$SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ as an $SU(6) \otimes U(1)_X$ subgroup

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An extension of the standard model to the local gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ as a family independent model is presented. The mass scales, the gauge boson masses, and the masses for the spin 1/2 particles in the model are studied. The mass differences between the up and down quark sectors, between the quarks and leptons, and between the charged and neutral leptons in one family are analyzed. The existence of two Dirac neutrinos for each family, one light and one very heavy, is predicted. By using experimental results from CERN LEP, SLC and atomic parity violation we constrain the mixing angle between the two neutral currents and the mass of the additional neutral gauge boson to be $-0.00015 \leqslant \sin\theta \leqslant 0$ and $1.5\ \mathrm{TeV} \leqslant M_{Z_2}$ at 95% C.L.

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I. INTRODUCTION

In spite of the remarkable experimental success of our leading theory of fundamental interactions, the so-called standard model (SM) based on the local gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \equiv G_{SM}$, with $SU(2)_L \otimes U(1)_Y$ hidden [1] and $SU(3)_c$ confined [2], it fails to explain several issues such as hierarchical fermion masses and mixing angles, charge quantization, CP violation, replication of families, etc. For example in the weak basis, before the symmetry is broken, the families in the SM are identical to each other; when the symmetry breaking takes place, the fermions acquire masses according to their experimental values and the three families become different. However in the SM there is no mechanism for explaining the origin of families or the fermion mass spectrum.

These drawbacks of the SM have led to a strong belief that the model is still incomplete and that it must be regarded as a low-energy effective field theory originating from a more fundamental one. That belief lies on strong conceptual indications for physics beyond the SM which have produced a variety of theoretically well motivated extensions of the model: left-right symmetry, grand unification, supersymmetry, superstring inspired extensions, etc. [3].

At present, however, there are no definitive experimental facts that point toward what lies beyond the SM and the best approach may be to depart from it as little as possible. In this regard, $SU(3)_L \otimes U(1)_X$ as a flavor group has been considered several times in the literature, first as a family independent theory [4], and then as a family structure [5,6] which, among its best features, provides with an alternative to the problem of the number N of families, in the sense that anomaly cancelation is achieved when N is a multiple of three; further, from the condition of $SU(3)_c$ asymptotic freedom which is valid only if the number of families is less than five, it follows that in those models N is equal to 3.

Over the past decade two three family models based on the $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ local gauge group have received special attention. In one of them the three known left-handed lepton components for each family are associated to three $SU(3)_L$ triplets as $(\nu_l, l^-, l^+)_L$, where l_L^+ is related to the right-handed isospin singlet of the charged lepton l_L^- in the SM [5]. In the other model the three $SU(3)_L$ lepton triplets are of the form $(\nu_l, l^-, \nu_l^c)_L$ where ν_{lL}^c is related to the right-handed component of the neutrino field ν_{lL} [6]. In the first model anomaly cancelation implies quarks with exotic electric charges $-4/3$ and 5/3, while in the second one the exotic particles have only ordinary electric charges.

A recent analysis of the $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ local gauge theory (hereafter the 331 group) has shown that, by restricting the fermion field representations to particles without exotic electric charges and by paying due attention to anomaly cancelation, six different models are obtained; while by relaxing the condition of nonexistence of exotic electric charges, an infinite number of models can be generated [7]. Four of the six models without exotic electric charges are family models for quarks and leptons in which at least one of the 3 families is treated differently. The remaining two models are one family or family independent models. One of them, which was analyzed in Ref. [8], is an E_6 subgroup. The other one, studied in this paper, is a subgroup of $SU(6) \otimes U(1)_X$, a new electroweak-strong unification group. For this last model we will do some phenomenological calculations in order to set the different mass scales and calculate the masses for all the spin 1/2 particles in one family.

This paper is organized as follows. In the next section we introduce the model as an anomaly free theory based on the local gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ and show that it is a subgroup of an electroweak-strong unification group

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based on the gauge structure $SU(6)\otimes U(1)_X$. In Sec. III we describe the scalar sector needed to break the symmetry and to produce masses to the fermion fields in the model. In Sec. IV we study the gauge boson sector paying special attention to the neutral currents present in the model and their mixing. In Sec. V we analyze the fermion mass spectrum. In Sec. VI we use experimental results in order to constrain the mixing angle between the two neutral currents and the mass scale of the new neutral gauge boson. In the last section we summarize the model, do a comparison between the model in Ref. [8] and the present one and state our conclusions. A technical Appendix on the diagonalization of the 4×4 mass matrix for the charged leptons in the model is presented at the end.

II. THE FERMION CONTENT OF THE MODEL

A. $SU(3)_c\otimes SU(3)_L\otimes U(1)_X$ as a one-family anomaly free model

In what follows we assume that the electroweak gauge group is $SU(3)_L\otimes U(1)_X$ which contain $SU(2)_L\otimes U(1)_Y$ as a subgroup. We also assume that the left handed quarks (color triplets) and left-handed leptons (color singlets) transform as the $\overline{3}$ and 3 representations of $SU(3)_L$ respectively. We have an $SU(3)_c$ vectorlike as in the SM and, contrary to most of the 331 models in the literature [5,6], in our model the anomalies cancel individually in each family as it is done for the SM.

The most general expression for the electric charge generator in $SU(3)_L\otimes U(1)_X$ is a linear combination of the three diagonal generators of the gauge group

$$Q=aT_{3L}+\frac{2}{\sqrt{3}}bT_{8L}+XI_3, \tag{1}$$

where $T_{iL}=\lambda_{iL}/2,\lambda_{iL}$ being the Gell-Mann matrices for $SU(3)_L$ normalized as $\mathrm{tr}(\lambda_{iL}\lambda_{jL})=2\delta_{ij}$, and $I_3=Dg(1,1,1)$ is the diagonal 3×3 unit matrix. Choosing $a=1$ gives the usual isospin of the electroweak interactions. X and b are arbitrary parameters to be fixed next.

We start by defining two $SU(3)_L$ triplets

$$\chi_L=\begin{pmatrix} d\\ u\\ q\end{pmatrix}_L, \quad \psi_L=\begin{pmatrix} \nu_e\\ e^-\\ l\end{pmatrix}_L,$$

where q_L and l_L are $SU(2)_L$ singlet exotic quark and lepton fields, respectively. Now, if the $(SU(3)_L,U(1)_X)$ quantum numbers for χ_L and ψ_L are $(\overline{3},X_\chi)$ and $(3,X_\psi)$, respectively, then by using Eq. (1) we have the relationship

$$X_\chi+X_\psi=Q_q+Q_l=-1/3, \tag{2}$$

where Q_q and Q_l are the electric charges of the $SU(2)_L$ singlets q_L and l_L respectively, in units of the absolute value of the electron electric charge.

