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Entropies and Fisher Information Matrix for Extended Beta Distribution

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Abstract

The extended beta type 1 distribution has the probability density function proportional to $x^{\alpha-1}(1-x)^{\beta-1}\exp[-\sigma/x(1-x)]$, $0<x<1$. In this article, we derive the Fisher information matrix and entropies such as Rényi and Shannon for the extended beta type 1 distribution.

Msthematics Subject Classification: 33E99, 60E05

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1 Introduction

The random variable X is said to have an extended beta type 1 distribution, denoted by $X \sim \text{EB1}(\alpha,\beta;\sigma)$, if its probability density function(p.d.f.) is given by (Chaudhry et al. [2]),

$$f_{\text{EB1}}(x;\alpha,\beta,\sigma)=\frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta;\sigma)}\exp\left[-\frac{\sigma}{x(1-x)}\right],\quad 0<x<1, \tag{1}$$

where $\sigma>0$, and $B(p,q;\sigma)$ is the extended beta function defined by (Chaudhry et al. [2], Miller [4])

$$B(p,q;\sigma)=\int_0^1 t^{p-1}(1-t)^{q-1}\exp\left[-\frac{\sigma}{t(1-t)}\right]dt, \tag{2}$$

where $-\infty < p, q < \infty$ and $\text{Re}(\sigma) > 0$. For $\sigma = 0$ we must have $p > 0$, $q > 0$ and in this case the extended beta function reduces to the Euler's beta function. Further, replacing t by $1-t$ in (2), one can see that $B(p, q; \sigma) = B(q, p; \sigma)$. The rationale and justification for introducing this function are given in Chaudhry et al. [2] where several properties and a statistical application have also been studied. Miller [4] further studied this function and has given several additional results. The extended beta function has been used by Morán-Vásquez and Nagar [5] to express the density function of the product of two independent Kummer-gamma variables. Recently, Nagar, Morán-Vásquez and Gupta [7] have studied several properties of the extended beta distribution. A matrix variate generalization of the extended beta function is available in Nagar, Roldán-Correa and Gupta [6]. The extended matrix variate beta distribution has been studied by Nagar and Roldán-Correa [8].

In this article, we derive the Fisher information matrix, Rényi and Shannon entropies for the extended beta type 1 distribution. The distributions of the product of two independent random variables when at least one of them is extended beta type 1 are available in Nagar, Zarrazola and Sánchez [9].

2 Some Definitions and Preliminary Results

In this section we give some definitions and preliminary results which are used in subsequent sections.

Suppose Ω is a set with typical element ω, and and let A be a subset of Ω. The indicator function of A, denoted by $\mathbf{1}_A(\cdot)$, is defined by

$$\mathbf{1}_A(\omega) = \begin{cases} 1 & \text{if } \omega \in A, \\ 0 & \text{if } \omega \notin A. \end{cases}$$

The digamma function, denoted by $\psi(\cdot)$, is defined as the logarithmic derivative of the gamma function. That is

$$\psi(z) = \frac{d}{dz} \ln \Gamma(z).$$

The trigamma function, denoted $\psi_1(\cdot)$, is the derivative of the digamma function. That is

$$\psi_1(z) = \frac{d}{dz}\psi(z) = \frac{d^2}{dz^2} \ln \Gamma(z).$$

The polygamma function of order m, denoted by $\psi^{(m)}(\cdot)$, is defined as the $(m+1)$-th derivative of the logarithm of the gamma function. That is

$$\psi^{(m)}(z) = \frac{d^m}{dz^m}\psi(z) = \frac{d^{m+1}}{dz^{m+1}} \ln \Gamma(z).$$

For $m = 0$, the polygamma function reduces to a psi function and for $m = 1$ it slides to a trigamma function. For results and properties of these function the reader is referred to Askey and Roy [1].

