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Non-fundamental effective apparent power defined through an instantaneous power approach

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ABSTRACT

This paper proposes a new definition of non-fundamental effective apparent power based on an analysis of instantaneous power flows. This new instantaneous power approach for the calculation of the non-fundamental effective apparent power extends, and adapts for new electrical conditions, the procedure applied by IEEE Std. 1459 for the quantification of active and reactive power in single-phase systems.

This proposed approach is based on the analysis of per-phase and three-phase instantaneous power flows when a three-phase four-wire balanced non-linear load is connected to an ideal power network that supplies a set of positive-sequence fundamental voltages. The per-phase and three-phase instantaneous power flows caused by positive-, negative-, and zero-sequence harmonic load currents are analyzed. The results obtained for the load zero-sequence harmonic currents disagree with the results obtained when applying IEEE Std. 1459. As a consequence, a new definition of the effective quantities is proposed. A comparison between the new definitions and IEEE Std. 1459 definitions is made in the paper.

1. Introduction

The presence of non-linear loads connected to the electrical power network is increasingly common [1–8]. Industrial applications based on three-phase static converters and adjustable speed drivers are examples of three-phase balanced non-linear loads that produce distortion in the current waves. The flow of load harmonic currents in the power network, together with reactive and unbalanced current components, produces the following effects [3–8]: voltage distortion and flicker; low system efficiency; low power factor; excessive neutral current; increase in power losses; disturbances to other consumers; malfunction of electronic equipment; etc.

IEEE Std. 1459 [9] includes definitions for the measurement of electric power quantities under sinusoidal, non-sinusoidal, balanced, or unbalanced conditions. It is stated in the introduction that 'the new definitions were developed to give guidance with respect to the quantities that should be measured or monitored for revenue purposes, engineering economic decisions, and determination of major harmonic polluters'. Some works [10–13] discussing instruments for the measurement of electrical power quantities defined in IEEE Std. 1459 have appeared in recent years. In Ref. [14], the power quantities defined in IEEE Std. 1459 are reformulated in the time–frequency domain by using the discrete wavelet transform. IEEE Std. 1459 power magnitudes are also used for the detection of major sources of waveform distortion [15]; or for the definition of the reference currents of shunt-active power compensators [16,17].

Some of the definitions proposed in IEEE Std. 1459 are still under discussion. In Ref. [18], the author disagrees with the definition of non-fundamental power. In Ref. [19], the authors propose a new definition of the unbalance power using an instantaneous approach based on the well-established concepts used in IEEE Std. 1459 for the definition of active and reactive power in single-phase systems. This instantaneous approach is extended in this paper to electrical systems with balanced harmonic currents, and the result is a new definition of non-fundamental effective apparent power and, consequently, a new definition of effective apparent power.

There are several works in the literature in which instantaneous power flows are used in the analysis of electrical systems [20–24]. In Ref. [20], the instantaneous power of a three-phase four-wire distribution system with unbalanced, distorted source voltage, and an unbalanced load was analysed. Supply voltages and load currents are decomposed into several terms by using the symmetrical components and the complex Fourier series. The results of the performed analysis enable the definition of the reference currents

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of a shunt-active power filter that supplies to the load all the power terms that differ from the fundamental positive-sequence active power (P_1^+). No relation between instantaneous power flows and power magnitudes is carried out in Ref. [20]. In Ref. [21,22], instantaneous power flows are analyzed for three-phase unbalanced systems with sinusoidal voltages and currents. The sum of the amplitudes of the instantaneous power oscillations in each phase quantifies the various reactive powers identified in the circuit. In Ref. [23], the physical meaning of the reactive power in non-sinusoidal situations is analyzed and the instantaneous powers for various single-phase cases are detailed. Three-phase instantaneous power flows are also analyzed in Ref. [24]. In all cases, there is a relation between the amplitude of the oscillations and the reactive power magnitudes that quantify the power flow. Works presented in Refs. [22–24] preceded IEEE Std. 1459, but the analysis of the instantaneous power flows presented in Refs. [22–24] is not applied to the definition of the power magnitudes included in IEEE Std. 1459. This paper proposes a modification of the approach used in Refs. [19,22–24] for the definition of a new expression of the non-fundamental effective apparent power.

This paper starts with a summary of IEEE Std. 1459 power definitions used in later sections. In Section 3 the harmonic components of a balanced non-linear load are classified into positive-, negative-, and zero-sequence harmonics. The instantaneous power approach is then applied to each type of harmonic component, and a comparison with results obtained by means of IEEE Std. 1459 power definitions is made. Differences are found when the load demands zero-sequence harmonic currents. Section 4 presents the expressions of the non-fundamental effective apparent power and the effective apparent power obtained by means of the instantaneous power approach. Section 5 presents a comparison of the main electrical quantities of a three-phase balanced circuit using the approaches considered in the paper. The final section presents a summary.

2. Power quantities in IEEE Std. 1459

For three-phase four-wire electrical systems the effective voltage (V_e) is defined in (1) as a function of the rms voltages at the point of measurement: voltages from line to neutral ($V_a - V_b - V_c$) and voltages from line to line ($V_{ab} - V_{bc} - V_{ca}$). V_e is resolved into a fundamental effective voltage (V_{e1}) and a non-fundamental effective voltage (V_{eH}) as follows:

$$V_e = \sqrt{\frac{3(V_a^2 + V_b^2 + V_c^2) + (V_{ab}^2 + V_{bc}^2 + V_{ca}^2)}{18}} = \sqrt{V_{e1}^2 + V_{eH}^2}. \quad (1)$$

The subscript "1" represents the fundamental component of the voltages or currents, and subscript H identifies the harmonic components of the voltages or currents.

The effective current (I_e) is defined in (2) as a function of the phase ($I_a - I_b - I_c$) and neutral (I_n) rms load currents. I_e is resolved into a fundamental effective current (I_{e1}), and a non-fundamental effective current (I_{eH}) as follows:

$$I_e = \sqrt{\frac{I_a^2 + I_b^2 + I_c^2 + I_n^2}{3}} = \sqrt{I_{e1}^2 + I_{eH}^2}. \quad (2)$$

An explanation of why V_e and I_e are defined in this way appears in Refs. [8,25]. The effective apparent power (S_e) is defined as follows:

$$S_e = 3V_eI_e. \quad (3)$$

Replacing in (3) the effective voltage and current by their respective fundamental and non-fundamental effective terms, the fundamental effective apparent power (S_{e1}) and the non-fundamental effective apparent power (S_{eN}) can be calculated as follows:

$$S_{e1} = 3V_{e1}I_{e1}, \quad (4)$$

$$S_{eN}^2 = S_e^2 - S_{e1}^2 = (3V_{e1}I_{eH})^2 + (3V_{eH}I_{e1})^2 + (3V_{eH}I_{eH})^2. \quad (5)$$

S_{e1} and S_{eN} are resolved in IEEE Std. 1459 into other power magnitudes. A summary of IEEE Std. 1459 appears in Ref. [25].

As indicated in Ref. [8], S_{eN} evaluates the pollution due to harmonics and can be used to determine the size of the selective shunt-active power compensators when used for non-fundamental compensation. It is suggested in Ref. [3] that S_{eN} may become important if utilities levy some form of harmonic pollution surcharge based on this power magnitude for customers with an excessive harmonic current demand. These arguments reinforce the importance of defining S_{eN} correctly.

3. Instantaneous power approach to harmonic power quantification

The instantaneous power approach was at first applied in Ref. [19] for the calculation of unbalance power (S_{U1}) as defined in IEEE Std. 1459. The study performed in Ref. [19] for an unbalanced linear load that demands fundamental positive-, negative-, and zero-sequence current components results in an expression of the unbalance power $S_{U1\#}$ that differs from the expression presented in IEEE Std. 1459. This modification yields a new value of S_{e1} in Ref. [19]. The methodology proposed in Ref. [19] is now extended for analyzing the quantification of S_{eN} using the instantaneous power approach.