Now, in order to cancel the $[SU(3)_L]^3$ anomaly two more $SU(3)_L$ lepton triplets with quantum numbers $\{3,X_i\},i=1,2$, must be introduced (together with their corresponding right-handed charged components). Each one of those multiplets must include one $SU(2)_L$ doublet and one singlet of new exotic leptons. The quarks fields u_L^c,d_L^c and q_L^c color antitriplets and $SU(3)_L$ singlets, with $U(1)_X$ quantum numbers X_u,X_d and X_q respectively, must also be introduced in order to cancel the $[SU(3)_c]^3$ anomaly. Then the hypercharges X_α with $\alpha=\chi,\psi,1,2,u,d,q,\ldots$ are fixed using Eqs. (1), (2) and the anomaly constraint equations coming from the vertices $[SU(3)_C]^2U(1)_X$, $[SU(3)_L]^2U(1)_X$, $[\mathrm{grav}]^2U(1)_X$ and $[U(1)_X]^3$, where $[\mathrm{grav}]^2U(1)_X$ stands for the gravitational anomaly [9]. These equations are

$$[SU(3)_c]^2U(1)_X{:}3X_\chi+X_u+X_d+X_q=0,$$

$$[SU(3)_L]^2U(1)_X{:}3X_\chi+X_\psi+X_1+X_2=0,$$

$$[\mathrm{grav}]^2U(1)_X{:}9X_\chi+3X_u+3X_d+3X_q+3X_\psi+3X_1+3X_2+\sum_{singl}X_{ls}=0,$$

$$[U(1)_X]^3{:}9X_\chi^3+3X_u^3+3X_d^3+3X_q^3+3X_\psi^3+3X_1^3+3X_2^3+\sum_{singl}X_{ls}^3=0,$$

where X_{ls} are the hypercharges of the right-handed charged lepton singlets needed in order to have a consistent field theory.

In order to fix the parameter b, let us use for q_L an exotic up type quark U of electric charge $Q_q=Q_U=2/3$. This fixes $b=1/2$, which implies $Q_l=-1$ and then l_L will be an exotic electron E_L^-. Then we have

$$\chi_L=\begin{pmatrix} d\\ u\\ U\end{pmatrix}_L, \quad \psi_L=\begin{pmatrix} \nu_e\\ e^-\\ E_1^-\end{pmatrix}_L.$$

In the former analysis we have used in the symmetry breaking chain $SU(3)_L\to SU(2)_L\otimes U(1)_Z$ the branching rule $3\to2(1/6)+1(-1/3)$, where the numbers in parenthesis are the new Z hypercharge values. Then, by using the electric charge generator in Eq. (1) that we now may write as $Q=T_{3L}+Z+X$, we have that $X_\chi=1/3$, $X_u=X_U=-2/3$ and $X_d=1/3$. For those values the anomaly $[SU(3)_c]^2U(1)_X$ is automatically canceled. Using the same argument we obtain $X_\psi=-2/3$. Then, the anomaly constraint equations imply $X_1+X_2=-1/3$, $\Sigma_{singl}X_{ls}^3+3X_1^3+3X_2^3=20/9$ and $\Sigma_{singl}X_{ls}=3$. By demanding for leptons of electric charges ±1 and 0 only, we must introduce, for the simplest solution, three equivalent singlets with hypercharges $X_{ls}=1$, $l=1,2,3$. Then $X_1=1/3$ and $X_2=-2/3$.

We thus end up with the following anomaly free multiplet structure:

$\chi_L = \begin{pmatrix} d \\ u \\ U \end{pmatrix}_L$	d_L^c	u_L^c	U_L^c
$\left(3,\bar{3},\frac{1}{3}\right)$	$\left(\bar{3},1,\frac{1}{3}\right)$	$\left(\bar{3},1,-\frac{2}{3}\right)$	$\left(\bar{3},1,-\frac{2}{3}\right)$

$\psi_L = \begin{pmatrix} \nu_e \\ e^- \\ E_1^- \end{pmatrix}_L$	$\psi_{1L} = \begin{pmatrix} E_2^+ \\ N_1^0 \\ \nu_e^c \end{pmatrix}_L$	$\psi_{2L} = \begin{pmatrix} N_2^0 \\ E_2^- \\ E_3^- \end{pmatrix}_L$	e_L^+	E_{1L}^+	E_{3L}^+
$\left(1,3,-\frac{2}{3}\right)$	$\left(1,3,\frac{1}{3}\right)$	$\left(1,3,-\frac{2}{3}\right)$	(1,1,1)	(1,1,1)	(1,1,1)

The numbers in parenthesis refer to the $[SU(3)_C, SU(3)_L, U(1)_X]$ quantum numbers, respectively.

B. $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ as an $SU(6)\otimes U(1)_X$ subgroup

In a model based on a local gauge group $SU(6)\otimes U(1)_\alpha$, the minimal choice of chiral multiplets that are free of the $[SU(6)]^3$ anomaly is $\bar{6}\oplus\bar{6}\oplus 15$ [10]. If we want to apply this group structure to the model in Sec. II A, three $SU(6)$ singlets with identical hypercharges α_1 must be introduced. The anomaly constraint equations read now

$$[SU(6)]^2 U(1)_\alpha : \alpha_6 + \alpha_6' + 4\alpha_{15} = 0,$$

$$[\mathrm{grav}]^2 U(1)_\alpha : 6\alpha_6 + 6\alpha_6' + 15\alpha_{15} + 3\alpha_1 = 0,$$

$$[U(1)_\alpha]^3 : 6\alpha_6^3 + 6\alpha'^3_6 + 15\alpha_{15}^3 + 3\alpha_1^3 = 0,$$

where α_6, α_6' and α_{15} are the $U(1)_\alpha$ hypercharges for the representations $\bar{6}$, $\bar{6}$ and 15, respectively. Since *a priori* there is no way to distinguish between the two $\bar{6}$ representations, we assume that $\alpha_6 = \alpha_6'$ and so the solution to the former set of equations is $\alpha_1 = 3\alpha_{15} = -\frac{3}{2}\alpha_6 \equiv \alpha$, where α is a free parameter.

Using for the symmetry breaking chain $SU(6)\to SU(3)_c \otimes SU(3)_L \otimes U(1)_\beta$ the branching rules $\bar{6}\to(\bar{3},1)(-\beta)+(1,3)(\beta)$ and $15\to(\bar{3},1)(2\beta)+(1,3)(-2\beta)+(3,\bar{3})(0)$ [11], we see immediately that the model in Sec. II A for the gauge group $SU(3)_C\otimes SU(3)_L\otimes U(1)_X$ is a subgroup of the gauge group $SU(6)\otimes U(1)_\alpha$ as far as we identify $\alpha = X = 1$. The particle content of the irreducible representations in $SU(6)$ are

$$\bar{6} = \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \\ \nu_e \\ e^- \\ E_1^- \end{pmatrix}_L, \quad \begin{pmatrix} U_1^c \\ U_2^c \\ U_3^c \\ N_2^0 \\ E_2^- \\ E_3^- \end{pmatrix}_L; \quad 15 = \begin{pmatrix} 0 & d_1^c & -d_2^c & d_1 & u_1 & U_1 \\ -d_1^c & 0 & d_3^c & d_2 & u_2 & U_2 \\ d_2^c & -d_3^c & 0 & d_3 & u_3 & U_3 \\ -d_1 & -d_2 & -d_3 & 0 & \nu_e^c & E_2^+ \\ -u_1 & -u_2 & -u_3 & -\nu_e^c & 0 & N_1^0 \\ -U_1 & -U_2 & -U_3 & -E_2^+ & -N_1^0 & 0 \end{pmatrix}_L;$$

$$1 = e_L^+, \qquad E_{1L}^+, \qquad E_{3L}^+.$$

Since our $SU(6)\otimes U(1)_X$ is not a subgroup of E_6, the model introduced here is not related to the superstring derived flipped $SU(6)$ model [12].