The Laguerre polynomial of degree n is defined by the sum (Gradshteyn and Ryzhik [3, Sec. 8.97]),

$$L_n(x) = \sum_{k=0}^{n} \frac{(-1)^k}{k!} \binom{n}{k} x^k,$$

where $\binom{n}{k}$ is the binomial coefficient. The first few Laguerre polynomials are $L_0(x) = 1$, $L_1(x) = -x+1$, $L_2(x) = (x^2 - 4x + 2)/2$ and $L_3(x) = (-x^3 + 9x^2 - 18x + 6)/6$.

The r-th order derivative of the the Laguerre polynomial $L_n(x)$ is given by

$$\frac{d^r}{dx^r} L_n(x) = (-1)^r L_{n-r}^{(r)}(x)$$

where $L_n^{(\alpha)}(x)$ is the generalized Laguerre polynomial of degree n defined by the sum

$$L_n^{(\alpha)}(x) = \sum_{k=0}^{n} \frac{(-1)^k}{k!} \binom{n+\alpha}{n-k} x^k.$$

The generating function for the Laguerre polynomials is given by

$$\frac{\exp[-xt/(1-t)]}{1-t} = \sum_{n=0}^{\infty} t^n L_n(x), \quad |t| < 1. \tag{3}$$

Replacing $\exp(-\sigma/t)$ and $\exp[-\sigma/(1-t)]$ by their respective series expansions involving Laguerre polynomials by using (3), namely,

$$\exp\left(-\frac{\sigma}{t}\right) = \exp(-\sigma) t \sum_{n=0}^{\infty} L_n(\sigma)(1-t)^n, \quad |t| < 1,$$

and

$$\exp\left(-\frac{\sigma}{1-t}\right) = \exp(-\sigma)(1-t) \sum_{m=0}^{\infty} L_m(\sigma) t^m, \quad |t| < 1,$$

in (2) and integrating t by using beta integral, Miller [4, Eq. 2.3] has given an an alternative representation for $B(p, q; \sigma)$ as

$$B(p, q; \sigma) = \exp(-2\sigma) \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} L_m(\sigma) L_n(\sigma) B(p+m+1, q+n+1), \tag{4}$$

where $p > -1$ and $q > -1$. Now, differentiating (4) appropriately, it is not difficult to observe that

$$\frac{\partial B(p,q;\sigma)}{\partial p} = \exp(-2\sigma)\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} L_m(\sigma)L_n(\sigma)B(p+m+1,q+n+1) \times [\psi(p+m+1)-\psi(p+q+m+n+2)],$$

$$\frac{\partial B(p,q;\sigma)}{\partial q} = \exp(-2\sigma)\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} L_m(\sigma)L_n(\sigma)B(p+m+1,q+n+1) \times [\psi(q+n+1)-\psi(p+q+m+n+2)],$$

$$\frac{\partial B(p,q;\sigma)}{\partial \sigma} = -\exp(-2\sigma)\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} L_m(\sigma)L_n(\sigma)B(p+m+1,q+n+1) \times \left[2+\frac{L_{m-1}^{(1)}(\sigma)}{L_m(\sigma)}\mathbf{1}_{\mathbb{N}_1}(m)+\frac{L_{n-1}^{(1)}(\sigma)}{L_n(\sigma)}\mathbf{1}_{\mathbb{N}_1}(n)\right],$$

$$\frac{\partial^2 B(p,q;\sigma)}{\partial p^2} = \exp(-2\sigma)\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} L_m(\sigma)L_n(\sigma)B(p+m+1,q+n+1) \times [\{\psi(p+m+1)-\psi(p+q+m+n+2)\}^2 + \psi_1(p+m+1)-\psi_1(p+q+m+n+2)],$$

$$\frac{\partial^2 B(p,q;\sigma)}{\partial q^2} = \exp(-2\sigma)\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} L_m(\sigma)L_n(\sigma)B(p+m+1,q+n+1) \times [\{\psi(q+n+1)-\psi(p+q+m+n+2)\}^2 + \psi_1(q+n+1)-\psi_1(p+q+m+n+2)],$$