The line-to-neutral voltages (v_{zh}) and the line currents (i_{zh}) described in Section 3.2.3 in IEEE Std. 1459 for a three-phase non-sinusoidal balanced system can be written as follows:

$$v_{zh} = \sqrt{2}\sum_{h=1}^{\infty} V_{zh}\sin(h(\omega_1 t - \varphi_z) + \alpha_{zh}), \quad (6)$$

$$i_{zh} = \sqrt{2}\sum_{h=1}^{\infty} I_{zh}\sin(h(\omega_1 t - \varphi_z) + \beta_{zh}), \quad (7)$$

where z denotes the three phases of the electrical system ($z = a, b, c$), h is the harmonic order, V_{zh} is the harmonic line-to-neutral rms voltage, φ_z is the phase-angle between line-to-neutral voltages ($\varphi_a = 0$, $\varphi_b = 2\pi/3$, $\varphi_c = 4\pi/3$), α_{zh} is the phase-angle of the hth harmonic voltage, ω_1 is the fundamental angular frequency, I_{zh} is the harmonic phase rms current, and β_{zh} is the phase-angle of the hth harmonic phase current. The fundamental positive-sequence voltage in phase a is selected as the phase origin.

Instantaneous power flows in three-phase four-wire circuits are analyzed for a three-phase balanced non-linear load connected through an ideal line to an ideal power network that only supplies fundamental positive-sequence voltages (V_1^+). The system under analysis is represented in Fig. 1, where the point of common

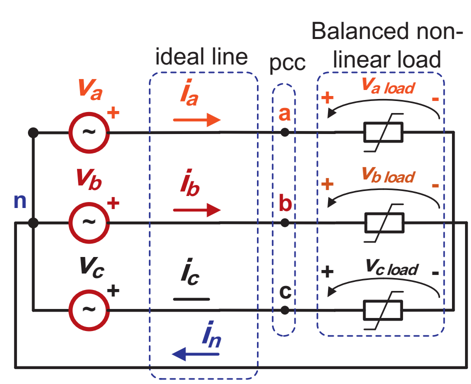

Fig. 1. Electrical system under analysis.

coupling of the load with the power network is designated as PCC, $V_{a1} = V_{b1} = V_{c1} = V_1^+$ and V_1^+ is the origin of angles ($\alpha_{z1} = 0$). Under these simplified conditions, the paper demonstrates that the expression of the non-fundamental effective apparent power obtained by means of the instantaneous power approach differs from the definition included in IEEE Std. 1459. Taking into account the previous assumptions, the line-to-neutral load voltages are expressed as follows:

$$v_{z1} = v_{z1}^+ = \sqrt{2}V_1^+ \sin(\omega_1 t - \varphi_z). \tag{8}$$

As indicated in Ref. [26], the relationship between the harmonic order of the three-phase balanced non-linear load current components and the rotation sequence, for $n = 0, 1, 2, \ldots, \infty$, is as follows:

- The three-phase phasors of harmonic orders $h = 3n + 1$ rotate in the positive-sequence $a - b - c$ $(0, -2\pi/3, -4\pi/3)$ with an angular frequency equal to $(3n + 1)\omega_1$.
- The three-phase phasors of harmonic orders $h = 3n + 2$ rotate in the negative-sequence $a - c - b$ $(0, -4\pi/3, -2\pi/3)$ with an angular frequency equal to $(3n + 2)\omega_1$.
- The three-phase phasors of harmonic orders $h = 3n + 3$ rotate in the zero-sequence with an angular frequency equal to $(3n + 3)\omega_1$. The three phasors are in phase and without rotation.

Fig. 2 shows the phasor diagrams for positive- (Fig. 2a), negative- (Fig. 2b), and zero-sequence (Fig. 2c) rotation harmonic load currents with respect to the origin of angles (V_1^+). The different variables, defined previously in (7) for the hth load harmonic current component, are represented by their rotation sequences. No direct current (DC) components are considered in the analysis because IEEE Std. 1459 states that significant DC components are rarely present in AC power systems. Furthermore, not even harmonics are common in power networks [27].

Three-phase systems should be considered as one single path through which electrical energy is transmitted to locations where it is converted into other forms of energy [9]. Assuming only fundamental positive-sequence voltage exists at the PCC, the instantaneous power delivered by the three-phase power network to the load (p) is calculated as the sum of the three per-phase instantaneous powers (p_z) as follows:

$$p = \sum_{\substack{z=a,b,c \\ h=1}}^{\infty} p_{zh} = v_{a1}^+ i_{ah} + v_{b1}^+ i_{bh} + v_{c1}^+ i_{ch}. \tag{9}$$

The instantaneous power per phase can be written as follows:

$$p_{zh} = 2V_1^+ \sin(\omega_1 t - \varphi_z) \sum_{h=1}^{\infty} I_{zh} \sin(h(\omega_1 t - \varphi_z) + \beta_{zh}). \tag{10}$$

If the three-phase load is linear and balanced, then all the line currents are equal to the fundamental positive-sequence current ($I_{z1} = I_{a1} = I_{b1} = I_{c1} = I_1^+$) and the phase shifts in the three phases are equal ($\beta_{a1} = \beta_{b1} = \beta_{c1} = \beta_1^+$). The instantaneous power flows per phase exist due to the product of V_1^+ and I_1^+ in each phase. From (10), with $h = 1$, the expression of the fundamental instantaneous power per phase can be written as follows:

$$p_{z1} = V_1^+ I_1^+ \cos(\beta_1^+)(1 - \cos(2\omega_1 t - 2\varphi_z)) - V_1^+ I_1^+ \sin(\beta_1^+) \times \sin(2\omega_1 t - 2\varphi_z). \tag{11}$$

Two terms are clearly distinguished in (11):

- The instantaneous per-phase fundamental positive-sequence active power (p_{zla}) as defined in (12), is formed by a time-invariant term and a sinusoidal oscillation. It is always positive and has a positive mean value. The flow of energy per phase is unidirectional from the supply to the load - and corresponds to the energy delivered by the generator to the load. The angular frequency of the sinusoidal oscillation is twice the fundamental angular frequency:

$$p_{z1a} = V_1^+ I_1^+ \cos(\beta_1^+)(1 - \cos(2\omega_1 t - 2\varphi_z)). \tag{12}$$

- The instantaneous per-phase fundamental positive-sequence reactive power (p_{z1q}), as defined in (13), appears only when the load includes some reactive parts. The current phase shift introduced by the reactive load is the cause of the sinusoidal oscillation that has a mean value equal to zero. It represents a bidirectional flow of energy between the supply and load, with an angular frequency of twice the fundamental angular frequency:

$$p_{z1q} = -V_1^+ I_1^+ \sin(\beta_1^+) \sin(2\omega_1 t - 2\varphi_z). \tag{13}$$

The sum of the three p_{z1a} is the instantaneous three-phase fundamental positive-sequence active power (p_{1a}^+). It is demonstrated in Ref. [19] that the sum results in a time-invariant value equal to the positive-sequence active power (P_1^+):

$$\sum_{z=a,b,c} p_{z1a} = p_{1a}^+ = \sum_{z=a,b,c} V_1^+ I_1^+ \cos(\beta_1^+)(1 - \cos(2\omega_1 t - 2\varphi_z)) = 3V_1^+ I_1^+ \cos\beta_1^+ = P_1^+. \tag{14}$$