III. THE SCALAR SECTOR

Our aim is to break the symmetry following the pattern

$$SU(3)_c\otimes SU(3)_L\otimes U(1)_X \rightarrow SU(3)_c\otimes SU(2)_L\otimes U(1)_Y \rightarrow SU(3)_c\otimes U(1)_Q$$

and give, at the same time, masses to the fermion fields in the model. With this in mind we introduce the following two Higgs scalars: $\phi_1(1,3,1/3)$ with a vacuum expectation value (VEV) aligned in the direction $\langle\phi_1\rangle=(0,0,V)^T$ and $\phi_2(1,3,1/3)$ with a VEV aligned as $\langle\phi_2\rangle=(0,v/\sqrt{2},0)^T$, with the hierarchy $V>v\sim 250$ GeV (the electroweak breaking scale). The scale of V is going to be fixed phenomenologically in Sec. VI.

Note that ϕ_1 and ϕ_2 have the same quantum numbers but they get VEVs at different mass scales. This is necessary in order to give a large mass to the exotic quark and a realistic mass to the known up quark via a mixing with the exotic quark, as we will see in Sec. V.

One more Higgs scalar $\phi_3(1,3,-2/3)$ is needed in order to give a mass to the down quark field in the model but, as we will discuss in Sec. V, the most convenient VEV for this new scalar is zero ($\langle\phi_3\rangle=0$).

IV. THE GAUGE BOSON SECTOR

In the model there are a total of 17 gauge bosons: One gauge field B^μ associated with $U(1)_X$, the 8 gluon fields associated with $SU(3)_c$ which remain massless after breaking the symmetry, and other 8 associated with $SU(3)_L$ which, for $b=1/2$ in Eq. (1), can be written as

$$\frac{1}{2}\lambda_\alpha A^\mu_\alpha=\frac{1}{\sqrt{2}}\begin{pmatrix} D^\mu_1 & W^{+\mu} & K^{+\mu}\\ W^{-\mu} & D^\mu_2 & K^{0\mu}\\ K^{-\mu} & \bar{K}^{0\mu} & D^\mu_3\end{pmatrix},$$

where $D^\mu_1=A^\mu_3/\sqrt{2}+A^\mu_8/\sqrt{6}$, $D^\mu_2=-A^\mu_3/\sqrt{2}+A^\mu_8/\sqrt{6}$, and $D^\mu_3=-2A^\mu_8/\sqrt{6}$. λ_i, $i=1,2,\ldots,8$, are the eight Gell-Mann matrices normalized as mentioned in Sec. II A.

After breaking the symmetry with $\langle\phi_1\rangle+\langle\phi_2\rangle$ and using for the covariant derivative for triplets $D^\mu=\partial^\mu - i(g/2)\lambda_\alpha A^\mu_\alpha - ig'XB^\mu$, we get the following mass terms for the charged gauge bosons: $M^2_{W^\pm}=(g^2/4)v^2$, $M^2_{K^\pm}=(g^2/2)V^2$, and $M^2_{K^0(\bar{K}^0)}=(g^2/2)(V^2+v^2/2)$. For the three neutral gauge bosons we get mass terms of the form

$$M=V^2\left(\frac{g'B^\mu}{3}-\frac{gA^\mu_8}{\sqrt{3}}\right)^2+\frac{v^2}{8}\left(\frac{2g'B^\mu}{3}-gA^\mu_3+\frac{gA^\mu_8}{\sqrt{3}}\right)^2.$$

By diagonalizing M we get the physical neutral gauge bosons which are defined through the mixing angle θ and Z_μ, Z'_μ by

$$Z^\mu_1=Z_\mu\cos\theta+Z'_\mu\sin\theta,$$

$$Z^\mu_2=-Z_\mu\sin\theta+Z'_\mu\cos\theta,$$

where

$$\tan(2\theta)=-\frac{v^2(1-2S^2_W)\sqrt{3-4S^2_W}}{4C^4_W V^2-v^2(1-2S^4_W)}. \tag{3}$$

The photon field A^μ and the fields Z_μ and Z'_μ are given by

$$A^\mu=S_WA^\mu_3+C_W\left[\frac{T_W}{\sqrt{3}}A^\mu_8+(1-T^2_W/3)^{1/2}B^\mu\right],$$

$$Z^\mu=C_WA^\mu_3-S_W\left[\frac{T_W}{\sqrt{3}}A^\mu_8+(1-T^2_W/3)^{1/2}B^\mu\right],$$

$$Z'^\mu=-(1-T^2_W/3)^{1/2}A^\mu_8+\frac{T_W}{\sqrt{3}}B^\mu. \tag{4}$$

$S_W=\sqrt{3}g'/\sqrt{3g^2+4g'^2}$ and C_W are the sine and cosine of the electroweak mixing angle respectively and $T_W=S_W/C_W$. We can also identify the Y hypercharge associated with the SM gauge boson as

$$Y^\mu=\left[\frac{T_W}{\sqrt{3}}A^\mu_8+(1-T^2_W/3)^{1/2}B^\mu\right]. \tag{5}$$

A. Charged currents

The Hamiltonian for the charged currents can be written as

$$\begin{aligned}H^{CC}=\frac{g}{\sqrt{2}}[&W^+_\mu(-\overline{u_L}\gamma^\mu d_L+\overline{\nu_{eL}}\gamma^\mu e^-_L+\overline{N^0_{2L}}\gamma^\mu E^-_{2L}\\ &+\overline{E^+_{2L}}\gamma^\mu N^0_{1L})+K^+_\mu(-\overline{U_L}\gamma^\mu d_L+\overline{\nu_{eL}}\gamma^\mu E^-_{1L}\\ &+\overline{N^0_{2L}}\gamma^\mu E^-_{3L}+\overline{E^+_{2L}}\gamma^\mu \nu^c_{eL})+K^0_\mu(-\overline{U_L}\gamma^\mu u_L\\ &+\overline{N^0_{1L}}\gamma^\mu\nu^c_{eL}+\overline{E^-_{2L}}\gamma^\mu E^-_{3L}+\overline{e^-_L}\gamma^\mu E^-_{1L})]+\text{H.c.},\end{aligned} \tag{6}$$

which implies that the interactions with the $K^\pm$ and $K^0(\bar{K}^0)$ bosons violate the lepton number and the weak isospin. Notice also that the first two terms in the previous expression constitute the charged weak current of the SM as far as we identify $W^\pm$ as the $SU(2)_L$ charged left-handed weak bosons.