$$\frac{\partial^2 B(p,q;\sigma)}{\partial \sigma^2} = \exp(-2\sigma)\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} L_m(\sigma)L_n(\sigma)B(p+m+1,q+n+1) \times \Bigg[4+4\frac{L_{m-1}^{(1)}(\sigma)}{L_m(\sigma)}\mathbf{1}_{\mathbb{N}_1}(m)+4\frac{L_{n-1}^{(1)}(\sigma)}{L_n(\sigma)}\mathbf{1}_{\mathbb{N}_1}(n) + 2\frac{L_{m-1}^{(1)}(\sigma)L_{n-1}^{(1)}(\sigma)}{L_m(\sigma)L_n(\sigma)}\mathbf{1}_{\mathbb{N}_1}(m)\mathbf{1}_{\mathbb{N}_1}(n) + \frac{L_{m-2}^{(2)}(\sigma)}{L_m(\sigma)}\mathbf{1}_{\mathbb{N}_2}(m)+\frac{L_{n-2}^{(2)}(\sigma)}{L_n(\sigma)}\mathbf{1}_{\mathbb{N}_2}(n)\Bigg],$$

$$\frac{\partial^2 B(p,q;\sigma)}{\partial q\,\partial p} = \exp(-2\sigma)\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} L_m(\sigma)L_n(\sigma)B(p+m+1,q+n+1) \times [\{\psi(p+m+1)-\psi(p+q+m+n+2)\} \times \{\psi(q+n+1)-\psi(p+q+m+n+2)\} - \psi_1(p+q+m+n+2)],$$

$$\frac{\partial^2 B(p,q;\sigma)}{\partial \sigma\,\partial p} = -\exp(-2\sigma)\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} L_m(\sigma)L_n(\sigma)B(p+m+1,q+n+1) \times \{\psi(p+m+1)-\psi(p+q+m+n+2)\} \times \left[2+\frac{L_{m-1}^{(1)}(\sigma)}{L_m(\sigma)}\mathbf{1}_{\mathbb{N}_1}(m)+\frac{L_{n-1}^{(1)}(\sigma)}{L_n(\sigma)}\mathbf{1}_{\mathbb{N}_1}(n)\right],$$

$$\frac{\partial^2 B(p,q;\sigma)}{\partial \sigma\,\partial q} = -\exp(-2\sigma)\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} L_m(\sigma)L_n(\sigma)B(p+m+1,q+n+1) \times \{\psi(q+n+1)-\psi(p+q+m+n+2)\} \times \left[2+\frac{L_{m-1}^{(1)}(\sigma)}{L_m(\sigma)}\mathbf{1}_{\mathbb{N}_1}(m)+\frac{L_{n-1}^{(1)}(\sigma)}{L_n(\sigma)}\mathbf{1}_{\mathbb{N}_1}(n)\right],$$

where

$$\mathbb{N}_j = \{j, j+1, \ldots\}.$$

3 Entropies

In this section, exact forms of Renyi and Shannon entropies are determined for the extended beta type 1 distribution.

Let $(\mathcal{X},\mathcal{B},\mathcal{P})$ be a probability space. Consider a p.d.f. f associated with $\mathcal{P}$, dominated by $\sigma-$finite measure μ on $\mathcal{X}$. Denote by $H_{SH}(f)$ the well-known Shannon entropy introduced in Shannon [12]. It is define by

$$H_{SH}(f) = -\int_{\mathcal{X}} f(x)\ln f(x)\,d\mu. \tag{5}$$

One of the main extensions of the Shannon entropy was defined by Rényi [11]. This generalized entropy measure is given by

$$H_R(\eta, f) = \frac{\ln G(\eta)}{1-\eta} \qquad (\text{for } \eta > 0 \text{ and } \eta \neq 1), \tag{6}$$

where

$$G(\eta) = \int_{\mathcal{X}} f^{\eta} d\mu.$$

The additional parameter η is used to describe complex behavior in probability models and the associated process under study. Rényi entropy is monotonically decreasing in η, while Shannon entropy (5) is obtained from (6) for $\eta \uparrow 1$. For details see Nadarajah and Zografos [10], Zografos and Nadarajah [14] and Zografos [13].