The sum of the three p_{z1q} is the instantaneous three-phase fundamental positive-sequence reactive power (p_{1q}^+). The sum results in a nil power flow at any time instant:

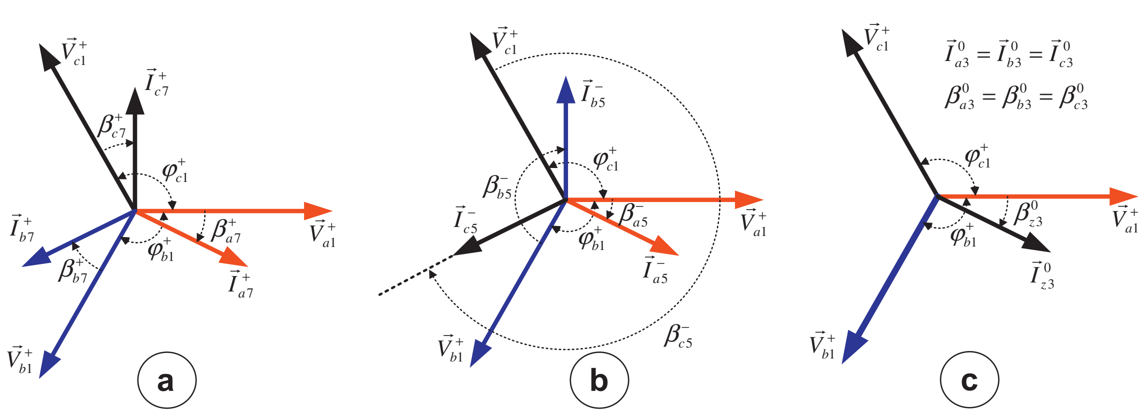

Fig. 2. Phasor diagrams with representation of the different phase shifts for the balanced non-linear studied cases: (a) positive-sequence rotation harmonics; (b) negative-sequence rotation harmonics; and (c) zero-sequence rotation harmonics.

$$\sum_{z=a,b,c} p_{z1q} = p_{1q}^{+} = \sum_{z=a,b,c} -V_1^{+} I_1^{+} \sin(\beta_1^{+}) \sin(2\omega_1 t - 2\varphi_z) = 0. \quad (15)$$

Because any mean value or sinusoidal oscillation characterizes p_{1q}^{+}, the power flow is quantified by the fundamental positive-sequence reactive power (Q_1^{+}), with Q_1^{+} defined as equal to the sum of the three amplitudes of p_{z1q} [19]:

$$Q_1^{+} = 3V_1^{+} I_1^{+} \sin\beta_1^{+} = \max|p_{a1q}| + \max|p_{b1q}| + \max|p_{c1q}|. \quad (16)$$

This definition applies the methodology used in IEEE Std. 1459 for the definition of reactive power in single-phase systems with linear loads ([9], p. 3, Sections 3.1.1.1–3.1.1.3); and is also used in Ref. [6] for the calculation of Q_1^{+}. The same instantaneous power approach is applied in Ref. [19] for the quantification of S_{U1}, which includes the power oscillations produced by fundamental negative- and zero-sequence load currents. The analysis of the three-phase power oscillations performed in Ref. [19] yields a new expression of S_{U1}, denoted as $S_{U1\#}$, and which differs from the expression proposed in IEEE Std. 1459.

The same method is applied now to a three-phase balanced non-linear load connected to an ideal power network that supplies V_1^{+} at the PCC. The expression of the instantaneous power flow for phase b for a three-phase four-wire electrical system connected to a balanced non-linear load appears in Ref. [24] (Eq. (34)). From (10), and for $h \geqslant 1$, the general equation of the per-phase instantaneous power is as follows:

$$p_{zh} = V_1^{+} \sum_{h=1}^{\infty} I_{zh} \{\cos[(h-1)(\omega_1 t - \varphi_z) + \beta_{zh}] - \cos[(h+1) \times (\omega_1 t - \varphi_z) + \beta_{zh}]\}. \quad (17)$$

The per-phase instantaneous power of the hth load harmonic current component includes two parts:

- The first part is a time-invariant term that is constant for a defined load, corresponding to the product of V_1^{+} and the rms value of the hth load harmonic current component (I_{zh}).
- The second part is a time-variant term that includes the two sinusoidal terms between brackets, with angular frequencies $(h-1)\omega_1$ and $(h+1)\omega_1$.

The interaction of these sinusoidal terms produces different waveforms depending on the harmonic sequence. In the following subsections, the per-phase and three-phase instantaneous power flows are analyzed for positive-, negative-, and zero-sequence rotation harmonic currents. The analysis will conclude with new definitions of S_{eN} and S_e that follow and expand the methodology used in IEEE Std. 1459 for the definition of the reactive power in single-phase systems with linear loads.

3.1. Power flows due to positive-sequence rotation harmonic current components

The positive-sequence rotation harmonic currents demanded by the balanced non-linear load correspond to $h = 3n + 1$ and $n > 0$. These harmonic currents have the same phase-sequence as V_1^{+} ($a - b - c$). The superscript "+" is used to identify the currents and powers in this case. The most important positive-sequence harmonic currents present in the power network are the 7th ($n = 2$), and the 13th ($n = 4$). Substituting $h = 3n + 1$ in (17), the per-phase instantaneous power due to positive-sequence harmonic currents (p_{zh}^{+}) is calculated as follows:

$$p_{zh}^{+} = V_1^{+} \sum_{\substack{n=1 \\ h=3n+1}}^{\infty} I_{zh}^{+} \{\cos[(3n)(\omega_1 t - \varphi_z) + \beta_{zh}^{+}] - \cos[(3n+2) \times (\omega_1 t - \varphi_z) + \beta_{zh}^{+}]\}. \quad (18)$$

The two sinusoidal terms between brackets have $(3n)\omega_1$ and $(3n+2)\omega_1$ angular frequencies, respectively, with a nil average value in both cases. Therefore, the power flows produced by load positive-sequence harmonic currents are considered as non-efficient power flows. The three-phase instantaneous power caused by positive-sequence harmonic current components (p_{h-3P}^{+}) corresponds to the sum of the three instantaneous powers calculated using (18):

$$p_{h-3P}^{+} = \sum_{z=a,b,c} p_{zh}^{+} = \sum_{z=a,b,c} \left[V_1^{+} \sum_{\substack{n=1 \\ h=3n+1}}^{\infty} I_{zh}^{+} \{\cos[(3n)(\omega_1 t - \varphi_z) + \beta_{zh}^{+}] - \cos[(3n+2)(\omega_1 t - \varphi_z) + \beta_{zh}^{+}]\} \right].$$

Expanding the terms between brackets, the three-phase instantaneous power of the hth load positive-sequence harmonic current component (p_h^{+}) is written as follows:

$$\begin{aligned} p_h^{+} = V_1^{+} I_h^{+} [& \{\cos[3n\omega_1 t + \beta_{ah}^{+}] + \cos[3n\omega_1 t + \beta_{bh}^{+}] + \cos[3n\omega_1 t + \beta_{ch}^{+}]\} \\ & - \{\cos[(3n+2)\omega_1 t + \beta_{ah}^{+}] + \cos[(3n+2)\omega_1 t + \beta_{ah}^{+}] \\ & + \cos[(3n+2)\omega_1 t + 2\pi/3 + \beta_{bh}^{+}] + \cos[(3n+2)\omega_1 t \\ & - 2\pi/3 + \beta_{ch}^{+}]\}]. \end{aligned} \quad (20)$$

