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Game Theory Based Distributed Model Predictive Control for a Hydro-Power Valley Control

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Abstract: Hydro-power valleys are large scale systems used to power energy production. The stored water is also used for navigation and agriculture purposes. Recently, the control of hydro-power valleys has been formulated as a centralized optimal control problem. However, the scale of the systems make unfeasible real time implementations for centralized controllers. In this work we propose the use of the game theory to formulate a distributed model predictive control scheme to control an hydro-power valley. The proposed control scheme is tested by using a power reference tracking scenario as a test-bed.

1. INTRODUCTION

A hydro-power valley (HPV) is a large-scale system whose main objective is to produce electric power from water stored in lakes and/or reaches. In this kind of systems the water flows across a duct equipped with a turbine, this movement converts the potential energy of the water into mechanical energy, which is then converted into electric energy. The electric power has to be produced considering that the water is also used for ancillary services like navigation and irrigation, and also the levels cannot be higher than a predefined threshold in order to avoid floods Faille (2009). All these restrictions plus the power requirements must be accomplished with an adequate control scheme.

As in almost all large-scale systems, the most common control scheme employed for controlling the HPV comprises a proportional-integral (PI) controller with disturbance feed-forward installed on each individual power plant Setz et al. (2008). However, the use of local PI controllers in a HPV does not guarantee an efficient use of the stored water, and in presence of disturbances the performance of the entire system could be compromised. For tackling these issues, multivariate control structures have been proposed for controlling HPV systems. Often, these are optimal control schemes (see Xiaohong et al. (1999) and the references therein). In this way, centralized model predictive control (MPC) schemes for controlling HPV have been proposed Setz et al. (2008).

Since an HPV is a large-scale system, a centralized MPC may become impractical, inflexible, and unsuitable, because it may require to exchange large amounts of information with high computational load associated Li et al. (2005); Negenborn (2007); Camponogara et al. (2002); Necoara et al. (2008); Doan et al. (2008); Camponogara and Talukdar (2007); Du et al. (2001); Venkat et al. (2007). Therefore, distributed model predictive control (DMPC) schemes were implemented to deal with large-scale MPC problems given their capabilities to divide a complex problem into several sub-problems. The use of DMPC schemes reduces the computational load and the information exchange required for the implementation of real-time large scale MPC Li et al. (2005); Negenborn (2007).

Several DMPC approaches have been presented in the literature (see Camponogara et al. (2002); Doan et al. (2008); Necoara et al. (2008); Rantzer (2009) for examples). Often, these approaches required the system to be stable and controllable, but it restricted the applicability of the proposed methods. Moreover, the reviewed approaches may force the subsystems to cooperate, often without taking into account whether the cooperative behavior gives some benefit to the subsystems, and might steer the subsystems to operating points where they do not perceive any benefit in terms of the local cost function. Considering these issues, game theory arises as an alternative to formulate and characterize the problem of not being able to determine when the subsystems should cooperate or not.

Game theory is a mathematical method to analyze calculated circumstances where the success of an individual is based upon the choices of the others (see Von Neumann et al. (1947) for a more detailed definition of game theory). The first ideas of applying game theory to the DMPC problem were proposed in Du et al. (2001); Li et al. (2005). In such approaches the DMPC problem was formulated as a non-cooperative game and it was demonstrated that the solution converged to the Nash equilibrium point of the game. In Rantzer (2009) related the DMPC problem with game theory using the cooperative game approach presented in Von Neumann et al. (1947). In these approaches the Lagrange multipliers used in the dual decomposition methods were conceived as prices in a market mechanisms (see Necoara et al. (2008) for details), allowing to achieve mutual agreements among subsystems. Other approaches related with the formulation of the DMPC problem as a game have been presented in Muñoz de la Peña et al. (2009); Maestre et al. (2011b). In Muñoz de la Peña et al. (2009); Maestre et al. (2011a,c,b) some algorithms based on cooperative games for solving the DMPC problem were proposed. In Venkat et al. (2006b,a) the authors presented examples where the convergence of the solution of the DMPC to a Nash equilibrium point produced an unstable closed-loop behavior. In addition, the DMPC schemes based on cooperative games required the solution of more than one optimization problem at each sample time, increasing the computational burden of the DMPC schemes.

In order to tackle these drawbacks, in this work is assumed that subsystems "bargain" to each other in order to (jointly) decide which strategy is best with respect to their mutual benefit. Such assumption is based on the fact that in a HPV the power-plants are coordinated in order to provide the required power. The DMPC problem for the HPV is then reformulated as a n-person bargaining game based on the concepts presented in Nash (1950b,a, 1953) about such games. A similar formulation of DMPC has previously been presented in Valencia et al. (2011); Alvarado et al. (2011).

The outline of this manuscript is as follows: In Section 2 the HPV model is introduced. In Section 3 the mathematical framework of non-symmetric game theory and the formulation of the DMPC as a non-symmetric game are presented. In Section 4 a DMPC based on game theory is formulated for an HPV. In Section 5 simulation results are presented. Finally, the concluding remarks are given in Section 6.