B. Neutral currents

The neutral currents $J_\mu(EM)$, $J_\mu(Z)$ and $J_\mu(Z')$ associated with the Hamiltonian $H^0 = eA^\mu J_\mu(EM) + (g/C_W)Z^\mu J_\mu(Z) + (g'/\sqrt{3})Z'^\mu J_\mu(Z')$ are

$$J_\mu(EM) = \frac{2}{3}\overline{u}\gamma_\mu u - \frac{1}{3}\overline{d}\gamma_\mu d + \frac{2}{3}\overline{U}\gamma_\mu U - \overline{e^-}\gamma_\mu e^- - \overline{E_1^-}\gamma_\mu E_1^- - \overline{E_2^-}\gamma_\mu E_2^- - \overline{E_3^-}\gamma_\mu E_3^-$$

$$= \sum_f q_f \overline{f}\gamma_\mu f,$$

$$J_\mu(Z) = J_{\mu,L}(Z) - S_W^2 J_\mu(EM),$$

$$J_\mu(Z') = T_W J_\mu(EM) - J_{\mu,L}(Z'), \qquad (7)$$

where $e = gS_W = g'C_W\sqrt{1-T_W^2/3} > 0$ is the electric charge, q_f is the electric charge of the fermion f in units of e, $J_\mu(EM)$ is the electromagnetic current, and the left-handed currents are

$$J_{\mu,L}(Z) = \frac{1}{2}(\overline{u_L}\gamma_\mu u_L - \overline{d_L}\gamma_\mu d_L + \overline{\nu_{eL}}\gamma_\mu \nu_{eL} - \overline{e_L^-}\gamma_\mu e_L^- + \overline{N_2^0}\gamma_\mu N_2^0 - \overline{E_2^-}\gamma_\mu E_2^-)$$

$$= \sum_f T_{3f}\overline{f_L}\gamma_\mu f_L,$$

$$\begin{aligned} J_{\mu,L}(Z') = {} & S_{2W}^{-1}(-\overline{d_L}\gamma_\mu d_L + \overline{\nu_{eL}}\gamma_\mu \nu_{eL} + \overline{E_{2L}^+}\gamma_\mu E_{2L}^+ + \overline{N_{2L}^0}\gamma_\mu N_{2L}^0) + T_{2W}^{-1}(-\overline{u_L}\gamma_\mu u_L + \overline{e_L^-}\gamma_\mu e_L^- \\ & + \overline{N_{1L}^0}\gamma_\mu N_{1L}^0 + \overline{E_{2L}^-}\gamma_\mu E_{2L}^-) - T_W^{-1}(-\overline{U_L}\gamma_\mu U_L + \overline{E_{1L}^-}\gamma_\mu E_{1L}^- + \overline{\nu_{eL}^c}\gamma_\mu \nu_{eL}^c + \overline{E_{3L}^-}\gamma_\mu E_{3L}^-) \\ = {} & \sum_f T_{9f}\overline{f_L}\gamma_\mu f_L, \end{aligned} \qquad (8)$$

where $S_{2W} = 2S_WC_W$, $T_{2W} = S_{2W}/C_{2W}$, $C_{2W} = C_W^2 - S_W^2$, $\overline{N_2^0}\gamma_\mu N_2^0 = \overline{N_{2L}^0}\gamma_\mu N_{2L}^0 + \overline{N_{2R}^0}\gamma_\mu N_{2R}^0 = \overline{N_{2L}^0}\gamma_\mu N_{2L}^0 - \overline{N_{2L}^{0c}}\gamma_\mu N_{2L}^{0c} = \overline{N_{2L}^0}\gamma_\mu N_{2L}^0 - \overline{N_{1L}^0}\gamma_\mu N_{1L}^0$, and similarly $\overline{E_2}\gamma_\mu E_2 = \overline{E_{2L}^-}\gamma_\mu E_{2L}^- - \overline{E_{2L}^+}\gamma_\mu E_{2L}^+$. In this way $T_{3f} = Dg(1/2,-1/2,0)$ is the third component of the weak isospin and $T_{9f} = Dg(S_{2W}^{-1}, T_{2W}^{-1}, -T_W^{-1})$ is a convenient 3×3 diagonal matrix, acting both of them on the representation 3 of $SU(3)_L$. Notice that $J_\mu(Z)$ is just the generalization of the neutral current present in the SM. This allows us to identify Z_μ as the neutral gauge boson of the SM, which is consistent with Eqs. (4) and (5).

The couplings of the mass eigenstates Z_1^μ and Z_2^μ are given by

$$\begin{aligned} H^{NC} &= \frac{g}{2C_W}\sum_{i=1}^{2} Z_i^\mu \sum_f \{\overline{f}\gamma_\mu[a_{iL}(f)(1-\gamma_5) + a_{iR}(f)(1+\gamma_5)]f\} \\ &= \frac{g}{2C_W}\sum_{i=1}^{2} Z_i^\mu \sum_f \{\overline{f}\gamma_\mu[g(f)_{iV} - g(f)_{iA}\gamma_5]f\}, \end{aligned}$$

where

$$a_{1L}(f) = \cos\theta(T_{3f} - q_f S_W^2) - \frac{g'\sin\theta C_W}{g\sqrt{3}}(T_{9f} - q_f T_W),$$

$$a_{1R}(f) = -q_f S_W\left(\cos\theta S_W - \frac{g'\sin\theta}{g\sqrt{3}}\right),$$

$$a_{2L}(f) = -\sin\theta(T_{3f} - q_f S_W^2) - \frac{g'\cos\theta C_W}{g\sqrt{3}}(T_{9f} - q_f T_W),$$

$$a_{2R}(f) = q_f S_W\left(\sin\theta S_W + \frac{g'\cos\theta}{g\sqrt{3}}\right) \qquad (9)$$

and

$$g(f)_{1V} = \cos\theta(T_{3f} - 2S_W^2 q_f) - \frac{g'\sin\theta}{g\sqrt{3}}(T_{9f}C_W - 2q_f S_W),$$

$$g(f)_{2V} = -\sin\theta(T_{3f} - 2S_W^2 q_f) - \frac{g'\cos\theta}{g\sqrt{3}}(T_{9f}C_W - 2q_f S_W),$$

$$g(f)_{1A} = \cos\theta T_{3f} - \frac{g'\sin\theta}{g\sqrt{3}}T_{9f}C_W,$$

$$g(f)_{2A} = -\sin\theta T_{3f} - \frac{g'\cos\theta}{g\sqrt{3}}T_{9f}C_W. \qquad (10)$$

The values of g_{iV}, g_{iA} with $i = 1,2$ are listed in Tables I and II. As we can see, in the limit $\theta = 0$ the couplings of Z_1^μ to the ordinary leptons and quarks are the same as in the SM, due to this we can test the new physics beyond the SM predicted by this particular model.

V. FERMION MASSES

The Higgs scalars introduced in Sec. III not only break the symmetry in an appropriate way, but produce the following mass terms for the fermions of the model.