Lemma 3.1. *Let* $g(\alpha, \beta, \sigma) = \lim_{\eta \to 1} h(\eta)$, *where*

$$h(\eta) = \frac{\mathrm{d}}{\mathrm{d}\eta} B\left(\eta(\alpha-1)+1, \eta(\beta-1)+1; \eta\sigma\right).$$

Then,

$$\begin{aligned} g(\alpha, \beta, \sigma) = \exp(-2\sigma) \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} & L_m(\sigma) L_n(\sigma) B(\alpha+m+1, \beta+n+1) \\ \times \Bigg[& -\frac{L_{m-1}^{(1)}(\sigma)}{L_m(\sigma)} \mathbf{1}_{\mathbb{N}_1}(m) - \frac{L_{n-1}^{(1)}(\sigma)}{L_n(\sigma)} \mathbf{1}_{\mathbb{N}_1}(n) - 2\sigma \\ & + (\alpha-1)\psi(\alpha+m+1) + (\beta-1)\psi((\beta+n+1) \\ & - (\alpha+\beta-2)\psi(\alpha+\beta+m+n+2) \Bigg], \end{aligned} \tag{7}$$

where $\psi(\cdot)$ *is the digamma function.*

Proof. By using (4), we expand $B\left(\eta(\alpha-1)+1, \eta(\beta-1)+1; \eta\sigma\right)$ in series involving Laguerre polynomials as

$$h(\eta) = \frac{\mathrm{d}}{\mathrm{d}\eta} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \Delta_{m,n}(\eta) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left[\frac{\mathrm{d}}{\mathrm{d}\eta} \Delta_{m,n}(\eta) \right], \tag{8}$$

where

$$\Delta_{m,n}(\eta) = \exp(-2\eta\sigma) L_m(\eta\sigma) L_m(\eta\sigma) \frac{\Gamma[\eta(\alpha-1)+m+2]\Gamma[\eta(\beta-1)+n+2]}{\Gamma[\eta(\alpha+\beta-2)+m+n+4]}.$$

Now, differentiating the logarithm of $\Delta_{m,n}(\eta)$ w.r.t. to η, one obtains

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}\eta} \Delta_{m,n}(\eta) = \Delta_{m,n}(\eta) \Bigg[& -\frac{\eta L_{m-1}^{(1)}(\eta\sigma)}{L_m(\eta\sigma)} \mathbf{1}_{\mathbb{N}_1}(m) - \frac{\eta L_{n-1}^{(1)}(\eta\sigma)}{L_n(\eta\sigma)} \mathbf{1}_{\mathbb{N}_1}(n) - 2\sigma \\ & + (\alpha-1)\psi(\eta(\alpha-1)+m+2) + (\beta-1)\psi(\eta(\beta-1)+n+2) \\ & - (\alpha+\beta-2)\psi(\eta(\alpha+\beta-2)+m+n+4) \Bigg]. \end{aligned} \tag{9}$$

Finally, substituting (9) in (8) and taking $\eta \to 1$, one obtains the desired result. □

Now, we derive the Rényi and the Shannon entropies for the extended beta type 1 distribution.

Theorem 3.1. *For the extended beta type 1 distribution defined by the p.d.f. (1), the Rényi and the Shannon entropies are given by*

$$H_R(\eta, f) = \frac{1}{1-\eta}\left[\ln B(\eta(\alpha-1)+1, \eta(\beta-1)+1; \eta\sigma) - \eta \ln B(\alpha, \beta; \sigma)\right] \quad (10)$$

and

$$H_{SH}(f) = -\frac{g(\alpha,\beta,\sigma)}{B(\alpha,\beta;\sigma)} + \ln B(\alpha,\beta;\sigma), \quad (11)$$

respectively, where $g(\alpha,\beta,\sigma)$ *is given by (7).*

Proof. For $\eta > 0$ and $\eta \neq 1$, using the p.d.f. of X given by (1), we have