2. HYDRO-POWER VALLEY MODELING

Consider the HPV shown in Figure 1. This HPV is composed by three lakes (L_m, $m = 1, 2, 3$) where the water is stored, a duct (U_1) that connects two lakes, a river with six dams (D_j, $j = 1, \dots, 6$), two turbines (T_p, $p = 1, 2$), and two turbine-pump devices (C_p). The stored water flows across ducts from one reservoir to another, or from the reservoir to power houses where the potential energy of the water is transformed into mechanical energy.

The river has a constant inflow q_{in} and a constant tributary flow q_{tr}. Moreover, each dam D_j is equipped with a turbine for electric power generation. They are located in the river and divided it into six reaches (R_j), where reaches R_1, R_2 and R_4, R_5 are connected with lakes L_1, L_3 through turbines T_1, T_2 and turbine-pumps C_1, C_2 respectively. Also, lakes L_1, L_2 are connected to each other by the duct U_1. This duct is only used for transmitting the water from one lake to another depending on the difference of the levels.

A model suitable for control purposes using the HPV of Figure 1 is derived in Savorgnan and Diehl (2011). This model is based on the following assumptions:

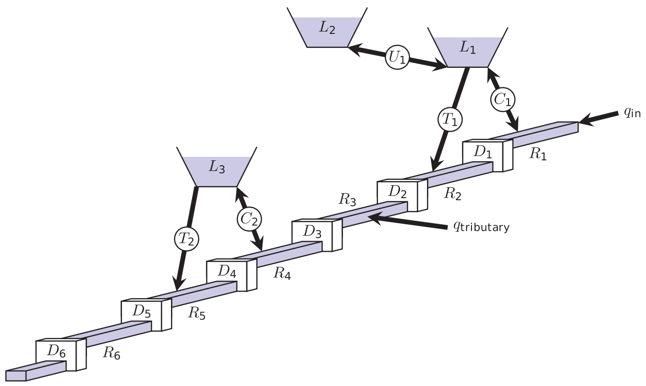

Fig. 1. Hydro-Power Valley used as a case of study.

- The ducts are connected at the bottom of the lakes (or at the bottom of the river bed).
- The cross sections of the reaches and of the lakes are rectangular.
- The width of the reaches varies linearly along them.
- The river bed slope is constant along every reach.

Based on these assumptions, the nonlinear, first-order Saint-Venant partial differential equations represent the state of the art for modeling one-dimensional river hydraulics with constant fluid density Setz et al. (2008). In these equations the hydraulic state of the river are described by two variables: the water depth $h(t, z)$ and the discharge $q(t, z)$, both varying as a function of space z and time t. Thus, the dynamics of each reach are given by Savorgnan and Diehl (2011); Setz et al. (2008)

$$\begin{aligned} 0 &= \frac{\partial q}{\partial z} + \frac{\partial s}{\partial t} \\ 0 &= \frac{1}{g}\frac{\partial}{\partial t}\left(\frac{q}{s}\right) + \frac{1}{2g}\frac{\partial}{\partial z}\left(\frac{q}{s}\right)^2 + \frac{\partial h}{\partial z} + I_f - I_o \end{aligned} \tag{1}$$

In Eq. (1), $q = q(t, z)$, $s = s(t, z)$, $h = h(t, z)$, $I_f = I_f(t, z)$, $I_o = I_o(t, z)$, where $s(t, z)$ is the wet surface, $I_f(t, z)$ is the friction slope, $I_o(t, z)$ is the river bed slope, and g is the gravitational acceleration. Since the cross sections of the reaches and lakes are assumed rectangular, the wet surface and the friction slope are given by Eqs. (2) and (3) respectively Savorgnan and Diehl (2011)

$$s(t, z) = w(z)h(t, z) \tag{2}$$

$$If(t, z) = \frac{q^2(t, z)(w(z) + 2h(t, z))^{\frac{4}{3}}}{k_{str}^2(w(z)h(t, z))^{\frac{10}{3}}} \tag{3}$$

where $w(z)$ is the river width, and k_{str} is the Gauckler-Manning-Strickler coefficient. For modeling the lakes, duct, turbines, and turbine-pumps elements, Eqs. (4)-(7) were used Savorgnan and Diehl (2011):

$$\frac{\partial h(t)}{\partial t} = \frac{q_{in}(t) - q_{out}(t)}{S} \tag{4}$$

$$q_{U1}(t) = S_{U1}\text{sign}(H(t))\sqrt{2g|H(t)|} \tag{5}$$

$$p_t(t) = k_t q_t(t)\Delta h_t(t) \tag{6}$$

$$p_C(t) = k_C(q_C(t))q_C(t)\Delta h_C(t) \tag{7}$$

where sign$(\cdot)$ is the sign function, S is the surface area of the lake, S_{U1} is the section of the duct, k_t is the turbine coefficient, $q_{in}(t)$, $q_{out}(t)$, are the input and output flows of the lakes respectively, $q_t(t)$ is the turbine discharge, $\Delta h_t(t)$, $\Delta h_C(t)$ are the heads of the turbine and the turbine-pump respectively, $q_{U1}(t)$ is the flow across the duct U_1; $p_t(t)$, $p_C(t)$ are the power generated by the turbines and the power generated or consumed by the turbine-pump elements respectively,