A. Masses for the up quark sector

For the quark sector we can write the following Yukawa terms:

TABLE I. The $Z_1^\mu \to \overline{f}f$ couplings.

f	$g(f)_{1V}$	$g(f)_{1A}$
d	$\left(-\frac{1}{2}+\frac{2S_W^2}{3}\right)[\cos\theta-\sin\theta/(4C_W^2-1)^{1/2}]$	$-\frac{1}{2}(\cos\theta-\sin\theta/[2(4C_w^2-1)^{1/2}])$
u	$\cos\theta\left(\frac{1}{2}-\frac{4S_W^2}{3}\right)+\frac{\sin\theta}{(4C_W^2-1)^{1/2}}\left(\frac{1}{2}+\frac{S_W^2}{3}\right)$	$\frac{1}{2}\cos\theta+\sin\theta(1/2-S_W^2)/(4C_W^2-1)^{1/2}$
U	$-\frac{4S_W^2\cos\theta}{3}-\sin\theta\left(1-\frac{7}{3}S_W^2\right)/(4C_W^2-1)^{1/2}$	$-C_W^2\sin\theta/(4C_W^2-1)^{1/2}$
e^-	$\cos\theta\left(-\frac{1}{2}+2S_W^2\right)-\frac{\sin\theta}{(4C_W^2-1)^{1/2}}\left(\frac{1}{2}+S_W^2\right)$	$-\frac{\cos\theta}{2}-\frac{\sin\theta}{(4C_W^2-1)^{1/2}}\left(\frac{1}{2}-S_W^2\right)$
E_1^-,E_3^-	$2\cos\theta S_W^2+\frac{\sin\theta}{(4C_W^2-1)^{1/2}}(1-3S_W^2)$	$C_W^2\sin\theta/(4C_W^2-1)^{1/2}$
E_2^-	$\cos\theta(-1+2S_W^2)-\frac{S_W^2\sin\theta}{(4C_W^2-1)^{1/2}}$	$-C_W^2\sin\theta/(4C_W^2-1)^{1/2}$
ν_e,N_2^0	$\frac{1}{2}[\cos\theta-\sin\theta/(4C_W^2-1)^{1/2}]$	$\frac{1}{2}[\cos\theta-\sin\theta/(4C_W^2-1)^{1/2}]$
ν_e^c	$C_W^2\sin\theta/(4C_W^2-1)^{1/2}$	$C_W^2\sin\theta/(4C_W^2-1)^{1/2}$
N_1^0	$-\frac{1}{2}\cos\theta-\frac{\sin\theta}{(4C_W^2-1)^{1/2}}(1/2-S_W^2)$	$-\frac{1}{2}\cos\theta-\frac{\sin\theta}{(4C_W^2-1)^{1/2}}(1/2-S_W^2)$

$$\mathcal{L}_Y^Q=\chi_L^T C(h_u\phi_2 u_L^c+h_U\phi_1 U_L^c+h_{uU}\phi_2 U_L^c+h_{Uu}\phi_1 u_L^c) + \text{H.c.}, \tag{11}$$

where h_η, $\eta=u,U,uU,Uu$, are Yukawa couplings of order one and C is the charge conjugate operator. From Eq. (11) we get for the up quark sector a mass matrix in the basis $(u,U)_L$ of the form

$$M_{uU}=\begin{pmatrix} h_u v/\sqrt{2} & h_{uU}v/\sqrt{2} \\ h_{Uu}V & h_U V\end{pmatrix}. \tag{12}$$

For the particular case $h_u=h_{uU}=h_{Uu}=h_U\equiv h$, the mass eigenvalues of the previous matrix are $m_u=0$ and $m_U=h(V+v/\sqrt{2})$. Since there is not a physical reason for the Yukawa couplings to be equal, let us calculate the mass eigenvalues as a function of $|M_{uU}|\equiv(h_u h_U-h_{uU}h_{Uu})$ the determinant of the Yukawa couplings, and in the expansion v/V. The algebra shows that $m_U\simeq h_U V+h_u v/\sqrt{2}-|M_{uU}|v^2/\sqrt{2}Vh_U$ and $m_u\simeq v|M_{uU}|(1+h_{uU}h_{Uu}v/\sqrt{2}Vh_U^3)/\sqrt{2}h_U$, so a large mass is generated for the exotic up quark and for the other one, associated with the ordinary up quark, a mass lowered by the determinant $|M_{uU}|$ which we expect to be smaller than one.

B. Lepton masses

For the lepton sector we have the following Yukawa terms:

$$\begin{aligned}\mathcal{L}_Y^l=&\epsilon_{abc}[\psi_L^a C(h_1\psi_{1L}^b\phi_1^c+h_2\psi_{1L}^b\phi_2^c)+\psi_{1L}^a C(h_3\psi_{2L}^b\phi_1^c \\ &+h_4\psi_{2L}^b\phi_2^c)]+\psi_L^T C\phi_1^*(h_5 e_L^+ +h_6E_{1L}^+ +h_7E_{3L}^+) \\ &+\psi_{2L}^T C\phi_1^*(h_8 e_L^+ +h_9E_{1L}^+ +h_{10}E_{3L}^+)+\psi_L^T C\phi_2^*(h_{11}e_L^+ \\ &+h_{12}E_{1L}^+ +h_{13}E_{3L}^+)+\psi_{2L}^T C\phi_2^*(h_{14}e_L^+ +h_{15}E_{1L}^+ \\ &+h_{16}E_{3L}^+)+\text{H.c.},\end{aligned} \tag{13}$$

where a,b,c are $SU(3)_L$ tensor indices and the Yukawas are again of order one.

1. Masses for the neutral leptons

For the neutral leptons Eq. (13) produce, in the basis (ν_e,ν_e^c,N_1,N_2), the mass matrix

$$M_N=\begin{pmatrix} 0 & -h_2v/\sqrt{2} & h_1V & 0 \\ -h_2v/\sqrt{2} & 0 & 0 & h_4v/\sqrt{2} \\ h_1V & 0 & 0 & -h_3V \\ 0 & h_4v/\sqrt{2} & -h_3V & 0\end{pmatrix}. \tag{14}$$

The eigenvalues for this matrix are

$$m_1=\pm\sqrt{\frac{1}{2}(A+\sqrt{A^2-4D^2})},$$

$$m_2=\pm\sqrt{\frac{1}{2}(A-\sqrt{A^2-4D^2})},$$

where $A=V^2(h_1^2+h_3^2)+(v^2/2)(h_2^2+h_4^2)$ and $D^2=V^2v^2(h_1h_4-h_2h_3)/2$. These values imply for this model that there are two Dirac neutrinos for each family. For the particular case $D=0$ we find $m_1=\pm V\sqrt{(h_1^2+h_3^2)+(v^2/2V^2)(h_2^2+h_4^2)}$ and $m_2=0$, which means a massless Dirac neutrino and, in the limit $V\gg v$, a very massive one. In the case $D\neq 0$ and $|D|\ll|A|$, an expansion of $m_i, i=1,2$, in powers of D^2/A^2 gives $m_1=\pm\sqrt{A}(1-D^2/2A^2+\cdots)\approx\pm V\sqrt{h_1^2+h_3^2}$ and $m_2=\pm D/\sqrt{A}(1+D^2/2A^2+\cdots)\approx\pm v(h_1h_4-h_2h_3)/\sqrt{2(h_1^2+h_3^2)}$ (in the limit $V\gg v$), which means that the mass of the light neutrino is suppressed with respect to the scale v by the small value of $h_1h_4-h_2h_3\sim D$. Notice that $D=0$ if we impose in our model the symmetry $\phi_1\leftrightarrow\pm\phi_2$ which implies $h_1=\pm h_2$ and $h_3=\pm h_4$.

