MATHEMATICS PRESERVICE TEACHERS' ARGUMENTATION

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This research deals with the preservice teachers' dialogic argumentations when presenting geometry tasks to their colleagues, during discussion sessions previous to teaching to children. An argumentation analysis tool is used that complement Toulmin's analysis proposal and that includes features related to mathematical logic, rhetoric and dialectic features. We propose both a representation for the dialogic argumentation and a way to identify its structural qualities.

INTRODUCTION

Due to the complexity of teachers' argumentation in the classroom, that do not let 'uniquely' follows the deduction rules of Aristotelian logic but recurs to 'persuasion' (Perelman, 1997), it is required diverse skills, specifically, to argue during teaching (Ufer, Heinze & Reiss, 2008). Several studies showed, that not only students have problems in this field (Reiss, Heinze, Kessler, Rudolph-Albert & Renkl, 2007), but also prospective and in-service teachers (Barkai, Tsamir, Tirosh & Dreyfus, 2002). The interest of the paper is to study preservice teachers' argumentations when explaining geometry tasks.

FRAMEWORK

In this research 'dialogic argumentation' is assumed as "social and collaborative process necessary to solve problems and advance knowledge" (Duschl & Osborne, 2002, p. 41). The dialogic argumentation is close related to 'communicative acts' that give and ask for reasons (Habermas, 1999; Toulmin, 2007) that includes not only logic-substantive features but rhetoric and dialectic put into play by preservice teachers while presenting geometry tasks to their fellow colleagues to explain, to teach and to convince.

The dialogic argumentations are analyzed in regard to structural qualities: logic-substantive, rhetoric and dialectic (Habermas, 1999); for representing the structure, it is used Knipping (2008) proposal. The warrants used by preservice teachers are presented as: a priori, empiric, institutional and evaluative (Nardi, Biza & Zachariades, 2011). The argumentative sequence, either progressive or retro-progressive, (Van Eemeren, Grootendorst & Henkemans, 2006) considers the natural way in which teachers give and ask for reasons.

CONTEXT AND METHODOLOGY

The research context is the course 'Teaching Practice', offered to preservice teachers in the program of mathematics in the School of Education, Antioquia University,

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Medellín, Colombia. This course spans for a year and a half. During the first year, the preservice teachers design and choose geometry tasks, solve and present them to their fellow colleagues, who criticized the presentations; in the remaining term, the teachers acted as teachers in the classroom. This paper informs about the first year. The preservice teachers were interviewed just after they presented the tasks to their colleagues. Interviews were recorded, transcribed and analyzed searching for: (1) formal argumentation structure (Knipping, 2008); (2) epistemological and pedagogical nature of reasoning (Nardi, Biza & Zachariades, 2011), and (3) argumentation sequences and interaction patterns (Clark & Sampson, 2008).

ANALYSIS AND RESULTS

We discussed two argument segments belonging to two preservice teachers who presented the solutions of two geometric tasks to their colleagues. The first argument responds to: How would you explain to your colleagues how to find the value of angle h? The second argument responds to the question: How would you teach the Pythagorean Theorem to ninth graders? In what follows the two preservice teachers' arguments are presented, the first argument belonging to Jhoanne (J), the second to Maria (M). It is shown the questions (Q) and the ensuing answers. The first question includes the graph as data. The pieces of the argument are signaled with a numeral located to the left (L1 means line one corresponding to the argument segment).

L1-L2 Q: How would you explain to your colleagues the way to find the value of angle h?

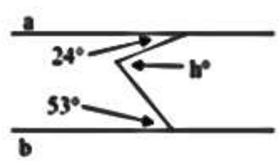


Figure 1: Graph for the question (Berg, Fuglestad, Goodchild & Sriraman, 2012, p. 682)

Jhoanne proceeds as follows:

L3-L4 J: In order to find out the value of angle h, first I prolong the lines in such a way that cut in A and D, the parallel lines a and b.

The argumentation motivated by the questions requires geometry knowledge organized in a sequence with the intention to explain. Figure 1 presents some data: the values of angles, two parallel lines and the nomination of the angle whose value is to be found. These data underline geometric knowledge that Jhoanne must know. In L3 Jhoanne uses an argumentative indicator -*first*- followed by a narrative in first person. Additionally, uses the modal qualifier: *in such a way that* (L4-5), uses the apriori epistemological warrant to extend the lines toward A and D. Next Jhoanne affirms:

L5-L7 J: Then by the properties of angles among paralell lines, I look for the angles alternate-interior as A and F, D and B, by the property of the sum of the measures of the internal angles of a triangle, which is 180°.

In this segment Jhoanne uses two a priori-epistemological warrants (Nardi, et al., 2011) corresponding to a property: alternate-interior angles among parallel lines and a theorem: the sum of the measures of internal angles in a triangle is 180º. Jhoanne uses an argumentation indicator -*then*- (L5), and a modal qualifier -*which is*- (L7). She proceeds:

L8-L9 J: We find the measure of angle C by the definition of plane angle we have that C°+ h°= 180° where C°=103° thus h = 77°.