$$k_C(q_C(t)) = \begin{cases} k_{tC} \text{ if } q_C(t) \geq 0 \\ k_{pC} \text{ if } q_C(t) < 0 \end{cases}$$

is the turbine-pump coefficient, k_{tC}, k_{pC} are the gains of the turbine-pump devices in turbine or pump mode respectively, $q_C(t)$ is the flow in the turbine-pump elements, and $H(t) = h_{L2}(t) - h_{L1}(t) + h_{U1}$, with $h_{L1}(t)$, $h_{L2}(t)$ the levels of the lakes 1 and 2 respectively, and h_{U1} the height difference of the duct.

Although Eqs. (1)-(7) describe the dynamic behavior of the HPV, this model is unsuitable for control purposes. In order to obtain a suitable model, a spatial discretization of Eq. (1) is required. The expressions of the resultant model are given in Savorgnan and Diehl (2011). Let $h_{\mathrm{L}m}(t)$ denote the level of the m-th lake. Let $q_{\mathrm{T}p}$, $q_{\mathrm{C}p}$ denote the inflow of the p-th turbine and p-th turbine-pump device respectively. For the reach R_j, let $Q_{\mathrm{R}j} = [q_{1j}(t), \ldots, q_{N_{\mathrm{x}}j}(t)]$ and $h_{\mathrm{R}j} = [h_{1j}(t), \ldots, h_{(N_{\mathrm{x}}+1)j}(t)]$ denote the vector of outflows and the vector of levels at each spatial partition, being N_{x} the number of spatial partitions of the reach. Also, let $q_{\mathrm{R}j}$ denote the outflow of the j-th turbine at the corresponding dam. Then, the inputs $u(t)$ and the states $x(t)$ of the HPV can be defined as

$$\begin{aligned} u(t) &= [q^T_{\mathrm{T}p}(t), q^T_{\mathrm{C}p}(t), q^T_{\mathrm{R}j}(t)]^T, \\ x(t) &= [h^T_{\mathrm{L}m}(t), Q^T_{\mathrm{R}j}(t), h^T_{\mathrm{R}j}(t)]^T \end{aligned}$$

$p = 1, 2$; $j = 1, 2, \ldots, 6$; $m = 1, 2, 3$. Based on this definition of states and inputs the proposed DMPC scheme is formulated.

3. DISTRIBUTED MODEL PREDICTIVE CONTROL AS A BARGAINING GAME

Let start the current Section introducing the concept of game. A game is defined as the tuple $G = (N, \{\Omega_i\}_{i\in N}, \{\phi_i\}_{i\in N})$, where $N = \{1, \ldots, M\}$ is the set of players, Ω_i is a finite set of possible actions of player i, and $\phi_i : \Omega_1 \times \ldots \times \Omega_M \longrightarrow \mathbb{R}$ is the pay-off function of the i-th player Akira (2005). If it is assumed that the players are able to "bargain" in order to achieve a common goal, the game G can be analyzed as a bargaining game following the Nash theories about such games. A bargaining game is a situation involving a set of players who have the opportunity to collaborate for mutual benefit by an agreement on a joint plan of action Nash (1950b, 1953). If an agreement is not possible, the players carry out an alternative plan determined by the information locally available. The benefit perceived by the player when an agreement is not possible is called disagreement point (see Peters (1992) for an in deep discussion about bargaining games).

Assume that the whole system can be decomposed into M subsystems such that the dynamic model of each subsystem is given by the linear state equation Camponogara et al. (2002); Doan et al. (2008); Du et al. (2001); Necoara et al. (2008)

$$\begin{aligned} x_i(k+1) &= \sum_{l=1}^{M} A_{il} x_l(k) + B_{il} u_l(k) \\ y_i(k) &= \sum_{l=1}^{M} C_{il} x_l(k) + D_{il} u_l(k) \end{aligned} \tag{8}$$

where A_{il}, B_{il}, C_{il}, and D_{il} are sub-matrices of A, B, C, and D respectively. Often, a quadratic cost function Venkat et al. (2006b) is used to measure the performance of the system (note that it can also be interpreted as the total energy). Using Eq. (8), this cost function becomes

$$L(\widetilde{x}(k), \widetilde{u}(k)) = \sum_{i=1}^{M} \phi_i(\widetilde{u}(k); x(k)) \tag{9}$$

where $\widetilde{x}(k) = [x^T(k), \ldots, x^T(k+N_{\mathrm{p}})]^T$, $\widetilde{u}(k) = [u^T(k), \ldots, u^T(k+N_{\mathrm{u}}), \ldots, u^T(k+N_{\mathrm{p}})]^T$, being N_{u}, N_{p} the control and prediction horizon respectively, with $N_{\mathrm{u}} \leq N_{\mathrm{p}}$, and

