Mackendrick: A Maple Package Oriented to Symbolic Computational Epidemiology

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Abstract. A Maple Package named *Mackendrick* is presented. Such package is oriented to symbolic computational epidemiology.

1 Introduction

We present here, the maple package *Mackendrick* which we have constructed for the solution of certain problems in symbolic computational epidemiology. Our package does not incorporate any kind of element of artificial intelligence, but for some of the problems that we solved, will be very funny to have some computer algebra system with artificial intelligence. The problems that we can solve here are linear problems but such problems only can be solved using computer algebra, due the involved calculations are very tedious and long as to be implemented by hand using pen and paper only.

Our emblematic problems are situations of spatial propagation of directly transmitted diseases when boundary conditions are involved at the form of endemic boundaries from where the disease spreads towards the interior of the habitat. More over, we consider here the extra complication that arises from the inclusion of the effects of heterogeneity of contact between individuals.

A fundamental epidemiological magnitude is the well know basic reproductive rate, denoted R_0. The principal function of the our package *Mackendrick* is the computation of the explicit analytical form of the R_0 for certain spatial models of disease diffusion with heterogeneity effects. We need here, computer algebra, because that it is required is a symbolic expression for R_0 and not a number or a graphic. Due, our package is constructed under maple platform, then our package has numerical and graphical computational power too.

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The package is loaded with

```
restart:
with(Mackendrick);
```

and the notification is

```
[dielou, difumemoestra, memo, memoyf, mysol,prosize,sir,veneco,
venecomemo];
```

which is the list of procedures that are contained within *Mackendrick*.
In the following sections, the commands of *Mackendrick* are presented.

2 The Command *mysol*

For example, the procedure *mysol* solves the following problem:

$$\frac{\partial}{\partial t}u(r,t) - \frac{\delta_1\left(\frac{\partial}{\partial r}u(r,t) + r\frac{\partial^2}{\partial r^2}u(r,t)\right)}{r}$$
$$-\delta_2\int_0^t \frac{M_0(t-\tau)\left(\frac{\partial}{\partial r}u(r,\tau) + r\frac{\partial^2}{\partial r^2}u(r,\tau)\right)}{r}d\tau$$
$$-(\beta_1 S_0 - \gamma_1)u(r,t) - \beta_2 S_0\int_0^t u(r,\tau)M_1(t-\tau)\,d\tau +$$
$$\gamma_2\int_0^t u(r,\tau)M_2(t-\tau)\,d\tau = 0 \qquad (1)$$

with the boundary condition

$$u(a,t) = \mu_b e^{-\eta t}. \qquad (2)$$

The procedure *mysol* needs as inputs the specifical forms of the functions $M_0(t)$, $M_1(t)$ and $M_2(t)$. Here we present two cases.

2.1 Without Memory

For example when

$$M_0(t) = 0, M_1(t) = 0, M_2(t) = 0, \qquad (3)$$

and with the instructions

```
M0:=0:M1:=0:M2:=0:
mysol(M0,M1,M2);
```

Mackendrick produces the following solution [1]

$$u\left(r,t\right)=\frac{\mu_b e^{-\eta\,t} J_0\left(\sqrt{\lambda\left(-\eta\right)}r\right)}{J_0\left(\sqrt{\lambda\left(-\eta\right)}a\right)}+\sum_{i=1}^{1}\sum_{n=1}^{\infty}\frac{-2\,e^{S_{i,n}t}\mu_b J_0\left(\frac{\alpha_n r}{a}\right)\alpha_n}{\left(S_{i,n}+\eta\right)\left(J_1\left(\alpha_n\right)\right)a^2\left(\frac{d}{dS_{i,n}}\lambda\left(S_{i,n}\right)\right)}. \tag{4}$$

The corresponding basic reproductive rate is given by

$$R_0=\frac{\beta_1 S_0 a^2}{\gamma_1 a^2+{\alpha_n}^2\delta_1}. \tag{5}$$

The function $\lambda(s)$ at (4) has the form

$$\lambda\left(s\right)=-\frac{s-\beta_1 S_0+\gamma_1}{\delta_1}, \tag{6}$$

and the parameters denoted $S_{i,n}$ at (4) are the solutions of the equation on s

$$-\frac{s-\beta_1 S_0+\gamma_1}{\delta_1}=\frac{{\alpha_n}^2}{a^2}, \tag{7}$$

where α_n are the zeroes of the Bessel function $J_0(x)$ [2].