$$\begin{aligned} G(\eta) &= \int_0^1 [f_{\mathrm{EB1}}(x;\alpha,\beta,\sigma)]^\eta \,\mathrm{d}x \\ &= \frac{1}{[B(\alpha,\beta;\sigma)]^\eta}\int_0^1 x^{\eta(\alpha-1)}(1-x)^{\eta(\beta-1)} \exp\left[-\frac{\eta\sigma}{x(1-x)}\right] \mathrm{d}x \\ &= \frac{B(\eta(\alpha-1)+1, \eta(\beta-1)+1; \eta\sigma)}{[B(\alpha,\beta;\sigma)]^\eta}, \end{aligned}$$

where the last line has been obtained by using (2). Now, taking logarithm of $G(\eta)$ and using (6), we get (10). The Shannon entropy (11) is obtained from (10) by taking $\eta \uparrow 1$ and using L'Hopital's rule. □

4 Fisher Information Matrix

In this section we calculate the Fisher information matrix for the extended beta type 1 distribution. The information matrix plays a significant role in statistical inference in connection with estimation, sufficiency and properties of variances of estimators. For a given observation x, the Fisher information matrix for the extended beta type 1 distribution is defined as

$$-\begin{bmatrix} E\left(\frac{\partial^2 \ln L(\alpha,\beta,\sigma)}{\partial\alpha^2}\right) & E\left(\frac{\partial^2 \ln L(\alpha,\beta,\sigma)}{\partial\beta\partial\alpha}\right) & E\left(\frac{\partial^2 \ln L(\alpha,\beta,\sigma)}{\partial\sigma\,\partial\alpha}\right) \\ E\left(\frac{\partial^2 \ln L(\alpha,\beta,\sigma)}{\partial\beta\,\partial\alpha}\right) & E\left(\frac{\partial^2 \ln L(\alpha,\beta,\sigma)}{\partial\beta^2}\right) & E\left(\frac{\partial^2 \ln L(\alpha,\beta,\sigma)}{\partial\beta\,\partial\sigma}\right) \\ E\left(\frac{\partial^2 \ln L(\alpha,\beta,\sigma)}{\partial\sigma\,\partial\alpha}\right) & E\left(\frac{\partial^2 \ln L(\alpha,\beta,\sigma)}{\partial\sigma\partial\beta}\right) & E\left(\frac{\partial^2 \ln L(\alpha,\beta,\sigma)}{\partial\sigma^2}\right) \end{bmatrix},$$

where $L(\alpha, \beta, \sigma) = \ln f_{\mathrm{EB1}}(x; \alpha, \beta, \sigma)$. From (1), the natural logarithm of $L(\alpha, \beta, \sigma)$ is obtained as

$$\ln L(\alpha, \beta, \sigma) = -\ln B(\alpha, \beta; \sigma) + (\alpha - 1)\ln x + (\beta - 1)\ln(1 - x) - \frac{\sigma}{x(1 - x)},$$

where $0 < x < 1$.

Now, noting that the expected value of a constant is the constant itself and observing that all the second order partial derivatives of $\ln L(\alpha, \beta, \sigma)$ are constants, we have

$$\begin{aligned} E\left(\frac{\partial^2 \ln L(\alpha, \beta, \sigma)}{\partial \alpha^2}\right) &= \frac{\partial^2 \ln L(\alpha, \beta, \sigma)}{\partial \alpha^2} \\ &= \frac{1}{[B(\alpha, \beta; \sigma)]^2}\left[\frac{\partial B(\alpha, \beta; \sigma)}{\partial \alpha}\right]^2 - \frac{1}{B(\alpha, \beta; \sigma)}\frac{\partial^2 B(\alpha, \beta; \sigma)}{\partial \alpha^2}, \end{aligned}$$