$$\begin{aligned} \phi_i(\widetilde{u}(k); x(k)) =& \widetilde{u}^T(k) Q_{uui} \widetilde{u}(k) + 2x^T(k) Q_{xui} \widetilde{u}(k) \\ &+ x^T Q_{xxi} x(k) \end{aligned} \tag{10}$$

with $Q_{uui} \geq 0$, for $i = 1, \ldots, M$ obtained with the solution of the Local Lyapunov equation. In Eq. (10) the notation $\phi_i(\widetilde{u}(k); x(k))$ indicates that the argument of ϕ_i is $\widetilde{u}(k)$ and $x(k)$ is a parameter of ϕ_i. Clearly, ϕ_i is a positive-definite quadratic function of $\widetilde{u}(k)$ and thus it is convex in $\widetilde{u}(k)$ (For the sake of simplicity of notation we will not indicate the dependence of ϕ_i on $x(k)$ explicitly in the remainder of this text and thus write $\phi_i(u(k))$ instead $\phi_i(u(k); x(k))$).

In the formulation of the DMPC, there are a set of subsystems $N = \{1, \ldots, M\}$ determined by the system decomposition of Eq. (8), a set of feasible control actions $\Omega = \Pi_{i=1}^{M} \Omega_i$, being Ω_i the feasible set for the whole system determined by the physical and operational constraints, determining a set of possible values for the cost function $\phi_i(\widetilde{u}(k))$ for each subsystem i, and a set of cost functions $\{\phi_1(\widetilde{u}(k)), \ldots, \phi_M(\widetilde{u}(k))\}$ defining the interests of each subsystem, all of them depending on the decision of the remaining subsystems. Therefore, the DMPC can be analyzed as a game $G_{\mathrm{DMPC}} = (N, \{\Omega_i\}_{i\in N}, \{\phi_i\}_{i\in N})$. Moreover, since in a DMPC scheme the subsystems are able to communicate to each other, it is possible to assume that they are also able to bargain. Hence, the game G_{DMPC} can be analyzed as a bargaining game, and the outcome of an DMPC can be also computed and characterized as a solution of such games. However, the axiomatic bargaining game theory presented in Nash (1950a,b, 1953); Peters (1992) has been developed for games with a static decision environment, which is not the case of the DMPC, where the decision environment changes accordingly to the dynamic equations modeling the behavior of the system. So, the original bargaining game theory has to be extended in order to cover games with time-varying decision space. Then, in this paper the concept of discrete-time dynamic bargaining game is introduced.

A discrete-time dynamic bargaining game refers to a situation where at each time step a static bargaining game (S, d) is solved depending on the dynamic evolution of the decision environment. It is determined by a state vector $x(k) \in \mathbb{R}^n$ and by an input vector $u(k) \in \mathbb{R}^m$, with $x(k) \in \mathbb{X}$ and $u(k) \in \mathbb{U}$, $\mathbb{X}$ and $\mathbb{U}$ being the feasible sets for $x(k)$ and $u(k)$ respectively. In this game, we assume that the feasible set and/or the disagreement point can change with time. Mathematically, a discrete-time dynamic bargaining game for N is defined as a sequence of games $\{(\Theta(0), \eta(0)), (\Theta(1), \eta(1)), \ldots\}$, denoted by $\{(\Theta(k), \eta(k))\}_{k=0}^{\infty}$, where for $k = 1, 2, 3, \ldots$, $\Theta(k)$ is a non-empty closed subset of $\mathbb{R}^M$; and $\eta(k) \in \text{int}(\Theta(k))$, $\eta(k)$ is the disagreement point. Also, there exists functions $f_i \in \mathbb{R}^{n_i}, g_i \in \mathbb{R}^{z_i}, h_i \in \mathbb{R},\ i = 1, \ldots, M$, determining the dynamic evolution of the decision environment, the feasible set, and the disagreement point of player i such that:

$$\begin{aligned} x_i(k+1) &= f_i(x(k), u(k)) \\ \Theta_i(k+1) &= g_i(x(k), u(k), \Theta(k)) \\ \eta_i(k+1) &= h_i(x(k), u(k), \eta(k)) \end{aligned}$$

with $x_i(k) \in \mathbb{X}_i$; $\mathbb{X}_i \subset \mathbb{X}$; z_i the dimension of the feasible set of player i, and $u(k) = [u_1^T(k), \ldots, u_M^T(k)]$ the vector of actions taken by the players affecting the decision environment. Finally, there exists a tuple $\phi(x(k), u(k)) \in \mathbb{R}^M$ such that $\phi(x(k), u(k)) \in \Theta(k)$, being $\phi_i(x(k), u(k))$ the profit function of the i-th player.