2. Masses for the charged leptons

For the charged leptons Eq. (13) produces, in the basis (e,E_1,E_2,E_3), the mass matrix

$$M_{eE}=\begin{pmatrix} h_{11}v/\sqrt{2} & h_{12}v/\sqrt{2} & -h_1V & h_{13}v/\sqrt{2} \\ h_5V & h_6V & h_2v/\sqrt{2} & h_7V \\ h_{14}v/\sqrt{2} & h_{15}v/\sqrt{2} & -h_3V & h_{16}v/\sqrt{2} \\ h_8V & h_9V & h_4v/\sqrt{2} & h_{10}V \end{pmatrix}, \tag{15}$$

and we must find the eigenvalues for the matrix $M_{eE}M_{eE}^T\equiv M^2(v,V)$.

Let us first consider the particular case $M^2(v=0,V)$ (see the Appendix). In this case we have first that $\det M^2(v=0,V)=0$, so, there is a zero mass eigenstate that we may identify as the electron (for the first family for example). Second, M^2 has three eigenvalues different from zero and of the order V^2, which means that at the first stage of the symmetry breaking chain

$$SU(3)_c\otimes SU(3)_L\otimes U(1)_X \xrightarrow{\langle\phi_1\rangle} SU(3)_c\otimes SU(2)_L\otimes U(1)_Y,$$

the three exotic charged leptons get heavy masses of order $V\gg v\sim 250$ GeV (the electroweak breaking mass scale).

Now, for $v\neq 0$ and by taking profit of $v/V\ll 1$, we can diagonalize $M^2(v,V)$ using matrix perturbation theory up to first order in the perturbation [13]. In the Appendix we show that, at this order, three eigenvalues of M^2 are of the order V^2 and the eigenvalue corresponding to the mass of the lightest charged lepton is given by

$$m_e^2\simeq\frac{h_1^2H^2+h_3^2J^2-2h_1h_3K^2}{2(h_1^2+h_3^2)}v^2,$$

where $H^2=h_{14}^2+h_{15}^2+h_{16}^2$, $J^2=h_{11}^2+h_{12}^2+h_{13}^2$ and $K^2=h_{11}h_{14}+h_{12}h_{15}+h_{13}h_{16}$. Notice that with Yukawa couplings of the order one $H^2\approx J^2\approx K^2$, and m_e is suppressed with respect to the scale v by a small difference of Yukawa couplings, namely h_1-h_3. It is worth noticing that the value of m_e is independent of the symmetry $\phi_1\leftrightarrow\pm\phi_2$ which makes zero the mass of the light neutrino.

TABLE II. The $Z_2^\mu\to\overline{f}f$ couplings.

	$g(f)_{2V}$	$g(f)_{2A}$
d	$\left(\frac{1}{2}-\frac{2S_W^2}{3}\right)[\sin\theta+\cos\theta/(4C_W^2-1)^{1/2}]$	$\frac{1}{2}(\sin\theta+\cos\theta/[2(4C_w^2-1)^{1/2}])$
u	$-\sin\theta\left(\frac{1}{2}-\frac{4S_W^2}{3}\right)+\frac{\cos\theta}{(4C_W^2-1)^{1/2}}\left(\frac{1}{2}+\frac{S_W^2}{3}\right)$	$-\frac{1}{2}\sin\theta+\cos\theta(1/2-S_W^2)/(4C_W^2-1)^{1/2}$
U	$\frac{4S_W^2\sin\theta}{3}-\cos\theta\left(1-\frac{7}{3}S_W^2\right)/(4C_W^2-1)^{1/2}$	$-C_W^2\cos\theta/(4C_W^2-1)^{1/2}$
e^-	$\sin\theta\left(-\frac{1}{2}+2S_W^2\right)-\frac{\cos\theta}{(4C_W^2-1)^{1/2}}\left(\frac{1}{2}+S_W^2\right)$	$\frac{\sin\theta}{2}-\frac{\cos\theta}{(4C_W^2-1)^{1/2}}\left(\frac{1}{2}-S_W^2\right)$
E_1^-,E_3^-	$-2\sin\theta S_W^2+\frac{\cos\theta}{(4C_W^2-1)^{1/2}}(1-3S_W^2)$	$C_W^2\cos\theta/(4C_W^2-1)^{1/2}$
E_2^-	$-\sin\theta(-1+2S_W^2)-\frac{S_W^2\cos\theta}{(4C_W^2-1)^{1/2}}$	$-C_W^2\cos\theta/(4C_W^2-1)^{1/2}$
ν_e,N_2^0	$-\frac{1}{2}[\sin\theta+\cos\theta/(4C_W^2-1)^{1/2}]$	$-\frac{1}{2}[\sin\theta+\cos\theta/(4C_W^2-1)^{1/2}]$
ν_e^c	$C_W^2\cos\theta/(4C_W^2-1)^{1/2}$	$C_W^2\cos\theta/(4C_W^2-1)^{1/2}$
N_1^0	$\frac{1}{2}\sin\theta-\frac{\cos\theta}{(4C_W^2-1)^{1/2}}(1/2-S_W^2)$	$\frac{1}{2}\sin\theta-\frac{\cos\theta}{(4C_W^2-1)^{1/2}}(1/2-S_W^2)$

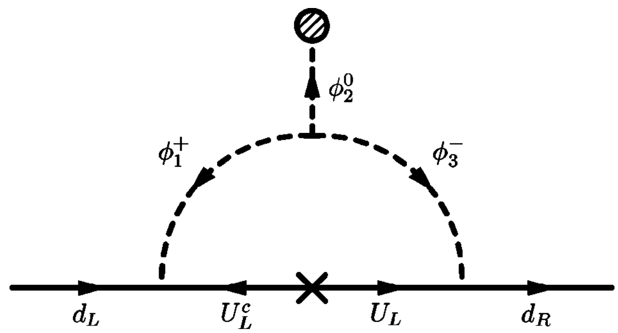

FIG. 1. A One-loop diagram contributing to the radiative generation of the down quark mass.

C. Mass for the down quark

The down quark does not get a mass at zero level. There are two alternatives if we want to produce a mass for this field: (1) introducing a third Higgs scalar $\phi_3(1,3,-2/3)$ with a VEV $\langle\phi_3\rangle=(v'/\sqrt{2},0,0)^T$, where $v'\sim v$, will immediately produce a tree level mass term $m_d=h_d v'/\sqrt{2}$; (2) introducing the same Higgs scalar $\phi_3(1,3,-2/3)$ but without a VEV ($\langle\phi_3\rangle=0$ as mentioned in Sec. III) produces a one-loop radiative mass for the down quark *d*. Figure 1 shows one of the diagrams contributing to this mass. A similar diagram is obtained by simultaneously replacing ϕ_1^+ by ϕ_2^+ and ϕ_2^0 by ϕ_1^0 in Fig. 1.

In both cases ϕ_3 does not introduce new mass terms for the leptons in the model. From the two alternatives discussed above, we prefer the second one since it is the only one generating a natural small mass value for the down quark *d*. For the first alternative we must explain either why $v'\ll v$ or why h_d, the Yukawa coupling for the down quark, is much less than one.