$$\begin{aligned} E\left(\frac{\partial^2 \ln L(\alpha, \beta, \sigma)}{\partial \beta^2}\right) &= \frac{\partial^2 \ln L(\alpha, \beta, \sigma)}{\partial \beta^2} \\ &= \frac{1}{[B(\alpha, \beta; \sigma)]^2}\left[\frac{\partial B(\alpha, \beta; \sigma)}{\partial \beta}\right]^2 - \frac{1}{B(\alpha, \beta; \sigma)}\frac{\partial^2 B(\alpha, \beta; \sigma)}{\partial \beta^2}, \end{aligned}$$

$$\begin{aligned} E\left(\frac{\partial^2 \ln L(\alpha, \beta, \sigma)}{\partial \sigma^2}\right) &= \frac{\partial^2 \ln L(\alpha, \beta, \sigma)}{\partial \sigma^2} \\ &= \frac{1}{[B(\alpha, \beta; \sigma)]^2}\left[\frac{\partial B(\alpha, \beta; \sigma)}{\partial \sigma}\right]^2 - \frac{1}{B(\alpha, \beta; \sigma)}\frac{\partial^2 B(\alpha, \beta; \sigma)}{\partial \sigma^2}, \end{aligned}$$

$$\begin{aligned} E\left(\frac{\partial^2 \ln L(\alpha, \beta, \sigma)}{\partial \alpha\, \partial \beta}\right) &= \frac{\partial^2 \ln L(\alpha, \beta, \sigma)}{\partial \alpha\, \partial \beta} \\ &= \frac{1}{[B(\alpha, \beta; \sigma)]^2}\left[\frac{\partial B(\alpha, \beta; \sigma)}{\partial \alpha}\right]\left[\frac{\partial B(\alpha, \beta; \sigma)}{\partial \beta}\right] \\ &\quad - \frac{1}{B(\alpha, \beta; \sigma)}\frac{\partial^2 B(\alpha, \beta; \sigma)}{\partial \alpha\, \partial \beta}, \end{aligned}$$

$$\begin{aligned} E\left(\frac{\partial^2 \ln L(\alpha, \beta, \sigma)}{\partial \alpha\, \partial \sigma}\right) &= \frac{\partial^2 \ln L(\alpha, \beta, \sigma)}{\partial \alpha\, \partial \sigma} \\ &= \frac{1}{[B(\alpha, \beta; \sigma)]^2}\left[\frac{\partial B(\alpha, \beta; \sigma)}{\partial \alpha}\right]\left[\frac{\partial B(\alpha, \beta; \sigma)}{\partial \sigma}\right] \\ &\quad - \frac{1}{B(\alpha, \beta; \sigma)}\frac{\partial^2 B(\alpha, \beta; \sigma)}{\partial \alpha\, \partial \sigma}, \end{aligned}$$

$$
\begin{aligned}
E\left(\frac{\partial^2 \ln L(\alpha,\beta,\sigma)}{\partial\beta\,\partial\sigma}\right) &= \frac{\partial^2 \ln L(\alpha,\beta,\sigma)}{\partial\beta\,\partial\sigma} \\
&= \frac{1}{[B(\alpha,\beta;\sigma)]^2}\Big[\frac{\partial B(\alpha,\beta;\sigma)}{\partial\beta}\Big]\Big[\frac{\partial B(\alpha,\beta;\sigma)}{\partial\sigma}\Big] \\
&\quad - \frac{1}{B(\alpha,\beta;\sigma)}\frac{\partial^2 B(\alpha,\beta;\sigma)}{\partial\beta\,\partial\sigma}.
\end{aligned}
$$

Finally, substituting explicit expressions for the first and second order partial derivatives of $B(\alpha,\beta;\sigma)$ by using results given in Section 2, we obtain