Let define the evolution of the disagreement point as

$$\eta_i(k+1) = \begin{cases} \eta_i(k) - \alpha(\eta_i(k) - \phi_i(\widetilde{u}(k))) \text{ if } \eta_i(k) > \phi_i(\widetilde{u}(k)) \\ \eta_i(k) + (\phi_i(\widetilde{u}(k)) - \eta_i(k)) \text{ if } \eta_i(k) < \phi_i(\widetilde{u}(k)) \end{cases}$$

$\forall i \in N$, with $0 < \alpha < 1$. Let Υ denote the set of values of $\phi_i(\widetilde{u}(k); x(k))$ such that $\widetilde{u}(k) \in \Omega$. Let the utopia point $\zeta_i(\Upsilon)$ be defined as $\zeta_i(\Upsilon) := \min\{\phi_i(\widetilde{u}(k)) \colon \phi_i(\widetilde{u}(k)) \in \Upsilon\}$ exist for every $i \in N$. Then, the game G_{DMPC} is a discrete-time dynamic bargaining game with $\Theta(k) = \Upsilon$ (the evolution of the decision environment is determined by the dynamics of the controlled system).

Until here the DMPC has been defined as a bargaining game. Now, a solution for such a game is derived based on the solution proposed in Nash (1950a,b, 1953); Peters (1992) for non-symmetric bargaining games.

the non-symmetric bargaining solution of a game G_{DMPC} at time step k can be computed in a centralized way as a solution of the maximization problem:

$$\max_{\widetilde{u}(k)} \sum_{i=1}^{M} w_i \log(\eta_i(k) - \phi_i(\widetilde{u}(k))) \tag{11}$$

subject to:

$$\eta_i(k) > \phi_i(\widetilde{u}(k)) \qquad \widetilde{u}(k) \in \Omega$$

with w_i a set of weights. Since a DMPC scheme only has horizontal communication, all the subsystems belong to the same layer. Although the definition of weighted hierarchy requires the selection of the weights for each subsystem, there are not guidelines for choosing their values. In the control theory field, the values of the weights can be arbitrarily selected as $w_i = \frac{1}{M},\ i = 1, \ldots, M$ Doan et al. (2008); Venkat et al. (2006a,b)).

The maximization problem of Eq. (11) can be solved in a distributed way by locally solving the system-wide control problem

$$\max_{\widetilde{u}_i(k)} \sum_{r=1}^{M} w_r \log(\eta_r(k) - \sigma_r(\widetilde{u}_i(k), \widetilde{u}_{-i}(k))) \tag{12}$$

Subject to:

$$\eta_r(k) > \sigma_r(\widetilde{u}_i(k), \widetilde{u}_{-i}(k)) \qquad \widetilde{u}_i(k) \in \Omega_i$$

with $\sigma_r(\widetilde{u}_i(k), \widetilde{u}_{-i}(k)) = \phi_R(\widetilde{u}(k))$ for $r = 1, \ldots, M$, where $\widetilde{u}_{-i}(k) = [\widetilde{u}_1^T(k), \ldots, \widetilde{u}_{i-1}^T(k), \widetilde{u}_{i+1}^T(k), \ldots, \widetilde{u}_M^T(k)]$.

Note that the maximization problem of Eq. (12) is equivalent to the maximization problem of Eq. (11), considering fixed $\widetilde{u}_{-i}(k)$ and optimizing only in the direction of $\widetilde{u}_i(k)$. This formulation allows each subsystem to take into account the effect of its decisions in the behavior of the whole system and to promote the cooperation among subsystems.

In order to implement the solution of a DMPC as a bargaining game, a negotiation model is proposed. A negotiation model is a sequence of steps for computing the outcome of a game. In the case of the DMPC game, the proposed algorithm solves such games in a distributed way. This algorithm is based on the negotiation model proposed by Nash (1953) for two-person games. The proposed steps for solving the DMPC game are:

(1) At time step k, each subsystem sends to the remaining subsystems the values of $x_i(k),\ \eta_i(k)$.
(2) With the information received, each subsystem solves the local optimization problem of Eq. (12).
(3) Let $\widetilde{u}_i^*(k)$ denote optimal control actions for subsystem $i,\ i = 1, \ldots, M$. If Eq. (12) is feasible, subsystem i selects the first control action of $\widetilde{u}_i^*(k)$. Otherwise, subsystem i selects the first control action of $\widetilde{u}_i(k)$, where $\widetilde{u}_i(k)$ is the initial condition of subsystem i at time step k for solving Eq. (12).
(4) Each subsystem updates its disagreement point based on Eq. (3).
(5) Each subsystem sends its updated control action and its updated disagreement point.
(6) Go to step 1.