VI. CONSTRAINS ON THE $(Z^\mu-Z'^\mu)$ MIXING ANGLE AND THE Z_2^μ MASS

To get bounds on $\sin\theta$ and M_{Z_2} we use experimental parameters measured at the Z pole from CERN e^+e^- collider (LEP), SLAC Linear Collider (SLC) and data from atomic parity violation as given in Table III [14,15]. The expression for the partial decay width for Z_1^μ in two fermions is

$$\Gamma(Z_1^\mu\to f\bar{f})=\frac{N_C G_F M_{Z_1}^3}{6\pi\sqrt{2}}\rho\left[\frac{3\beta-\beta^3}{2}[g(f)_{1V}]^2+\beta^3[g(f)_{1A}]^2\right](1+\delta_f)R_{QCD+QED}, \qquad (16)$$

where f is an ordinary SM fermion with $m_f\leqslant M_{Z_1}/2$, Z_1^μ is the physical gauge boson observed at LEP, $N_C=3(1)$ is the number of colors of quarks (leptons), $R_{QCD+QED}$ are the QCD and QED corrections and $\beta=\sqrt{1-4m_b^2/M_{Z_1}^2}$ is a kinematic factor. The factor δ_f contains the one loop vertex contributions and it is zero for all fermions except for the bottom quark for which it can be written as $\delta_b\approx10^{-2}(-m_t^2/(2M_{Z_1}^2)+1/5)$ [16]. The ρ parameter has two contributions, one of them is the oblique correction given by $\delta\rho\approx3G_F m_t^2/(8\pi^2\sqrt{2})$ and the other one is the tree level contribution due to the $(Z_\mu-Z'_\mu)$ mixing and can be parametrized as $\delta\rho_V\approx(M_{Z_2}^2/M_{Z_1}^2-1)\sin^2\theta$. Finally, $g(f)_{1V}$ and $g(f)_{1A}$ are the coupling constants of the Z_1^μ field with ordinary fermions and they are listed in Table I. In the following we will use [15] $m_t=174.3$ GeV, $\alpha_s(m_Z)=0.1192$, $\alpha(m_Z)^{-1}=127.938$, and $S_W^2=0.2333$.

TABLE III. Experimental data and SM values for the parameters.

	Experimental results	SM
Γ_Z (GeV)	2.4952 ± 0.0023	2.4963 ± 0.0016
Γ(had) (GeV)	1.7444 ± 0.0020	1.7427 ± 0.0015
$\Gamma(l^+l^-)$ (MeV)	83.984 ± 0.086	84.018 ± 0.028
R_e	20.804 ± 0.050	20.743 ± 0.018
$A_{FB}(e)$	0.0145 ± 0.0025	0.0165 ± 0.0003
R_b	0.21653 ± 0.00069	0.21572 ± 0.00015
R_c	0.1709 ± 0.0034	0.1723 ± 0.0001
$A_{FB}(b)$	0.0990 ± 0.0020	0.1039 ± 0.0009
$A_{FB}(c)$	0.0689 ± 0.0035	0.0743 ± 0.0007
$A_{FB}(s)$	0.0976 ± 0.0114	0.1040 ± 0.0009
A_b	0.922 ± 0.023	0.9348 ± 0.0001
A_c	0.631 ± 0.026	0.6683 ± 0.0005
A_s	0.82 ± 0.13	0.9357 ± 0.0001
$A_e(\mathcal{P}_\tau)$	0.1498 ± 0.0048	0.1483 ± 0.0012
$Q_W(Cs)$	$-72.06\pm0.28\pm0.34$	-73.09 ± 0.04

The effective weak charge in atomic parity violation, Q_W, can be expressed as a function of the number of protons (Z) and the number of neutrons (N) in the atomic nucleus in the form

$$Q_W=-2[(2Z+N)c_{1u}+(Z+2N)c_{1d}], \qquad (17)$$

where $c_{1q}=2g(e)_{1A}g(q)_{1V}$. The theoretical value for Q_W is given by [17]

$$Q_W(^{133}_{55}C_s)=-73.09\pm0.04+\Delta Q_W. \qquad (18)$$

ΔQ_W, which includes the contributions of new physics, can be written as [18]

$$\Delta Q_W=\left[\left(1+4\frac{S_W^2}{1-2S_W^2}\right)-Z\right]\delta\rho_V+\Delta Q'_W. \qquad (19)$$

The term $\Delta Q'_W$ is model dependent and it can be obtained for our model by using $g(e)_{1A}$ and $g(q)_{1V}$ from Table I. The value we obtain is

$$\Delta Q'_W=(8.49Z+5.59N)\sin\theta-(3.10Z+2.62N)\frac{M_{Z_1}^2}{M_{Z_2}^2}. \qquad (20)$$

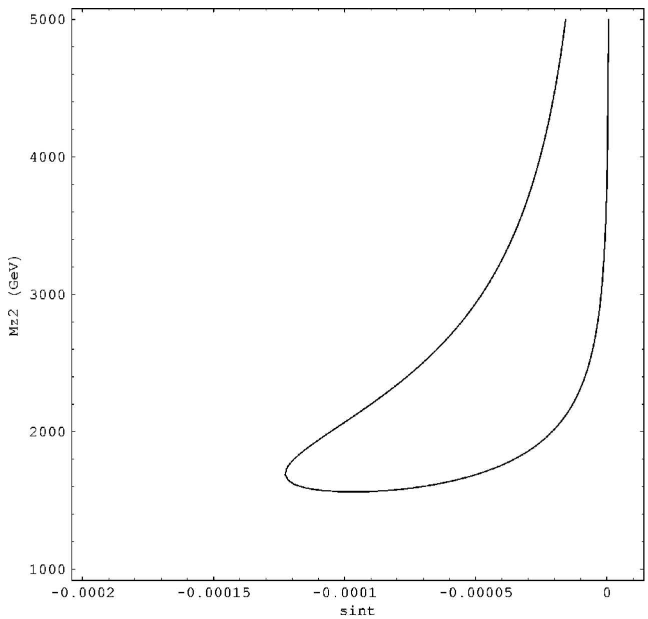

FIG. 2. Contour plot displaying the allowed region for sin θ vs M_{Z_2} at 95% C.L.

The discrepancy between the SM and the experimental data for ΔQ_W is given by [19]

$$\Delta Q_W = Q_W^{exp} - Q_W^{SM} = 1.03 \pm 0.44. \tag{21}$$

Introducing the expressions for Z pole observables in Eq. (16), with ΔQ_W in terms of new physics in Eq. (19) and using experimental data from LEP, SLC and atomic parity violation (see Table III), we do a χ^2 fit and we find the best allowed region for $\sin\theta$ vs M_{Z_2} at 95% C.L. In Fig. 2 we display the allowed region for $\sin\theta$ vs M_{Z_2} at 95% confidence level. The result is

$$-0.00015 \leqslant \sin\theta \leqslant 0,$$

$$1.5 \;\; \text{TeV} \leqslant M_{Z_2}, \tag{22}$$

which shows that the mass of the new neutral gauge boson is compatible with the bound obtained in $p\bar{p}$ collisions at the Fermilab Tevatron [20]. The negative value for θ obtained from phenomenology is consistent with the negative value on the front of Eq. (3), which by the way implies 60 TeV $\leqslant V < \infty$, where the ∞ upper limit is clearly seen from Fig. 2 ($\sin\theta \to 0$ when $M_{Z_2} \to \infty$).