The terms with $(3n)\omega_1$ angular frequencies are zero-sequence components because the phase shifts of the balanced non-linear load are equal ($\beta_{ah}^{+} = \beta_{bh}^{+} = \beta_{ch}^{+}$). The terms with $(3n + 2)\omega_1$ angular frequencies are negative-sequence components ($a - c - b$). Negative-sequence components are described in Ref. [28] as zero-sum components because the sum of the three sinusoidal terms is equal to zero at any time. This yields the following expression of the three-phase instantaneous power of the hth load positive-sequence harmonic current component:

$$p_h^{+} = 3V_1^{+} I_h^{+} \cos[3n\omega_1 t + \beta_{ah}^{+}]. \quad (21)$$

A simulation of the circuit described in Fig. 1 is performed using Matlab/Simulink for a balanced load that demands only the 7th harmonic current component. The base voltage is V_1^{+}, ($V_{\text{p.u.}} = V_z = V_1^{+}$), the base current is I_7^{+} ($I_{\text{p.u.}} = I_z = I_7^{+}$), and β_{z7}^{+} is established equal to zero. The per unit (p.u.) base apparent power in the simulation is defined as the product of the base voltage multiplied by the base current:

$$S_{\text{p.u.}} = V_1^{+} I_7^{+}. \quad (22)$$

From (18), the per-phase instantaneous power for the studied case is as follows:

$$p_{z7}^{+} = V_1^{+} I_{z7}^{+} \{\cos[6(\omega_1 t - \varphi_z)] - \cos[8(\omega_1 t - \varphi_z)]\}. \quad (23)$$

Fig. 3 shows the two sinusoidal terms of (23) and the sum of both the terms (p_{z7}^{+}) in the p.u. system for phase a (origin of angles):

The waveform represented in the top plot in Fig. 3 is the same as in the b and c phases because the three power flows are zero-sequence components. The waveform represented in the middle is the same in phases b and c but with a negative-sequence ($a - c - b$). The sum of the two terms is represented in the bottom plot in Fig. 3 and corresponds to (23) applied to phase a. The maximum apparent power is smaller than 2 ($p_{z7}^{+}|_{\max}$), while the minimum apparent power ($p_{z7}^{+}|_{\min}$) is equal to -2.

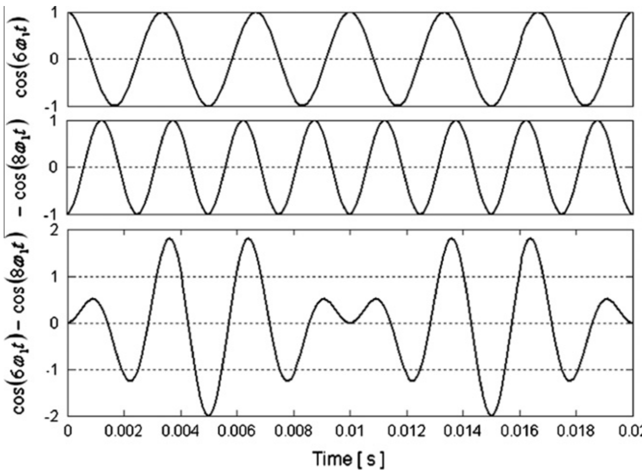

Fig. 3. Sinusoidal terms in (23) and p.u. instantaneous power flow in phase *a* for the 7th load harmonic current component.

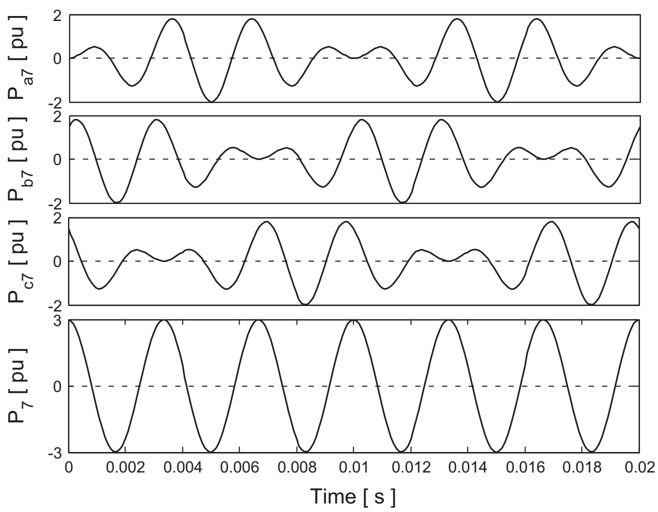

Fig. 4. Instantaneous p.u. power flows in phases *a–b–c* (p^+_{z7}) and the three-phase instantaneous power (p^+_7) for the 7th load harmonic current component.

The three-phase instantaneous power, calculated as the sum of the three per-phase instantaneous powers, is as follows:

$$p^+_7 = 3V^+_1 I^+_7 \cos(6\omega_1 t). \tag{24}$$

The three waveforms at the top of Fig. 4 are the per-phase instantaneous powers. The three waveforms are equal, with a phase shift equal to $2\pi/3$ and a negative sequence ($a-c-b$). The bottom waveform corresponds to (24) and is a sinusoidal signal with an angular frequency equal to $6\omega_1$ and a maximum value equal to $3V^+_1 I^+_7$.

Following the instantaneous approach presented in Ref. [19], the quantification of the non-fundamental effective apparent power ($S_{eN\#}$) is performed using the amplitude of the oscillating instantaneous three-phase power:

$$S_{eN\#} = \max|p^+_7| = 3V^+_1 I^+_7, \tag{25}$$

where the subscript "#" denotes the magnitudes obtained by means of the proposed instantaneous approach. The result obtained in (25) is also equal to the sum of the three per-phase apparent powers that multiplies the sinusoidal terms in (23). IEEE Std. 1459 uses the amplitude of the oscillating instantaneous power to define the reactive power in single-phase circuits ([9], Section 3.1.1.3).

Using (1)–(5) in the studied case, V_e is equal to V^+_1, I_e is equal to I^+_7, and S_{e1} is equal to zero. The value of S_{eN} following IEEE Std. 1459 is as follows:

$$S_{eN} = S_e = 3V^+_1 I^+_7. \tag{26}$$

For positive-sequence harmonic current components, the value of S_{eN} calculated by means of IEEE Std. 1459 in (26) is equal to the value calculated using the instantaneous approach ($S_{eN\#}$) in (25).

3.2. Power flows due to negative-sequence rotation harmonic current components

The instantaneous power approach is applied to negative-sequence harmonic currents demanded by the balanced non-linear load. These harmonic currents correspond to $h = 3n + 2$ and $n > 0$, with an $a-c-b$ sequence. The superscript "−" is used to identify the negative-sequence currents and powers. The most important negative-sequence harmonic currents present in the power network are the 5th ($n = 1$), and the 11th ($n = 3$). Substituting $h = 3n + 2$ in (17), the per-phase instantaneous power due to negative-sequence harmonic currents (p^-_{zh}) is calculated as follows:

$$p^-_{zh} = V^+_1 \sum_{\substack{n=0 \\ h=3n+2}}^{\infty} I^-_{zh}\{\cos[(3n+1)(\omega_1 t - \varphi_z) + \beta^-_{zh}] - \cos[(3n + 3)(\omega_1 t - \varphi_z) + \beta^-_{zh}]\}. \tag{27}$$

The two sinusoidal terms between brackets have $(3n + 1)\omega_1$ and $(3n + 3)\omega_1$ angular frequencies, respectively, with a nil average value, so the power flows represented by (27) are considered as non-efficient. The three-phase instantaneous power caused by negative-sequence harmonic current components (p^-_{h-3P}) corresponds to the sum of the three instantaneous powers calculated using (27):

$$p^-_{h-3P} = \sum_{z=a,b,c} p^-_{zh} = \sum_{z=a,b,c}\left[V^+_1 \sum_{\substack{n=0 \\ h=3n+2}}^{\infty} I^-_{zh}\{\cos[(3n+1)(\omega_1 t - \varphi_z) + \beta^-_{zh}] - \cos[(3n+3)(\omega_1 t - \varphi_z) + \beta^-_{zh}]\}\right]. \tag{28}$$