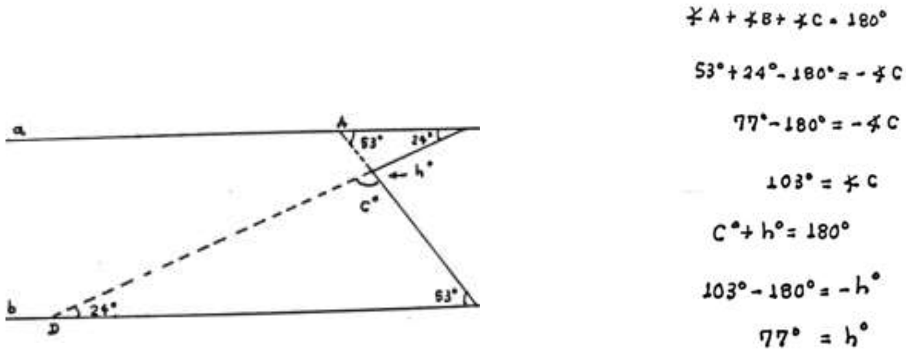


Figure 2: Ilustration and computing proposed by Jhoanne

Jhoanne uses manifold argumentation indicators: *we have that*, *where* and *thus* (van Eemeren, et al., 2006). When passing from L5-L7 to L8-L9, she uses an ilustration as a rhetoric resource (Perelman, 1997), and the procedure to find the unknown value is algebraic in nature. The second argumentative segment, corresponding to Maria, refers to the design of a class related to the Pythagorean Theorem.

L10 Q: How would you teach the Pythagorean Theorem to ninth graders?

Maria says:

L11-L15 M: What I understood […] is that I have to, more or less, propose a draft about planning an activity with ninth graders to teach them the Pythagorean Theorem, then I planned the activity as a guide, then I proposed a puzzle [tamgram like], […] and to arrive […] to the formal features of the Theorem.

In this segment, Maria first establish her argument conclusion, that deals with the teaching the Pythagorean Theorem to ninth graders (9º) using a puzzle, then she states the objective about discussing 'formaly' such Theorem. The sentence 'What I understood…' -first person- supposess *a communicative understanding* and *an action* as well (Habermas, 1999). Additionally, she uses '*more or less'* -a modal qualifier- and assumes her proposal as 'possible' and not as a definite statement.

L16-L20 M: […] Initially, as Carlos did, I would begin with some history, even though they [the kids] are in ninth grade, a story can be told to them about the Pythagorean Theorem […] because it is believed that Pythagoras discovered the theorem, but it was also known to ancient civilizations in Babylon and Egypt, the Pythagorean triads were also known to them…

In this segment, Maria says that she took into consideration her colleague Carlos`s proposal that accounts for the use of a rhetoric resource as the model (Perelman, 1997). Additionally, L3 serves as intersubjective evidence that refers to meaning negotiation by their colleagues. Within the first reasons expressed by Maria, appears an apriori institutional warrant (Nardi, et al., 2011), because she uses the history of the Theorem. Additionally, she expresses the modal qualifier '*though*' that refers to the likelihood of using a story as a resource to teach the kids. Telling a story is an evidence of the practical rationality or reasonableness (Habermas, 1999; Toulmin, 2007). The preservice teacher continues arguing:

L21-L26 M: […] As it is said there, a man called Pythagoras discovered an amazing fact regarding triangles, if a triangle as a right angle, so to speak, an angle whose measure is 90º and a square is constructed on each one of its legs; then the biggest square [refering to the square constructed on the hypotenuse] has exactly the same area as the other two squares together […]. The triangle's biggest side is called hypotenuse.

On one side, this segment manifests an a priori-epistemological warrant (Nardi, et al., 2011), because it resorts both to the statement of the Pythagorean Theorem and to the definition of a right triangle. On the other side, it uses the modal qualifier '*exactly*' (L25), because she is certain about her statement. Every fragment -L3 to L9- offers evidence on the use of the theoretical rationality in the dialogic argumentation (Habermas, 1999), which complements the practical rationality (Rigotti & Greco, 2009), and links the actions that are epistemological, teleological and communicative (Habermas, 1999) to the future teachers' argument.

L27-L30 M: So, I would begin with some templates more or less …the handouts would be the templates, that they [pupils] have to cut and they themselves can verify if the two squares are 'put togheter'; those that I constructed on the triangle legs, I would obtain the area [or the square constructed] on the hypotenuse.

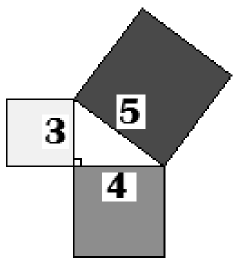
Maria begins her Pythagorean Theorem teaching proposal by using the puzzle to shape the rectangles over the legs and over the hypotenuse of a right triangle. The use of the puzzle puts into practice the practical rationality and manages to persuade her colleagues (Perelman, 1997), which links the puzzle activity to the Pythagorean Theorem.