The initial condition for solving (12) at time step $k+1$ are given by the shifted control input $\widetilde{u}_{oi}(k+1) = [\bar{u}_i^T(k+1), \ldots, \bar{u}_i^T(k+N_{\text{p}}), 0]$, where $\bar{u}_i(k)$ denotes the value of the control input to be applied in the i-th subsystem. Note that in the proposed negotiation model seems that there is not a negotiation process. However, the cost function

$$J(\widetilde{u}(k)) = \sum_{r=1}^{M} w_r \log(\eta_r(k) - \sigma_r(\widetilde{u}_i(k), \widetilde{u}_{-i}(k)))$$

allows every subsystem to have certain degree of coordination with the remaining subsystems. Thus, subsystem i is able to compute its optimal control inputs in a separated way from the information provided by the remaining subsystems. Furthermore, in comparison with the Lagrange multipliers based DMPC schemes, the proposed algorithm does not require an iterative process for computing the local control actions, decreasing the computational burden. The convexity and the feasibility of the proposed DMPC scheme are analyzed in Alvarado et al. (2011). Moreover, the whole system cost function is bounded by the sum of all disagreement points. So, if the disagreement point decreases and tends to zero, the cost function of the whole system also decreases and tends to zero guaranteeing stability. A formal definition of the stability conditions of the proposed control scheme is presented in Valencia (2012).

In the proposed negotiation model only one optimization problem should be solved. This allows to reduce the computational burden of the DMPC scheme associated with the communications among subsystems, and maybe with the solution of more than one optimization problem at each time step. In the next section a DMPC scheme based on game theory is formulated for a HPV.

4. GAME-THEORY-BASED CONTROL OF A HYDRO-POWER VALLEY

With the purpose of designing a MPC for the HPV depicted in Section 2, the power tracking scenario proposed in Savorgnan and Diehl (2011) is considered. In this scenario, power output of the system should follow a given reference while keeping the water levels in the lakes and at the dams as constant as possible. So, the global cost function considered for the DMPC is composed by two terms: the first term penalizes the 1-norm of the power tracking error, and the second term penalizes the 2-norm of the deviations of the levels in the lakes and in the dams from their steady state values.

The HPV can be defined by the linear discrete model of Eq. (8), where $A,\ B,\ C,\ D$ are the matrices resulting from the linearization of Eqs. (1)-(7), and $y(k) = [p(k), h_D^T(k)]^T$, with $p(k)$ the power produced by the HPV, and $h_D(k) = [h_{D1N_x}, h_{D2N_x}, h_{D3N_x}, h_{D4N_x}, h_{D5N_x}, h_{D6N_x}]$ the levels at the dams (only the levels in the last element of the spatial discretization of the reaches is considered to regulate the levels of the reaches). Note that the power produced by the HPV is piecewise defined with respect to $u(k)$ due to the turbine-pump elements. In order to overcome this issue in the linearization, constants k_{des1}, k_{des2} were introduced, virtual inputs $\bar{u}_1(k) \in [-q_{C1\text{pump}}, q_{C1\text{turb}}]$, $\bar{u}_2(k) \in [-q_{C2\text{pump}}, q_{C2\text{turb}}]$ were considered, and a gain compensation

$$u_p(k) = \begin{cases} \dfrac{k_{desp}}{k_{tCp}} \bar{u}_p(k) \text{ if } \bar{u}_p(k) \geq 0 \\ \dfrac{k_{desp}}{k_{pCp}} \bar{u}_p(k) \text{ if } \bar{u}_p(k) < 0 \end{cases}$$

was proposed, where $q_{C1\text{pump}}, q_{C2\text{pump}}, q_{C1\text{turb}}, q_{C2\text{turb}}$ are the maximum pumped flows and maximum turbine flows for the turbine-pump elements C_1, C_2 respectively, $p = 1, 2$ (the values of $q_{C1\text{pump}}, q_{C2\text{pump}}, q_{C1\text{turb}}, q_{C2\text{turb}}$ are given in Savorgnan and Diehl (2011)).

From Savorgnan and Diehl (2011), it is possible to divide the HPV of Figure 1 into eight subsystems:

- Subsystem 1: lakes L_1 and L_2, turbine T_1, and turbine-pump C_1.
- Subsystem 2: lake L_3, turbine T_2, and turbine-pump C_2.
- Subsystems 3-8: reaches R_1 to R_6 respectively.

Thus, the DMPC problem is formulated as follows Savorgnan and Diehl (2011):

$$\begin{aligned} &\min_{\widetilde{u}(k)} \lambda|\widetilde{p}_r(k) - \widetilde{y}_p(\widetilde{u}(k))| + \widetilde{u}^T(k) Q_{uu} \widetilde{u}(k) \\ &\qquad + h_D^T(k) Q_{ux} \widetilde{u}(k) + h_D^T(k) Q_{xx} h_D^T(k) \\ &\text{Subject to:} \\ &\qquad \widetilde{u}(k) \in \Omega \\ &\qquad u(k+\nu) = u(k+N_\text{u}),\ \forall N_\text{u} < \nu < N_\text{p} - 1 \end{aligned} \tag{13}$$