Expanding the terms between brackets, the three-phase instantaneous power of the hth load negative-sequence harmonic current component (p^-_h) is as follows:

$$\begin{aligned} p^-_h = V^+_1 I^-_h\, [&\{\cos[(3n+1)\omega_1 t + \beta^-_{ah}] + \cos[(3n+1)\omega_1 t - 2\pi/3 + \beta^-_{bh}] \\ &+ \cos[(3n+1)\omega_1 t + 2\pi/3 + \beta^-_{ch}]\} + \{\cos[(3n+3)\omega_1 t + \beta^-_{ah}] \\ &+ \cos[(3n+3)\omega_1 t + \beta^-_{bh}] + \cos[(3n+3)\omega_1 t + \beta^-_{ch}]\}]. \end{aligned} \tag{29}$$

The terms with $(3n + 1)\omega_1$ angular frequencies are positive-sequence components ($a-b-c$). They are zero-sum components because the sum of the three sinusoidal terms is equal to zero at any time [28]. The terms with $(3n + 3)\omega_1$ angular frequencies are zero-sequence components because the phase shifts of the balanced non-linear load are equal ($\beta^-_{ah} = \beta^-_{bh} = \beta^-_{ch}$). Simplifying (29), the three-phase instantaneous power of the hth load negative-sequence harmonic current component is expressed as follows:

$$p^-_h = -3V^+_1 I^-_h \cos[(3n+3)\omega_1 t + \beta^-_{ah}]. \tag{30}$$

The circuit in Fig. 1 is now simulated for a balanced load that demands only the 5th harmonic current component. The new base current is I^-_5 ($I_{p.u.} = I_z = I^-_5$) and β^-_{z5} is equal to zero. The base apparent power ($S_{p.u.5}$) is equal to $V^+_1 \cdot I^-_5$. From (27), the per-phase instantaneous power for the studied case is as follows:

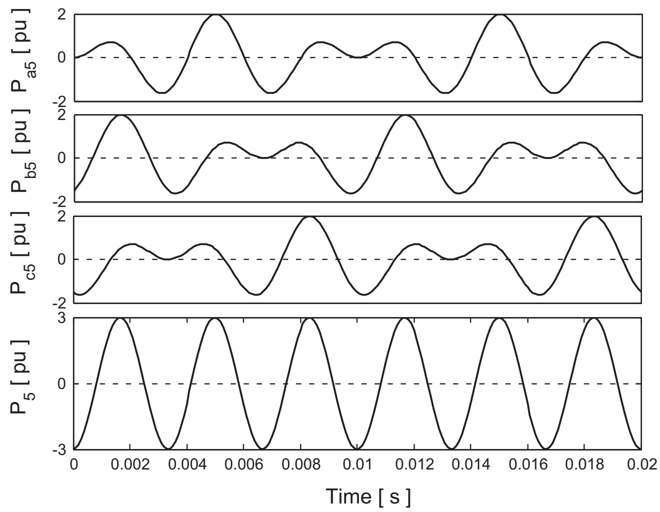

Fig. 5. Instantaneous p.u. power flows in phases a–b–c (p_{z5}^{-}) and the three-phase instantaneous power (p_{5}^{-}) for the 5th load harmonic current component.

$$p_{z5}^{-} = V_1^{+} I_{z5}^{-} \{\cos[4(\omega_1 t - \varphi_z)] - \cos[6(\omega_1 t - \varphi_z)]\}. \quad (31)$$

The three waveforms at the top of Fig. 5 are the p.u. per-phase instantaneous powers. The three waveforms are equal, with a phase shift equal to $2\pi/3$ and a negative sequence ($a - c - b$), with a maximum apparent power ($p_{z5}^{+}|_{\max}$) equal to 2 and a minimum apparent power ($p_{z5}^{+}|_{\min}$) near to -2. The three-phase instantaneous power in this case is equal to (32) and is represented by the bottom waveform in Fig. 5. It corresponds to a sinusoidal signal with an angular frequency equal to $6\omega_1$ and a maximum value equal to $3V_1^{+}I_5^{-}$:

$$p_5^{-} = \sum_{z=a,b,c} p_{z5}^{-} = -3V_1^{+} I_{z5}^{-} \cos(6\omega_1 t). \quad (32)$$

Following the instantaneous approach, the quantification of $S_{eN\#}$ is achieved by using the amplitude of the oscillating instantaneous three-phase power:

$$S_{eN\#} = \max|p_5^{-}| = 3V_1^{+} I_5^{-}. \quad (33)$$

The result obtained in (33) is also equal to the sum of the three per-phase apparent powers that multiplies the sinusoidal terms in (31). The value of S_{eN} following IEEE Std. 1459 is as follows:

$$S_{eN} = S_e = 3V_1^{+} I_5^{-}. \quad (34)$$

For negative-sequence harmonic current components, the value of S_{eN} calculated by means of IEEE Std. 1459 in (34) is equal to the value calculated following the instantaneous approach ($S_{eN\#}$) in (33).

3.3. Power flows due to zero-sequence rotation harmonic current components

The instantaneous power approach is applied to zero-sequence harmonic currents ($h = 3n + 3$ and $n > 0$) demanded by the balanced non-linear load. The superscript "0" is used to identify the currents and powers in this case. The most important zero-sequence harmonic currents present in the power network are the 3rd ($n = 0$) and 9th ($n = 2$). The zero-sequence harmonic currents flow through the lines and through the neutral wire, increasing the power losses in the system and producing harmful effects in the electric system: overcurrents and overheating in the neutral wire; increased losses in distribution transformers; etc. [29,30]. Replacing $h = 3n + 3$ in (17), the per-phase instantaneous power due to zero-sequence harmonic currents (p_{zh}^{0}) is calculated as follows:

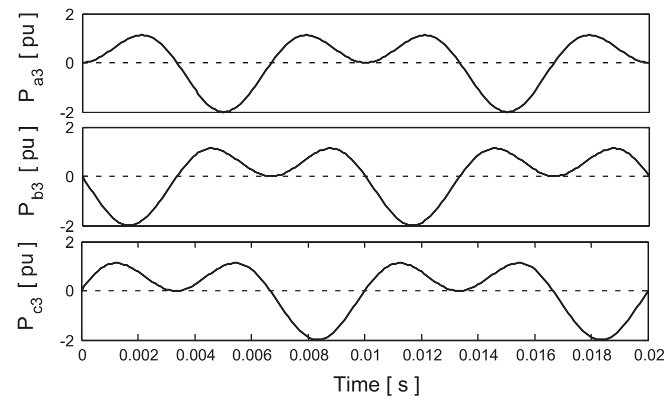

Fig. 6. Instantaneous p.u. power flows in phases a–b–c (p_{z3}^{0}) for the 3rd load harmonic current component.

$$p_{zh}^{0} = V_1^{+} \sum_{\substack{n=0 \\ h=3n+3}}^{\infty} I_{zh}^{0} \{\cos[(3n+2)(\omega_1 t - \varphi_z) + \beta_{zh}^{0}] - \cos[(3n + 4)(\omega_1 t - \varphi_z) + \beta_{zh}^{0}]\}. \quad (35)$$