$$
\begin{aligned}
E\left(\frac{\partial^2 \ln L(\alpha,\beta,\sigma)}{\partial\alpha^2}\right) &= \frac{\exp(-4\sigma)}{[B(\alpha,\beta;\sigma)]^2}\Big[\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} L_m(\sigma)L_n(\sigma)B(\alpha+m+1,\beta+n+1) \\
&\qquad \times [\psi(\alpha+m+1)-\psi(\alpha+\beta+m+n+2)]\Big]^2 \\
&\quad - \frac{\exp(-2\sigma)}{B(\alpha,\beta;\sigma)}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} L_m(\sigma)L_n(\sigma)B(\alpha+m+1,\beta+n+1) \\
&\qquad \times [\{\psi(\alpha+m+1)-\psi(\alpha+\beta+m+n+2)\}^2 \\
&\qquad + \psi_1(\alpha+m+1)-\psi_1(\alpha+\beta+m+n+2)], \\
E\left(\frac{\partial^2 \ln L(\alpha,\beta,\sigma)}{\partial\beta^2}\right) &= \frac{\exp(-4\sigma)}{[B(\alpha,\beta;\sigma)]^2}\Big[\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} L_m(\sigma)L_n(\sigma)B(\alpha+m+1,\beta+n+1) \\
&\qquad \times [\psi(\beta+n+1)-\psi(\alpha+\beta+m+n+2)]\Big]^2 \\
&\quad - \frac{\exp(-2\sigma)}{B(\alpha,\beta;\sigma)}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} L_m(\sigma)L_n(\sigma)B(\alpha+m+1,\beta+n+1) \\
&\qquad \times [\{\psi(\beta+n+1)-\psi(\alpha+\beta+m+n+2)\}^2 \\
&\qquad + \psi_1(\beta+n+1)-\psi_1(\alpha+\beta+m+n+2)], \\
E\left(\frac{\partial^2 \ln L(\alpha,\beta,\sigma)}{\partial\sigma^2}\right) &= \frac{\exp(-4\sigma)}{[B(\alpha,\beta;\sigma)]^2}\Big[\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} L_m(\sigma)L_n(\sigma)B(\alpha+m+1,\beta+n+1) \\
&\qquad \times \Big[2+\frac{L_{m-1}^{(1)}(\sigma)}{L_m(\sigma)}\mathbf{1}_{\mathbb{N}_1}(m)+\frac{L_{n-1}^{(1)}(\sigma)}{L_n(\sigma)}\mathbf{1}_{\mathbb{N}_1}(n)\Big]\Big]^2 \\
&\quad - \frac{\exp(-2\sigma)}{B(\alpha,\beta;\sigma)}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} L_m(\sigma)L_n(\sigma)B(\alpha+m+1,\beta+n+1) \\
&\qquad \times \Big[4+4\frac{L_{m-1}^{(1)}(\sigma)}{L_m(\sigma)}\mathbf{1}_{\mathbb{N}_1}(m)+4\frac{L_{n-1}^{(1)}(\sigma)}{L_n(\sigma)}\mathbf{1}_{\mathbb{N}_1}(n) \\
&\qquad + 2\frac{L_{m-1}^{(1)}(\sigma)L_{n-1}^{(1)}(\sigma)}{L_m(\sigma)L_n(\sigma)}\mathbf{1}_{\mathbb{N}_1}(m)\mathbf{1}_{\mathbb{N}_1}(n) \\
&\qquad + \frac{L_{m-2}^{(2)}(\sigma)}{L_m(\sigma)}\mathbf{1}_{\mathbb{N}_2}(m)+\frac{L_{n-2}^{(2)}(\sigma)}{L_n(\sigma)}\mathbf{1}_{\mathbb{N}_2}(n)\Big],
\end{aligned}
$$

$$
\begin{aligned}
E\left(\frac{\partial^2 \ln L(\alpha,\beta,\sigma)}{\partial\alpha\,\partial\beta}\right) = {} & \frac{\exp(-4\sigma)}{[B(\alpha,\beta;\sigma)]^2}\Big[\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} L_m(\sigma)L_n(\sigma)B(\alpha+m+1,\beta+n+1) \\
& \times [\psi(\alpha+m+1)-\psi(\alpha+\beta+m+n+2)]\Big] \\
& \times \Big[\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} L_m(\sigma)L_n(\sigma)B(\alpha+m+1,\beta+n+1) \\
& \times [\psi(\beta+n+1)-\psi(\alpha+\beta+m+n+2)]\Big] \\
& - \frac{\exp(-2\sigma)}{B(\alpha,\beta;\sigma)}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} L_m(\sigma)L_n(\sigma)B(\alpha+m+1,\beta+n+1) \\
& \times [\{\psi(\alpha+m+1)-\psi(\alpha+\beta+m+n+2)\} \\
& \times \{\psi(\beta+n+1)-\psi(\alpha+\beta+m+n+2)\} \\
& - \psi_1(\alpha+\beta+m+n+2)],
\end{aligned}
$$