where $\widetilde{p}_r(k) = [p_r(k), \ldots, p_r(k+N_\text{p})]$, with $p_r(k)$ the power references; $\widetilde{y}_p(\widetilde{u}(k)) = [p(x(k), u(k)), \ldots, p(x(k), u(k + N_\text{p} - 1))]$, with $p(x(k), u(k))$ the power produced by the HPV; $Q_{uu} = \bar{B}^T \bar{Q} \bar{B}$, $Q_{ux} = x^T(k) \bar{A}^T \bar{Q} \bar{B}$, $Q_{xx} = \bar{A}^T \bar{Q} \bar{A}$, and Ω is the feasible set composed by the input constraints and the mapping of the state constraints to input constraints, with $\bar{A},\ \bar{B}$ the resulting matrices from the prediction of $h_D(k)$ along N_p, $\bar{Q}$ the Q block diagonal matrix resulting form the division of the model, and $\lambda > 0$ a diagonal matrix. The power reference to be followed by the entire system is known 24 hours in advance and the inputs of the system can be changed every 30 minutes.

Let $\sigma_i(\widetilde{u}_i(k), \widetilde{u}_{-i}(k))$ be the local cost function of each subsystem, $\sigma_i(\widetilde{u}_i(k), \widetilde{u}_{-i}(k))$ defined as

$$\begin{aligned} \sigma_i(\widetilde{u}_i(k), \widetilde{u}_{-i}(k)) = \gamma|\widetilde{p}_r(k) - \widetilde{y}_p(\widetilde{u}_i(k), \widetilde{u}_{-i}(k))| \\ + [\widetilde{u}_i^T(k), \widetilde{u}_{-i}^T(k)] \bar{H}_i [\widetilde{u}_i^T(k), \widetilde{u}_{-i}^T(k)]^T \\ + 2\bar{F}_i [\widetilde{u}_i^T(k), \widetilde{u}_{-i}^T(k)]^T \end{aligned}$$

where $\bar{H}_i, \bar{F}_i$ are the resultant matrices of the permutation of the rows and columns of Q_{uu} and Q_{ux} respectively (the state dependence of $\sigma_i(\cdot)$ was omitted for notational convenience), and $\gamma \in \mathbb{R}$ a constant weight (the term $h_D^T(k) Q_{xx} h_D^T(k)$ was omitted because it is constant with respect to the decision variables, then it does not affect the result of the optimization). From Savorgnan and Diehl (2011), the state and input constraints are time independent and they only establishes lower and upper boundaries to the states and inputs. So, they are independent for each subsystems, i.e., there is not coupled constraints. Then, for the control of the HPV we have a game $G_\text{HPV} = \{N, \{\sigma_i(\widetilde{u}_i(k), \widetilde{u}_{-i}(k))\}_{i \in N}, \{\Omega_i\}_{i \in N}\}$, with $N = \{1, \ldots, 8\}$, in which all subsystems have the same goal: to minimize the power tracking error keeping the levels in the lakes and at the dams as close as possible to their steady state values. Hence, the game G_HPV can be analyzed and solved as a discrete-time dynamic bargaining game $\{(\Upsilon, \eta(k))\}_{k=0}^{\infty}$, with $\eta(k)$ defined as in Section 3. In the following section the simulation results are presented.

5. SIMULATION RESULTS

Based on the formulation presented in Section 4, a closed-loop simulation of the HPV described in Section 2 was performed along 24 hours (simulation time). In this simulation, $k_{des1} = \frac{3}{4}(k_{tC1} + k_{pC1})$, $k_{des2} = \frac{3}{4}(k_{tC2} + k_{pC2})$, $T_s = 1800s$ (30 minutes), $N_\text{p} = 48$ (corresponding to a day), $N_\text{u} = 32$, $w_{1,2} = \frac{0.4}{2}$, $w_{3-8} = \frac{0.6}{6}$ (the weights of subsystems 1 to 8), $\eta_i(0) = 1 \times 10^5$ (the initial disagreement point of subsystems 1 to 8), $\gamma = 50$, $Q = I$ (I being the identity matrix). The values of the parameters of Eqs. (1)-(7), as well as the lower and upper values of the inputs and the states were taken as proposed in Savorgnan and Diehl (2011).

Figure 2 shows the comparison between the power produced by the HPV and the power reference when the proposed DMPC scheme computes the inputs of each subsystem. This figure shows how the power produced by the HPV followed the power reference, satisfying one of the objectives proposed for the control scheme. However, there was an oscillation at the beginning of the experiment due to the transient generated by the change of power from

175 MW (equilibrium power) to the initial required power 150 MW.

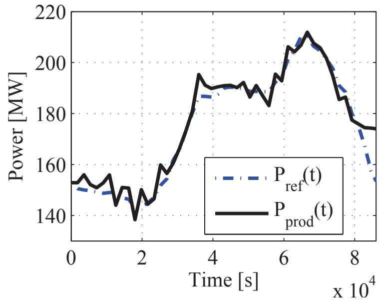

Fig. 2. Comparison between the power produced by the HPV with the power reference, when the proposed game-theory-based DMPC is used for computing the inputs of the subsystems

In order to maintain some power demand, the levels of the reaches and the lakes should be modified. In Figure 3 the behavior of the levels is presented. At the beginning of the simulation the lakes increased their levels due to the reduction of power from the equilibrium point to the set point (see first panel of Figure 3). When the required power was increased the lakes reached constant levels of water, achieving one of the system objectives. During the whole simulation the reaches maintained their levels as constant as possible (see second panel of Figure 3). If it is considered that the reaches also can be used for maritime traffic, maintaining constant their levels guarantees it. This condition was considered in the selection of the weights, by giving more importance to the reaches compared with the lakes; it is evidenced with the comparison: $\sum_{i=3}^{8} w_i > \sum_{i=1}^{2} w_i$.