The two sinusoidal terms between brackets have $(3n + 2)\omega_1$ and $(3n + 4)\omega_1$ angular frequencies, respectively, with a nil average value, so the power flows represented by (35) are considered as non-efficient. The three-phase instantaneous power caused by the zero-sequence harmonic current components (p_{h-3P}^{0}) corresponds to the sum of the three instantaneous powers calculated using (35):

$$p_{h-3P}^{0} = \sum_{z=a,b,c} p_{zh}^{0} = \sum_{z=a,b,c} \left[V_1^{+} \sum_{\substack{n=1 \\ h=3n+3}}^{\infty} I_{zh}^{0} \{\cos[(3n+2)(\omega_1 t - \varphi_z) + \beta_{zh}^{0}] - \cos[(3n+4)(\omega_1 t - \varphi_z) + \beta_{zh}^{0}]\} \right]. \quad (36)$$

Expanding the terms between brackets, the three-phase instantaneous power of the hth load zero-sequence harmonic current component (p_h^{0}) is as follows:

$$p_h^{0} = V_1^{+} I_h^{0} [\{\cos[(3n+2)\omega_1 t + \beta_{ah}^{0}] + \cos[(3n+2)\omega_1 t + 2\pi/3 + \beta_{bh}^{0}] + \cos[(3n+2)\omega_1 t - 2\pi/3 + \beta_{ch}^{0}]\} + -\{\cos[(3n+4)\omega_1 t + \beta_{ah}^{0}] + \cos[(3n+4)\omega_1 t - 2\pi/3 + \beta_{bh}^{0}] + \cos[(3n+4)\omega_1 t + 2\pi/3 + \beta_{ch}^{0}]\}] \quad (37)$$

The terms with $(3n + 2)\omega_1$ angular frequencies are negative-sequence components ($a - c - b$) and the terms with $(3n + 4)\omega_1$ angular frequencies are positive-sequence components ($a - b - c$). These are two sets of zero-sum components that always equal zero [28] when added. Simplifying (37), the three-phase instantaneous power of the hth load zero-sequence harmonic current component is equal to zero at any time instant:

$$p_h^{0} = 0. \quad (38)$$

Circuit represented in Fig. 1 is now simulated for a balanced load that demands only the 3rd harmonic current component. The new base current is I_3^{0} ($I_{\text{p.u.}} = I_z = I_3^{0}$) and β_{z3}^{0} equals zero. The base apparent power ($S_{\text{p.u.0}}$) is equal to ($V_1^{+}I_3^{0}$). From (35), the per-phase instantaneous power for the studied case is as follows:

$$p_{z3}^{0} = V_1^{+} I_{z3}^{0} \{\cos[2(\omega_1 t - \varphi_z)] - \cos[4(\omega_1 t - \varphi_z)]\}. \quad (39)$$

The three waveforms in Fig. 6 are the per-phase instantaneous powers. The three waveforms are equal, with a phase shift equal to $2\pi/3$ and a negative-sequence ($a - c - b$). The three-phase instantaneous power, calculated as the sum of the three per-phase instantaneous powers, is equal to zero.

The problem that arises is that it is impossible to apply the same instantaneous power approach used for positive- and negative-sequence harmonic currents because the three-phase instantaneous power for zero-sequence harmonic currents equals zero. Other examples of three-phase instantaneous power equalling zero are reported in Ref. [19], corresponding to a three-phase linear load that demands fundamental reactive positive-sequence currents, or fundamental zero-sequence currents with a PCC voltage that includes only V_1^+. It is shown in (15) and (16) that the calculation of Q_1^+ is achieved by summing the maximum value of the per-phase power oscillations.

The criterion applied in Ref. [19] when the three-phase instantaneous power is equal to zero is modified here because the per-phase power flows produced by the zero-sequence harmonic currents contain two sinusoidal terms with different angular frequencies – and the maximum of the sinusoidal function is not equal to the absolute value of the minimum. The calculation of S_{eN} using the instantaneous approach is made by adding the per-phase apparent power that multiplies the sinusoidal terms between brackets in (39). This calculation produces the following expression:

$$S_{eN*} = V_1^+ I_{a3}^0 + V_1^+ I_{b3}^0 + V_1^+ I_{c3}^0 = 3V_1^+ I_3^0. \tag{40}$$

If IEEE Std. 1459 is used in this case, then $I_{a3}^0 = I_{b3}^0 = I_{c3}^0 = I_3^0$ and $I_n = 3I_3^0$. Substituting in (2), the value of I_e is as follows:

$$I_e = \sqrt{\frac{(I_3^0)^2 + (I_3^0)^2 + (I_3^0)^2 + (3I_3^0)^2}{3}} = 2I_3^0. \tag{41}$$

The value of S_{eN} following IEEE Std. 1459 is as follows:

$$S_{eN} = S_e = 3V_1^+ 2I_3^0. \tag{42}$$

By a comparison of expressions (40) and (42), it is concluded that S_{eN} calculated using IEEE Std. 1459 in (42) is twice the value calculated in (40) following the instantaneous power approach ($S_{eN\#}$):

$$S_{eN} = 2S_{eN\#}. \tag{43}$$

This result confirms the statement made in Ref. [18] about '…the definition of non-fundamental power S is flawed'. From the previously obtained results a new expression of the effective apparent power is proposed in the next section in accordance with the studies performed using the instantaneous power approach.

4. Effective apparent power and non-fundamental effective apparent power based on the instantaneous power approach

The instantaneous power approach is used only in the IEEE Std. 1459 to study the active and reactive power of a single-phase linear system. The instantaneous power approach is extended in Ref. [19] to the three-phase systems that include linear unbalanced loads. The results obtained in Ref. [19] coincide with the power quantities defined in Ref. [9] for P_1^+ and Q_1^+ but differ in the definition of S_{U1}. Following the same methodology, the instantaneous power flows of balanced harmonic currents are analyzed. The instantaneous power flows include sinusoidal terms that produce complex oscillations, with different maximum and minimum values for the waveform. From the analysis performed, it is found that there are two types of three-phase oscillations:

- A sinusoidal oscillation that arises from the zero-sequence terms included in the equations that represent the per-phase power flows. This type of three-phase power flow is related to positive-and negative-sequence harmonic currents.
- A nil oscillation that arises from the sum of zero-sum sinusoidal terms (positive- and negative-sequences). This case appears in the analysis of the three-phase zero-sequence harmonic currents.

For positive- and negative-sequence harmonic currents, $S_{eN\#}$ is calculated using the amplitude of the oscillating instantaneous three-phase power, which yields the same result if the three per-phase apparent powers that multiply the sinusoidal terms are added. For zero-sequence harmonic currents, $S_{eN\#}$ is calculated following the new approach by adding the per-phase apparent power that multiplies the sinusoidal terms.

If more than one harmonic current component is demanded by the load, some interactions appear between instantaneous three-phase powers. Comparing (24) and (32), the angular frequency in both equations is the same, but the phase shift is 180° between both waveforms. If the rms values of the 5th and 7th harmonic current are equal, it results in a nil three-phase oscillation. Reduction of harmonic content in supply lines by means of interaction between single-phase and three-phase non-linear loads is described in Ref. [31]. As a consequence, the calculation of $S_{eN\#}$ using the proposed instantaneous power approach is made for all types of harmonics by adding the per-phase apparent power that multiplies the sinusoidal term of the instantaneous expression.