L31-L34 M: Just before finishing […] the work about what I just said, I would reach the formal definition, that I would do by showing them […] the triangle, thus the square's area constructed on one leg plus the square's area built on the other leg would be equal to the square's area of the hypotenuse.

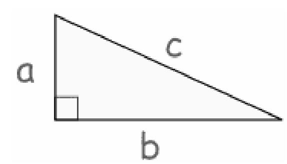
Later she performs Pythagorean Theorem verifications for particular cases. Maria employs the examples as a rhetoric resource, which let her to generalize (Perelman, 1997).

L35-L39 M: I would perform some verifications, if it works with an easy example, algebraically assigning numbers to the lenghts of both sides for the students to calculate the area with a simple operation, then one square measures three (3), the other square measures four (4), and the resulting square would measure five (5), there we show the solution.

L40-L44 M: Why would it be useful? If we know the side lengths of a right triangle, the Pythagorean Theorem would help us to find the length of the third side, but I would make them notice that it is only true for right triangles, that it is not true for every triangle. Then I would write it as an equation and there we would perform algebraic procedures using equations, just as it is shown in Figure 3.



Let's see if the areas are the same:
$3^2 + 4^2 = 5^2$
Computing we get:
$9 + 16 = 25$
Yes, it works!



$a^2 + b^2 = c^2$

Figure 3: Verification of the Pythagorean Theorem using an example

The tasks are formulated using questions, whose structure responds to: how is it argumented, whom the arguments are directed to and what is argumented. It can be said that the preservice teachers argue in first person (Habermas, 1999), and assume the 'leading' role. The roles of protagonist and antagonist give account of the 'progressive and retrogressive' arguments, based on 'argumentative indicators' (Van Hemeren, Grootendorst & Henkemans, 2006). On the other side, an advantage of the dialogic argumentation is that it favors solving the dichotomy between analytic and substantive argumentations (Toulmin, 2007), because preservice teachers not only use logic inference rules to communicate their knowledge but also rhetoric resources linked to illustrations, examples and models (Perelman, 1997). The latter can be seen between the first and the second segment of the arguments by the two preservice teachers which show 'density' in the dialogic argumentations that surpasses Toulmin's model (2007). A drawback of this report is that preservice teachers' dialogic argumentations, while teaching in the classroom to real pupils, are not discussed. On the other side, the argumentative indicators used let us to identify the warrants chosen by the preservice teachers. Table 1 shows the relationships among the argumentative indicators, modal qualifiers and warrants used by preservice teachers. When questions are used to generate a class planning, the preservice teachers use no absolute modal qualifiers, for instance: *more or less, such as, even though, it does not work for every case*. In terms of a pattern of interaction, we identify a protagonist role in Jhoanne dialogic argumentation, and the use of both explicit graphic reasoning (Figure 2) and verbal reasoning. The reasoning would not be explicit if logic-formal inference rules would be applied in an analytic argumentation. In the segment corresponding to Maria's argument, the argumentative sequence was not only accompanied by warrants linked to the geometric knowledge -a priori epistemological warrants-, but also by warrants linked to the history of the Pythagorean Theorem -institutional a priori warrant-.

		Argumentation indicators	Modal qualifiers	Warrants
Argumentative segment 1	L3	First	In such a Way	A priori-epistemological
	L4	And		
	L5	Then, I look for		
	L6	And, by		A priori-epistemological
	L7	Which is		
	L8	We find, by, we have		A priori-epistemological
	L9	Thus	Where	
Argumentative segment 2	L11		More or less	A priori-institutional
	L13	Then		
	L16	Initially	Though	A priori-institutional
	L22	So to speak		
	L23	And		
	L25		Exactly	
	L27		More or less	Empiric-personal
	L30	I would obtain		A priori-epistemological
	L31	I just said		A priori-epistemological
	L32	Thus		
	L34		Be equal to	
	L35		If it work	A priori-epistemological
	L41	But		
	L43	And		

Table 1: Relations among argumentation indicators, modal qualifiers and warrants

According to the relations, presented in Table 1, it can be stated that the first argument is devoted to explaining. The modal qualificators are sparse and express not only likelihood for the task proposed but also Maria's confidence in her solution to it. Meanwhile the second argument uses manifold modal qualificators that express Jhoanne' stance in regard to the likelihood for her solution to the task proposed. The auditorium was not an issue for the first argument, but it was for the second. In regard to the rhetoric resources, Jhoanne's argument uses only the illustration, while Maria's uses: illustration, model and example. The first argument use only of a priori epistemological warrants, while the second use a priori epistemological, empirical-personal as well as a priori-institutional. Additionally, the argumentation indicators facilitate the identification of the Toulmin's model components. If the question is linked to knowledge, Jhoanne's argument case, the modal qualificators give account of a structure close to the formal logic, but if the question is guided by a teaching intention, Maria's argument case, the modal qualificators point out to a doubt and establish a link to the reasonablennes.

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