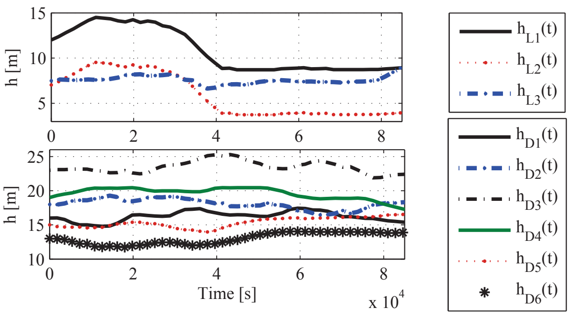

Fig. 3. Behavior of the levels in the lakes (first panel) and the levels at the dams (second panel) of the HPV. In both panels the levels are inside the values defined by the constraints, although the levels of the lakes (first panel) present large excursions before remaining constant, while the levels of the reaches remains as constant as possible.

The excursions of the levels of the lakes can be associated with the behavior of the control inputs (see Figure 4). Even though the control inputs remained inside the range defined by the constraints, the control actions of subsystems 1 and 2 had higher variations than the control actions of the remaining subsystems with respect to their local capability. This produced lager changes in the levels of the lakes than the levels of the reaches. Recall that subsystems 3 to 8 were power plants and subsystems 1 and 2 were ducts equipped with turbines and turbine-pump elements with less capability to produce electric power than the power plants.

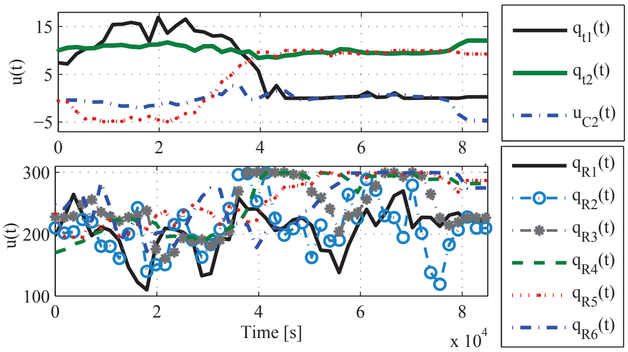

Fig. 4. Control actions applied to the subsystems. In the first panel the behavior of the control actions applied to subsystems 1 and 2 is presented. In the second panel the behavior of the control actions applied to subsystems 3 to 8 is presented. In both panels the control actions remain inside the range defined by the constraints of the control inputs.

Finally, in Figure 5 the evolution of the disagreement points is presented. In this Figure, the disagreement started at the same point but as they were evolving each subsystem had its own value indicating the non-symmetry of the game G_{HPV}. Figure 5 also shows a zoom between 4×10^4s and 7.5×10^4s, note that all the disagreement points decreased with low frequency oscillations. Such oscillations were associated to the decision process of each subsystem.

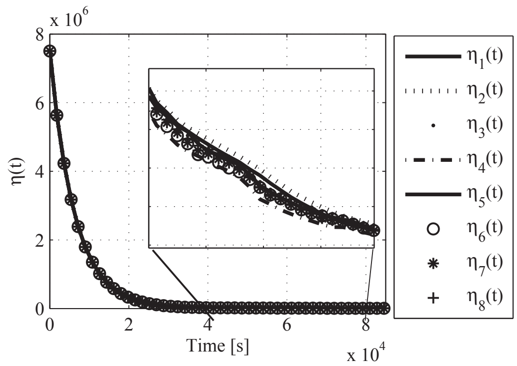

Fig. 5. Behavior of the disagreement points at the full simulation. This Figure shows an overall evolution and presents a detailed view that allows to evidence the non-symmetry of the game.

6. CONCLUDING REMARKS

In this work a distributed model predictive control scheme based on game theory was proposed for controlling an HPV. With this purpose, a model of an HPV suitable for control purposes was presented, and the mathematical framework for the formulation of the DMPC scheme as a non-symmetric bargaining game also was presented. This methodology allows the subsystems decide whether to cooperate or not based on the benefit perceived by the cooperative behavior. Moreover the disagreement point is reduced at each simulation step, forcing the subsystems to cooperate with reduced values of the cost function, improving the performance of the closed-loop system. In order to validate the proposed control scheme, a power reference tracking scenario was used as a testbed. In this scenario both the power tracking and the level regulation objectives were achieved.

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