2.2 With Exponential Memory

$$M_0(t)=e^{-\epsilon_0 t}, M_1(t)=e^{-\epsilon_1 t}, M_2(t)=e^{-\epsilon_2 t}, \tag{8}$$

and with instructions

```
M0:=exp(-epsilon[0]*t):M1:=exp(-epsilon[1]*t):M2:=exp(-epsilon[2]*t):
mysol(M0,M1,M2);
```

Mackendrick produces the following solution

$$u\left(r,t\right)=\frac{\mu_b e^{-\eta\,t} J_0\left(\sqrt{\lambda\left(-\eta\right)}r\right)}{J_0\left(\sqrt{\lambda\left(-\eta\right)}a\right)}+\sum_{i=1}^{4}\sum_{n=1}^{\infty}\frac{-2\,e^{S_{i,n}t}\mu_b J_0\left(\frac{\alpha_n r}{a}\right)\alpha_n}{\left(S_{i,n}+\eta\right)\left(J_1\left(\alpha_n\right)\right)a^2\left(\frac{d}{dS_{i,n}}\lambda\left(S_{i,n}\right)\right)}. \tag{9}$$

The corresponding basic reproductive rate is given by

$$R_0=\frac{S_0 a^2\epsilon_2\epsilon_0\left(\beta_1\epsilon_1+\beta_2\right)}{\epsilon_1\left({\alpha_n}^2\delta_1\epsilon_0\epsilon_2+{\alpha_n}^2\delta_2\epsilon_2+\gamma_2 a^2\epsilon_0+\gamma_1 a^2\epsilon_2\epsilon_0\right)}. \tag{10}$$

The function $\lambda(s)$ at (9) has the form

$$\lambda\left(s\right)=\left(s-\beta_1 S_0+\gamma_1-\frac{\beta_2 S_0}{s+\epsilon_1}+\frac{\gamma_2}{s+\epsilon_2}\right)\left(-\delta_1-\frac{\delta_2}{s+\epsilon_0}\right)^{-1}, \tag{11}$$

and the parameters denoted $S_{i,n}$ at (9) are the solutions of the equation on s

$$\sqrt{\left(s-\beta_1 S_0+\gamma_1-\frac{\beta_2 S_0}{s+\epsilon_1}+\frac{\gamma_2}{s+\epsilon_2}\right)\left(-\delta_1-\frac{\delta_2}{s+\epsilon_0}\right)^{-1}}=\frac{\alpha_n}{a}. \tag{12}$$

3 The Command *veneco*

The procedure *veneco* solves the following problem

$$\frac{d}{dt}X_i\left(t\right)-\frac{\beta\,nX_i\left(t\right)}{k}+\gamma\,X_i\left(t\right)-\frac{2\beta\,n\left(\sum_{j=1}^{k}\nu\,X_j\left(t\right)-\nu\,X_i\left(t\right)\right)}{k}=0. \quad (13)$$

with the instruction

```
veneco(n);
```

our Mackendrick gives the following form of the basic reproductive rate [3]

$$R_{0,k}=\frac{\beta\,n\left(1+2\,\nu\,k-2\,\nu\right)}{\gamma\,k} \quad (14)$$

4 Conclusions

We believe that the Maple package *Mackendrick* can be useful within the domain of symbolic computational epidemiology. Our *Mackendrick* can solve certain complex spatial epidemic models. The method of solution that *Mackendrick* incorporates is the Laplace transform technique with the application of the Bromwich integral and residue theorem for the realization of the inverse Laplace transform [4]. Also, *Mackendrick* involves certain theorem of Linear Algebra, which is presented in [3]. This theorem must be introduced *ad hoc* but it is possible that with the future development of artificial intelligence, such theorem can be proved directly by the computer algebra system that is the background of *Mackendrick* . We hope that at the future our package can be extended and applied to more numerous and complex problems in mathematical epidemiology.

References

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