Under the conditions established in the analysis, it seems clear that expression (2) is invalid if the instantaneous power approach is followed. Differences are produced by the inclusion of the neutral current in the calculation of (2). Using the instantaneous approach, the definition proposed for the effective current ($I_{e\#}$) is as follows:

$$I_{e\#} = \sqrt{\frac{I_a^2 + I_b^2 + I_c^2}{3}}. \tag{44}$$

For the sake of brevity, the study of an electrical system that includes harmonic voltages at the PCC using the instantaneous power approach is not included. The study results in an effective voltage ($V_{e\#}$) defined as follows:

$$V_{e\#} = \sqrt{\frac{V_a^2 + V_b^2 + V_c^2}{3}}. \tag{45}$$

With (44) and (45), the expression of the effective apparent power following the instantaneous approach ($S_{e\#}$) is as follows:

$$S_{e\#} = 3V_{e\#} I_{e\#} = \sqrt{V_a^2 + V_b^2 + V_c^2}\sqrt{I_a^2 + I_b^2 + I_c^2}. \tag{46}$$

The results obtained with (46) coincide with the results obtained by means of (25), (33), and (40). Following the definitions proposed in (44)-(46) the new effective voltage ($V_{e\#}$) and current ($I_{e\#}$) are as follows:

$$V_{e\#} = \sqrt{(V_1^+)^2 + (V_h^0)^2}, \tag{47}$$

$$I_{e\#} = \sqrt{(I_1^{+a})^2 + (I_h^0)^2}. \tag{48}$$

These new definitions yield new expressions of the effective apparent power ($S_{e\#}$) and the non-fundamental effective apparent power ($S_{eN\#}$) as follows:

$$\begin{aligned} S_{e\#}^2 &= 9[(V_1^+ I_1^{+a})^2 + (V_1^+ I_h^0)^2 + (V_h^0 I_1^{+a})^2 + (V_h^0 I_h^0)^2], \\ S_{e\#}^2 &= (P_1^+)^2 + S_{eN\#}^2, \end{aligned} \tag{49}$$

$$S_{eN\#}^2 = 9[(V_1^+ I_h^0)^2 + (V_h^0 I_1^{+a})^2 + (V_h^0 I_h^0)^2]. \tag{50}$$

As is performed in IEEE Std. 1459, $S_{eN\#}$ is resolved as follows in the current distortion power (D_{eI}), the voltage distortion power (D_{eV}), and the harmonic apparent power (S_{eH}):

$$D_{eI}^2=(3V_1^+I_h^0)^2, \tag{51}$$

$$D_{eV}^2=(3V_h^0I_1^+)^2, \tag{52}$$

$$S_{eH}^2=(3V_{eH}I_{eH})^2=(3V_h^0I_h^0)^2. \tag{53}$$

S_{eH} is resolved into the harmonic active power (P_H) and the harmonic distortion power (D_{eH}) as follows:

$$S_{eH}^2=(3V_{eH}I_{eH})^2=P_H^2+D_{eH}^2. \tag{54}$$

5. Comparison of the non-fundamental effective apparent power definitions

The electrical circuit represented in Fig. 7 is analyzed comparing IEEE Std. 1459 definitions with the quantities proposed using the instantaneous approach. The load is balanced and includes a linear part (denoted as R) in parallel with a non-linear distorting load (denoted as D). The linear load only demands fundamental positive-sequence active current components ($i_{1a}^{+a}, i_{1b}^{+a}, i_{1c}^{+a}$), with an rms value equal to I_1^{+a} in the three phases. The non-linear load only demands zero-sequence current components ($i_{ha}^0+i_{hb}^0+i_{hc}^0$), with an rms value equal to I_h^0 in the three phases. The neutral current is equal to the sum of the three harmonic zero-sequence current components ($i_{ha}^0+i_{hb}^0+i_{hc}^0$), with an rms value equal to $3I_h^0$.

The supply voltage includes the fundamental positive-sequence voltages ($v_{1a}^+, v_{1b}^+, v_{1c}^+$) plus several harmonic zero-sequence components ($v_{ha}^0, v_{hb}^0, v_{hc}^0$) that appear due to the flow of the harmonic zero-sequence current components through the distribution lines. Under this condition, the harmonic order of the current and voltage harmonic components is the same. The rms value of the fundamental positive-sequence voltages is V_1^+, while the rms value of the zero-sequence voltage components is V_h^0. Following IEEE Std. 1459 definitions, V_e is calculated by means (1) as follows:

$$V_e=\sqrt{(V_1^+)^2+\frac{1}{2}(V_h^0)^2}. \tag{55}$$

Replacing in (2) the values of the load currents defined previously, I_e is equal to:

$$I_e=\sqrt{(I_1^{+a})^2+4(I_h^0)^2}. \tag{56}$$

Substituting (55) and (56) into (3), and expanding terms, S_e is written as follows:

$$S_e^2=9\left[(V_1^+I_1^{+a})^2+4(V_1^+I_h^0)^2+\frac{1}{2}(V_h^0I_1^{+a})^2+2(V_h^0I_h^0)^2\right]. \tag{57}$$

The first term in (57) corresponds to P_1^+ because it is obtained using the product of the fundamental positive-sequence voltage with the fundamental positive-sequence active current. The remaining terms in (57) are part of S_{eN}:

$$S_{eN}^2=9\left[4(V_1^+I_h^0)^2+\frac{1}{2}(V_h^0I_1^{+a})^2+2(V_h^0I_h^0)^2\right]. \tag{58}$$

The first term in (58) includes the product of V_1^+ and I_h^0 and corresponds to the current distortion power (D_{eI}). Rearranging terms, D_{eI} can be expressed as follows:

$$D_{eI}^2=(3V_1^+(2I_h^0))^2 \tag{59}$$

where I_{eH} in (5) is equal to $2I_h^0$ in this case. The second term in (58) includes the product of V_h^0 and I_1^+ and corresponds to the voltage distortion power (D_{eV}). By rearranging terms, D_{eV} can be expressed as follows:

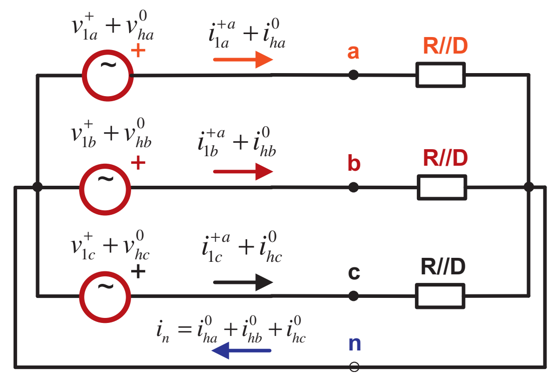

Fig. 7. Electrical power system under analysis.

$$D_{eV}^2=\left(3\left(\frac{1}{\sqrt{2}}V_h^0\right)I_1^+\right)^2, \tag{60}$$

where V_{eH} in (5) is equal to $(1/\sqrt{2})\cdot V_h^0$. The last term in (58) includes the product of V_h^0 and I_h^0 and corresponds to the harmonic apparent power (S_{eH}). By rearranging terms, S_{eH} can be expressed as follows:

$$S_{eH}^2=(3V_{eH}I_{eH})^2=2(3V_h^0I_h^0)^2. \tag{61}$$

Because (61) includes harmonic voltage and current components of the same order, S_{eH} is resolved in this case as follows:

$$S_{eH}^2=2(3V_h^0I_h^0)^2((\cos\theta_h^0)^2+(\sin\theta_h^0)^2). \tag{62}$$

The harmonic active power included in (62) is denoted as P_{H*} and is expressed as follows:

$$P_{H^*}=\sqrt{2}(3V_h^0I_h^0\cos\theta_h^0). \tag{63}$$

The subscript "$*$" denotes the magnitudes obtained by resolving IEEE Std. 1459 definitions that do not coincide with commonly accepted magnitudes. The other term in (62) corresponds to D_{eH} and in the case under analysis the term corresponds to a harmonic reactive power, denoted as Q_{H^*}, that is expressed as follows:

$$D_{eH}=Q_{H^*}=\sqrt{2}(3V_h^0I_h^0\sin\theta_h^0). \tag{64}$$

Expressions (63) and (64) do not coincide with the commonly accepted expressions of P_H and Q_H included in IEEE Std. 1459:

$$P_H=3\sum_{h\neq1}V_hI_h\cos\theta_h, \tag{65}$$

$$Q_H=3\sum_{h\neq1}V_hI_h\sin\theta_h. \tag{66}$$

The differences between (63) and (65), and (64) with (66), are produced by the factor $\sqrt{2}$ that is multiplying the remaining terms that appear in P_H and Q_H.

