory frame system. From this value, there are no other resonance regions evident in the total cross-section as seen in Fig. 9.1a and Fig. 9.1b. The parameters that give the best fit to the experimental data in this region are displayed in table 9.1.

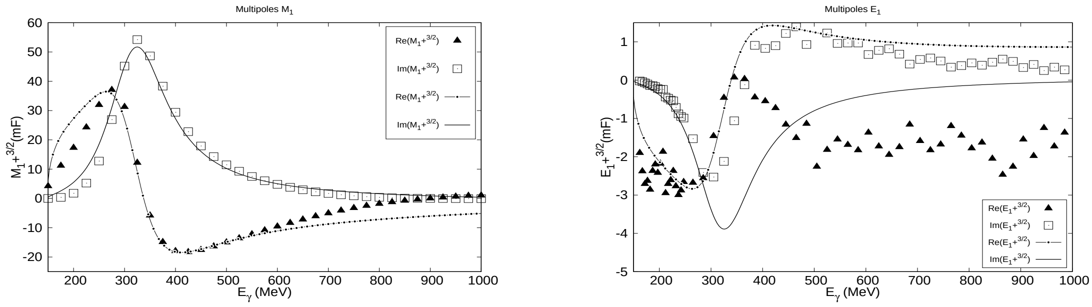
9.1.4. Electromagnetic Multipoles

In Fig. 9.2a and Fig. 9.2b we plot the *real* and *imaginary* parts of the multipoles $M_{1+}^{\frac{3}{2}}$, and $E_{1+}^{\frac{3}{2}}$, given by Eq. (8.112) and Eq. (8.113), respectively, by using the estimated parameters given in Table 9.1.

9.2. Conclusions

1. We have elaborated a model for photoproduction of pions (π^+ and π^0) on proton which is based on an Effective Lagrangian Approach (ELA) fulfilling chiral symmetry, gauge invariance, and crossing symmetry. The model includes the Born terms: nucleon, pion in flight, and Kroll-Rutherman, the vector meson exchanges: ρ and ω and, the nucleon resonances: $P_{33}(1232)$, $P_{11}(1440)$, $D_{13}(1520)$, $S_{11}(1535)$, $P_{33}(1600)$, $S_{11}(1650)$, and $P_{11}(1710)$.

2. The analysis of the spin-$\frac{3}{2}$ nucleon resonance electromagnetic vertex as well as the spin-$\frac{3}{2}$ field propagator are one of the main features considered in this work, which are treated consistently under the point transformation of the Ψ^μ field. We have expressed the electromagnetic vertex in terms of the covariant multipole decomposition, in analogy with the Dirac-Pauli decomposition of the nucleon form factor and then we have established a relation with the well-known normal parity



(a) (b)

Figure 9.2.: Calculated multipoles in mF of pion photoproduction off proton for different photon energies in the laboratory frame: (a) $M_{1+}^{3/2}$, (b) $E_{1+}^{3/2}$. The experimental data are taken from the Data Analysis Center of the George Washington University <`http://gwdac.phys.gwu.edu`>.

decomposition of the vertex, in the limit case of the spin-$\frac{3}{2}$ nucleon resonance on shell.

3. We have made use of the prescription that includes an absorptive one-loop self-energy correction to the spin-$\frac{3}{2}$ field propagator to reproduce the complex-mass prescription for its resonant form.

4. We have introduced form factors preserving the gauge invariance of the model, which give account of the structure effects of the composite particles and also permit to extend the energy range to include both, the second and the third resonance regions.

5. We have established a reliable set of parameters for the model in accordance with experimental data [1], in which the coupling constants, the magnetic moments, masses and widths of the nucleon resonances have been adjusted within suitable ranges by fitting to the experimental total cross-sections of the processes $\gamma\, p \to \pi^+\, n$ and $\gamma\, p \to \pi^0\, p$.