$$
\begin{aligned}
E\left(\frac{\partial^2 \ln L(\alpha,\beta,\sigma)}{\partial\alpha\,\partial\sigma}\right) = {} & -\frac{\exp(-4\sigma)}{[B(\alpha,\beta;\sigma)]^2}\Big[\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} L_m(\sigma)L_n(\sigma)B(\alpha+m+1,\beta+n+1) \\
& \times [\psi(\alpha+m+1)-\psi(\alpha+\beta+m+n+2)]\Big] \\
& \times \Big[\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} L_m(\sigma)L_n(\sigma)B(\alpha+m+1,\beta+n+1) \\
& \times \Big[2+\frac{L_{m-1}^{(1)}(\sigma)}{L_m(\sigma)}\mathbf{1}_{\mathbb{N}_1}(m)+\frac{L_{n-1}^{(1)}(\sigma)}{L_n(\sigma)}\mathbf{1}_{\mathbb{N}_1}(n)\Big]\Big] \\
& + \frac{\exp(-2\sigma)}{B(\alpha,\beta;\sigma)}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} L_m(\sigma)L_n(\sigma)B(\alpha+m+1,\beta+n+1) \\
& \times \{\psi(\alpha+m+1)-\psi(\alpha+\beta+m+n+2)\} \\
& \times \Big[2+\frac{L_{m-1}^{(1)}(\sigma)}{L_m(\sigma)}\mathbf{1}_{\mathbb{N}_1}(m)+\frac{L_{n-1}^{(1)}(\sigma)}{L_n(\sigma)}\mathbf{1}_{\mathbb{N}_1}(n)\Big],
\end{aligned}
$$

$$
\begin{aligned}
E\left(\frac{\partial^2 \ln L(\alpha,\beta,\sigma)}{\partial\beta\,\partial\sigma}\right) = {} & -\frac{\exp(-4\sigma)}{[B(\alpha,\beta;\sigma)]^2}\Big[\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} L_m(\sigma)L_n(\sigma)B(\alpha+m+1,\beta+n+1) \\
& \times [\psi(\beta+n+1)-\psi(\alpha+\beta+m+n+2)]\Big] \\
& \times \Big[\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} L_m(\sigma)L_n(\sigma)B(\alpha+m+1,\beta+n+1) \\
& \times \Big[2+\frac{L_{m-1}^{(1)}(\sigma)}{L_m(\sigma)}\mathbf{1}_{\mathbb{N}_1}(m)+\frac{L_{n-1}^{(1)}(\sigma)}{L_n(\sigma)}\mathbf{1}_{\mathbb{N}_1}(n)\Big]\Big] \\
& + \frac{\exp(-2\sigma)}{B(\alpha,\beta;\sigma)}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} L_m(\sigma)L_n(\sigma)B(\alpha+m+1,\beta+n+1) \\
& \times \{\psi(\beta+n+1)-\psi(\alpha+\beta+m+n+2)\} \\
& \times \Big[2+\frac{L_{m-1}^{(1)}(\sigma)}{L_m(\sigma)}\mathbf{1}_{\mathbb{N}_1}(m)+\frac{L_{n-1}^{(1)}(\sigma)}{L_n(\sigma)}\mathbf{1}_{\mathbb{N}_1}(n)\Big],
\end{aligned}
$$

5 Conclusion

In this article, we have derived the Fisher information matrix and entropies such as Rényi and Shannon for the extended beta type 1 distribution. We have used several definitions and results from special function to derive and express these results.

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