The expressions of $S_{e\#}$ and $S_{eN\#}$ in (49) and (50) contain the same terms as S_e and S_{eN} in (57) and (58); except that all the factors that multiply the terms between brackets are equal to one. P_H and Q_H included in the last term in (50) match with (65) and (66) respectively, so avoiding the erroneous factors found in (63) and (64).

6. Analysis of power quantities in a three-phase circuit

A three-phase circuit is analyzed using MathCad software to compare IEEE Std. 1459 voltage, current, and power magnitudes with the magnitudes derived from the instantaneous power approach proposed in this paper. The line to neutral supply voltages

Table 1
Voltages and currents for the three-phase balanced system under analysis.

h	Voltage phasors (V_{rms})	Current phasors (A_{rms})
1	$\vec{V}_{a1} = 230\angle 0^\circ$	$\vec{I}_{a1} = 10\angle 0^\circ$
	$\vec{V}_{b1} = 230\angle -120^\circ$	$\vec{I}_{b1} = 10\angle -120^\circ$
	$\vec{V}_{c1} = 230\angle +120^\circ$	$\vec{I}_{c1} = 10\angle +120^\circ$
3	$\vec{V}_{a3} = \vec{V}_{b3} = \vec{V}_{c3} = 34.5\angle 0^\circ$	$\vec{I}_{a3} = \vec{I}_{b3} = \vec{I}_{c3} = 1.5\angle 0^\circ$

Table 2
Magnitudes for the three-phase circuit under analysis.

	IEEE 1459	Instantaneous approach (#)
V_e (V)	231.3	232.5
$V_{e1} = V_1^+$ (V)		230
V_{eH} (V)	24.4	34.5
I_e (A)	10.44	10.11
$I_{e1} = I_1^+$ (A)		10
I_{eH} (A)	3	1.5
S_e (va)	7244	7055.2
$S_{e1} = S_1^+$ (va)		6900
P_1^+ (W)		6900
S_{eN} (va)	2206.5	1471.9
S_{eH} (va)	219.5	155.25
D_{eI} (va)	2070	1035
D_{eV} (va)	731.8	1035
P (W)		7055.2
$P_{A1} = P_{B1} = P_{C1}$ (W)		2300
P_H (W)		155.2

Table 3
Factors of variation between magnitudes for the circuit under analysis.

$V_e = 0.99\ V_{e\#}$		$V_{eH} = 0.71\ V_{eH\#}$
$I_e = 1.03\ I_{e\#}$		$I_{eH} = 2\ I_{eH\#}$
$S_e = 1.03\ S_{e\#}$		$S_{eN} = 1.5\ S_{eN\#}$
$D_{eI} = 2\ D_{eI\#}$	$D_{eV} = 0.71\ D_{eV\#}$	$S_{eH} = 1.41\ S_{eH\#}$

and the load currents used in the analysis appear in Table 1. The line to neutral rms supply voltages is 232.5 V, and the rms current lines equal 10.11 A in the three phases. The neutral current equals 4.5 A, and is produced only by the third harmonic current component.

Table 2 shows a comparison between the main electrical magnitudes in the system using the IEEE Std. 1459 approach and using the approach proposed in the paper. For the case of analysis, the following magnitudes defined in IEEE Std. 1459 are equal to zero: $I_1^-, I_1^0, V_1^-, V_1^0, Q_1^+, S_{U1}, P_1^-$, and P_1^0. Magnitudes that give the same value using both approaches are in the middle of the two columns. Current and power magnitudes calculated using the IEEE Std. 1459 definitions reveal in the example an overvaluation that varies between 3% for S_e to 100% for I_{eH} and D_{eI}. The value of I_{eH} does not correspond to any harmonic current component flowing in the circuit.

Table 3 presents the variation of the different magnitudes – while taking into account the approach used in their calculation. Effective voltages with the new approach are higher than the voltages calculated using IEEE Std. 1459. The factor $1/\sqrt{2}$ in (60) is responsible of an undervaluation of the zero-sequence harmonic voltages in IEEE Std. 1459 definitions. It results in a $D_{eV\#}$ that is greater when using the approach proposed in this paper than when calculated using IEEE Std. 1459.

Power factors are used as a merit factor of the electrical system. IEEE Std. 1459 defines the effective power factor (P_{Fe}) and the fundamental positive-sequence power factor (P_{F1}^+) as follows:

$$P_{Fe} = \frac{P}{S_e}, \tag{67}$$

$$P_{F1}^+ = \frac{P_1^+}{S_1^+}. \tag{68}$$

The total power factor (P_{FT}) introduced in Refs. [32,33] measures the relationship between the active power under ideal operating conditions (P_1^+) and S_e:

$$P_{FT} = \frac{P_1^+}{S_e}. \tag{69}$$

Using the instantaneous power approach, the effective power factor ($P_{Fe\#}$) and the total power factor ($P_{FT\#}$) are redefined using $S_{e\#}$ in (67) and (69). For the case under study, P_{F1}^+ is equal to one for both approaches. Following IEEE Std. 1459 definitions $P_{Fe} = 0.97$ and $P_{FT} = 0.95$; while $P_{Fe\#} = 1$ and $P_{FT\#} = 0.97$. Following the proposed definitions, only $P_{FT\#}$ shows that an inefficient phenomenon exists in the circuit. $P_{Fe\#}$ and $P_{FT\#}$ calculated using the proposed power quantities produce better results than those obtained using IEEE Std. 1459 definitions.

7. Conclusion

IEEE Std. 1459 applies an instantaneous power approach only for the quantification of active and reactive power in single-phase systems. The remaining IEEE Std. 1459 power quantities are obtained by resolving the effective voltage, current, and apparent power into different terms. As is demonstrated in this paper, the application of IEEE Std. 1459 definitions to an electrical circuit with a harmonic neutral current component yields an expression of the harmonic active power that disagrees with the expression included in the IEEE Std. 1459.

The instantaneous power approach is extended in this paper to propose a new definition of the non-fundamental effective apparent power and which avoids the problems in the harmonic active power calculation. The per-phase and the three-phase instantaneous power flows are analyzed for a three-phase four-wire balanced non-linear load connected to an ideal power network that supplies a set of positive-sequence fundamental voltages. The calculation of $S_{eN\#}$ using the instantaneous power approach is made by adding the per-phase apparent power that multiplies the sinusoidal term of the instantaneous expression.

The expression of $S_{eN\#}$ is compared with the expression obtained applying IEEE Std. 1459. $S_{eN\#}$ expressions for positive- and negative-sequence harmonic load currents coincide for both approaches. For zero-sequence harmonic load currents, the value of S_{eN} obtained by means of the IEEE Std. 1459 definition is twice the value obtained using the $S_{eN\#}$ instantaneous power approach.

A balanced three-phase circuit connected to a non-ideal supply voltage is analyzed. Voltage, current, and power magnitudes are calculated using IEEE Std. 1459 definitions for the instantaneous approach. The analysis shows that there is a disagreement between the harmonic active power defined in the IEEE Std. 1459 and the value obtained by resolving S_{eN}. The comparison between the different magnitudes demonstrates that current and power magnitudes in IEEE Std. 1459 are over-rated; while voltage magnitudes are under-rated.

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