VII. CONCLUDING REMARKS

We have presented an anomaly-free model based on the local gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ which is a subgroup of an electroweak-strong unification group $SU(6) \otimes U(1)_X$, does not studied in the literature so far. We break the gauge symmetry down to $SU(3)_c \otimes U(1)_Q$ and at the same time give masses to the fermion fields in the model in a consistent way by using three different Higgs scalars ϕ_i, $i=1,2,3$ with $\langle \phi_3 \rangle = 0$ and $\langle \phi_i \rangle \neq 0$, $i=1,2$ as stated in Sec. III. This Higgs fields and their VEVs set two different mass scales: $v \sim 250$ GeV$\ll V$, where $V \geqslant 60$ TeV. By using experimental results from LEP, SLC and atomic parity violation we bound the mixing angle and the mass of the additional neutral current to be $-0.00015 \leqslant \sin\theta < 0$ and 1.5 TeV$\leqslant M_{Z_2}$ at 95% C.L.

The mass spectrum of the fermion fields in the model arises as a consequence of the mixing between ordinary fermions and their exotic counterparts without the need of different scale Yukawa couplings. Conspicuously, the four neutral leptons in the model split as two Dirac neutrinos one heavy and the other one light.

Notice that all the new states are vector-like with respect to the SM quantum numbers. They consist of an isosinglet quark U of electric charge 2/3, a lepton isodoublet $(E_2^+, N_1^0)_L^T$ with ($N_{1L}^{0c} = N_{2L}^0$ in the weak basis), and two charged lepton isosinglets E_{1L}^+ and E_{3L}^+. In addition, a right-handed neutrino field is naturally included in the weak basis.

The model in Ref. [8] is also a one family model like the one presented here and uses the same value for the b parameter in the electric charge generator Q; so, the gauge boson content of both models is the same. Now, for the model in Ref. [8] there are 27 Weyl fields which are the same fields present in the fundamental representation of the electroweak-strong unification group E_6 [21]; in our model there are 30 Weyl fields which can be accommodated in the representations of a new electroweak-strong unification group $SU(6) \otimes U(1)_X$ which is not a subgroup of E_6, so the fermion content of the two models is different.

The low energy predictions of the two models are also different. Even though the low energy charged sector of both models looks alike, it is not so for the neutral sector. The model in Ref. [8] allows for a second neutral current associated with a mass as low as 600 GeV and predicts for each family the existence of a tiny mass Majorana neutrino and a Dirac neutrino with a mass of the order of the electroweak mass scale. For the model in this paper there is only a small mass Dirac neutrino in each family and the mass scale of the second neutral current is of the order of a few TeV. Also the scalar sector of the two models is quite different.

Finally we like to emphasize the way how the fermion masses in one family are generated in the model presented in this paper: First, all the exotic fermions get very large masses, while the lightest charged lepton acquires a mass at tree level of the order of $(h_1 - h_3)v$ (in the limit $V \gg v$), suppressed as a consequence of the mixing with exotic charged leptons. The up quark gets a mass at tree level suppressed also as a consequence of a mixing with a heavy exotic up type quark. The down quark gets its mass as a one-loop radiative correction. Finally there is a Dirac neutrino in each family with a mass suppressed by differences of products of Yukawa couplings. This unfamiliar way to generate masses might lead to explanations for the major trends in the masses of the fermions within one family (the first family for example).

ACKNOWLEDGMENTS

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APPENDIX

In this appendix we diagonalize perturbatively the mass matrix M_{eE} for the charged leptons which appears in Sec. V B 2. This matrix can be written as

$$M_{eE}=V\begin{pmatrix} h_{11}q & h_{12}q & -h_1 & h_{13}q \\ h_5 & h_6 & h_2q & h_7 \\ h_{14}q & h_{15}q & -h_3 & h_{16}q \\ h_8 & h_9 & h_4q & h_{10} \end{pmatrix},$$

where $q=v/(\sqrt{2}V)\ll 1$.

The matrix $M^2(v,V)=M_{eE}M_{eE}^T$ can be separated as $M^2=V^2[M_0^2+M^2(q)]$, where $V^2M_0^2=M^2(v=0,V)$ and

$$M_0^2=\begin{pmatrix} h_1^2 & 0 & h_1h_3 & 0 \\ 0 & A^2 & 0 & B^2 \\ h_1h_3 & 0 & h_3^2 & 0 \\ 0 & B^2 & 0 & C^2 \end{pmatrix},$$

$$M^2(q)=\begin{pmatrix} J^2q^2 & D^2q & K^2q^2 & E^2q \\ D^2q & h_2^2q^2 & F^2q & h_2h_4q^2 \\ K^2q^2 & F^2q & H^2q^2 & G^2q \\ E^2q & h_2h_4q^2 & G^2q & h_4^2q^2 \end{pmatrix}.$$

In M_0^2 and $M^2(q)$ the parameters are related to the Yukawas in the following way:

$$A^2=h_5^2+h_6^2+h_7^2,$$

$$B^2=h_5h_8+h_6h_9+h_7h_{10},$$

$$C^2=h_8^2+h_9^2+h_{10}^2,$$

$$D^2=-h_1h_2+h_5h_{11}+h_6h_{12}+h_7h_{13},$$

$$E^2=-h_1h_4+h_8h_{11}+h_9h_{12}+h_{10}h_{13},$$

$$F^2=-h_2h_3+h_5h_{14}+h_6h_{15}+h_7h_{16},$$

$$G^2=-h_3h_4+h_8h_{14}+h_9h_{15}+h_{10}h_{16},$$

$$H^2=h_{14}^2+h_{15}^2+h_{16}^2,$$

$$J^2=h_{11}^2+h_{12}^2+h_{13}^2,$$

$$K^2=h_{11}h_{14}+h_{12}h_{15}+h_{13}h_{16}.$$

The eigenvalues of the matrix $V^2M_0^2$ are

$$m_{01}^2=0,$$

$$m_{02}^2=(h_1^2+h_3^2)V^2,$$

$$m_{03}^2=(A^2+C^2-\sqrt{A^4+4B^4+C^4-2A^2C^2})V^2/2,$$

$$m_{04}^2=(A^2+C^2+\sqrt{A^4+4B^4+C^4-2A^2C^2})V^2/2.$$

We now rotate $M^2(q)$ using the matrix of eigenvectors of M_0^2. After the algebra is done we get, at first order in the expansion v^2/V^2, the following four eigenvalues:

$$m_1^2=[(h_1^2H^2+h_3^2J^2-2h_1h_3K^2)/[2(h_1^2+h_3^2)]]v^2,$$

$$m_2^2=m_{02}^2+O(v^2),$$

$$m_3^2=m_{03}^2+O(v^2),$$

$$m_4^2=m_{04}^2+O(v^2),$$

where m_1 corresponds to the mass of the lightest charged lepton and $O(v^2)$ stands for corrections of order v^2 to the squared masses of the exotic charged leptons.

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