6. By means of the established set of parameters we have tried to reproduce the electromagnetic multipoles $M_{1+}^{3/2}$, and $E_{1+}^{3/2}$, obtaining a qualitatively good agreement in the case of the $M_{1+}^{3/2}$ multipole. However, for the multipole $E_{1+}^{3/2}$, we obtain a partial agreement only at low energy.

7. We have estimated the magnetic (G_M) and electric (G_E) form factors of the $\Delta(1600)$ resonance by means of the proposed model. The value of the helicity amplitudes obtained from these form factors are in close agreement with the measured experimetal values given in Ref. [1].

8. The analysis we have made with spin-$\frac{3}{2}$ resonances may be extended to consider, in the future, resonances of higher spin such as $N(1675)$ and $N(1680)$, both with spin-$\frac{5}{2}$.

Appendices

A. Pion Field Quantization

A.1. Second Quantized Pion Field

The general normalized solution of the free-field Klein-Gordon equation is

$$\pi^{\pm}(x) = \int \frac{d^3\vec{q}}{(2\pi)^3 2\omega_{\vec{q}}} \left(a_{\mp}(\vec{q}) \mathrm{e}^{-iq\cdot x} + a^{\dagger}_{\pm}(\vec{q}) \mathrm{e}^{iq\cdot x} \right), \tag{A.1}$$

where $a_{\mp}(\vec{q})$ and $a^{\dagger}_{\pm}(\vec{q})$ are the *annihilation* and *creation* operators for a pion with charge $\mp$ and charge $\pm$, respectively, and $\omega_{\vec{q}} \equiv \sqrt{|\vec{q}|^2 + m_\pi^2}$.
For the neutral pion field,

$$\pi^{0}(x) = \int \frac{d^3\vec{q}}{(2\pi)^3 2\omega_{\vec{q}}} \left(a_{0}(\vec{q}) \mathrm{e}^{-iq\cdot x} + a^{\dagger}_{0}(\vec{q}) \mathrm{e}^{iq\cdot x} \right), \tag{A.2}$$

On the other hand, the *contractions* of the field operator $\pi^{\alpha}(x)$ $(\alpha = \pm, 0)$ with *external* states are given by

$$\pi^{\alpha}\overbracket{(x)|\vec{q}}\rangle = \mathrm{e}^{-iq\cdot x} \quad \text{and} \quad \langle\overbracket{\vec{q}|\pi^{\alpha}}(x) = \mathrm{e}^{iq\cdot x}, \tag{A.3}$$

from which, for example, $\langle\overbracket{\vec{q}|\partial_\mu\pi^{\alpha}}(x) = iq_\mu\, \mathrm{e}^{iq\cdot x}$.

A.2. Pion Field Propagator

The propagator is given by the *time-ordered product* ($\mathscr{T}$) of the field operators [27]

$$\langle 0|\mathscr{T}\pi^{\alpha}(x)\pi^{\alpha\dagger}(y)|0\rangle = \int \frac{d^4q}{(2\pi)^4} D_F(p) e^{-iq\cdot(x-y)}, \tag{A.4}$$

where

$$D_F(p) \equiv \frac{i}{q^2 - m_\pi^2 + i\epsilon}, \tag{A.5}$$

is the Feynman propagator in momentum space representation.
Then, by taking into account that the πNN coupling is chosen to be PV, the propagator that appears actually in the amplitudes is given by

$$\langle 0|\mathscr{T}\partial_\mu\pi^{\alpha}(x)\pi^{\alpha\dagger}(y)|0\rangle = \int \frac{d^4q}{(2\pi)^4} D_F(p)(-iq_\mu) e^{-iq\cdot(x-y)}, \tag{A.6}$$

etc.

B. Photon Field Quantization

B.1. Second Quantized Photon Field

For the photon field $A^\mu(x)$,

$$A^\mu(x) = \sum_\lambda \int \frac{d^3\vec{k}}{(2\pi)^3 2|\vec{k}|} \left(a_\lambda(\vec{k}) \epsilon^\mu_\lambda \, \mathrm{e}^{-ik\cdot x} + a^\dagger_\lambda(\vec{k}) \epsilon^{\mu *}_\lambda \, \mathrm{e}^{ik\cdot x} \right), \tag{B.1}$$

where ϵ^μ_λ is the *polarization* vector, which we take as

$$\epsilon^\mu_\lambda = \frac{1}{\sqrt{2}}(0; -\lambda, -i, 0), \tag{B.2}$$

with $\lambda = \pm 1$.
Similar to the pion field, the *contractions* of the field operator $A^\mu(x)$ with *external* states are given by

$$\overbrace{A^\mu(x)|\vec{k}}, \lambda\,\rangle = \epsilon^\mu_\lambda \, \mathrm{e}^{-ik\cdot x} \quad \text{and} \quad \partial_\rho \overbrace{A^\mu(x)|\vec{k}}, \lambda\,\rangle = -ik_\rho \epsilon^\mu_\lambda \, \mathrm{e}^{-ik\cdot x}. \tag{B.3}$$

B.2. Vector Meson Field Propagator

The *massive* vector field is much like the photon field, and the propagator is given by the *time-ordered product* of the field operators [30]

$$\langle 0 | \mathscr{T} V^\mu(x) V^\nu(y) | 0 \rangle = \int \frac{d^4k}{(2\pi)^4} \Delta^{\mu\nu}_F(k) e^{-ik\cdot(x-y)}, \tag{B.4}$$

where

$$\Delta^{\mu\nu}_F(k) \equiv -\frac{i(g^{\mu\nu} - k^\mu k^\nu / m_V^2)}{k^2 - m_V^2 + i\epsilon}, \tag{B.5}$$

is the Feynman propagator in momentum space representation.

C. Spin-$\frac{1}{2}$ Field Quantization

C.1. Second Quantized Dirac Field

For the spin-$\frac{1}{2}$ nucleon and resonant fields

$$\psi(x) = \sum_s \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{M_X}{E_p} \left(b_s(\vec{p}) u_s(p) \mathrm{e}^{-ip\cdot x} + d_s^\dagger(\vec{p}) v_s(p) \mathrm{e}^{ip\cdot x} \right), \tag{C.1}$$

where s is the spin *projection*, the operators $b_s(\vec{p})$ and $d_s^\dagger(\vec{p})$ *annihilate* and *create* a Dirac particle of given spin, respectively, and $E_p \equiv \sqrt{|\vec{p}|^2 + M_X^2}$.
The *contractions* of the field operator $\psi(x)$ with *external* states are given by

$$\psi(x)|\vec{p}, s\,\rangle = u_s(p)\mathrm{e}^{-ip\cdot x} \quad \text{and} \quad \langle \vec{q}, s\,|\bar{\psi}(x) = \bar{u}_s(p)\mathrm{e}^{ip\cdot x}, \tag{C.2}$$

where the four-component *spinor* $u_s(p)$ is given by

$$u_s(p) = \sqrt{\frac{E_p + M_X}{2M_X}} \begin{pmatrix} \chi_s \\ \frac{\vec{\sigma}\cdot\vec{p}}{E_p+M_X}\chi_s \end{pmatrix}. \tag{C.3}$$

C.2. Dirac Field Propagator

The propagator for the spin-$\frac{1}{2}$ is given by the Dirac propagator

$$\langle 0|\mathscr{T}\psi(x)\bar{\psi}(y)|0\rangle = d(\partial) \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - M_X^2 + i\epsilon} e^{-ip\cdot(x-y)}, \tag{C.4}$$

where the operator $d(\partial)$ is given by

$$d(\partial) \equiv i\not{\partial} + M_X. \tag{C.5}$$

In *momentum space*, the Feynman propagator becomes

$$S_F(p) = \frac{i(\not{p} + M_X)}{p^2 - M_X^2 + i\epsilon}. \tag{C.6}$$

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