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Department of Economics Working Paper Series

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Working Paper 2011-14
July 2011

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Macroeconomic Implications of the Underground Sector: Challenging the Double Business Cycle Approach

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Abstract

Within the literature on business cycles featuring shadow economic activities, there is an approach based on the arguable premise that fluctuations in the official and unofficial sectors are negatively correlated. The present paper develops a real business cycle model that does not impose such an assumption. To do so, preferences are characterized so that regular and irregular labor are additively separable. Furthermore, leisure time is spent on both irregular work effort and non-market activities. Simulations are conducted to examine the performance of the model economy and to compare the resulting cyclical features with related empirical findings. In addition, computational experiments allow to analyze the effects of different tax structures, enforcement rates and tastes for irregular labor on the volatility and comovements of aggregate variables. These simulations and experiments overall offer a more comprehensive view of the cyclical implications of the shadow economy.

Keywords: Underground economy; Shadow economy; Business cycles; Dynamic stochastic general equilibrium models

JEL codes: E26, E32, H26, O17

[*]I am greatly indebted to my adviser, Christian Zimmermann, for his valuable comments and support. Also, I would like to thank seminar participants at the University of Connecticut. Any errors are entirely mine.

1 Introduction

This paper explores some of the macroeconomic implications of the existence of an underground sector using a general equilibrium model of aggregate fluctuations. Such a sector comprises the production of legal goods by legally registered firms, but either it is not reported to the tax authority or it is conducted with workers that are not legally registered. In addition to firms' non-compliance of tax and labor regulations, the existence of a shadow economy allows individuals to 'work off the books' in economies in which the employer reports official labor income to the tax enforcement agency as a matter of course. Given these circumstances, two sectors with possibly different equilibrium wage rates and other dissimilar characteristics coexist, allowing for wage adjustment and both households and firms making labor choices in response to technology and fiscal policy shocks.

There exist a few equilibrium business cycle models addressing shadow economic activities. These models usually share some features in common with the household production literature (see Benhabib, Rogerson, & Wright, 1991; Greenwood & Hercowitz, 1991; McGrattan, Rogerson, & Wright, 1997), with the main difference being that they consider commodities produced in the official and unofficial sectors as substitutes that are tradeable in the market. Some of these models have been built based on the arguable premise that fluctuations in both sectors are negatively correlated. Examples of the so-called double business cycle approach are Busato and Chiarini (2004) and Russo (2008). While the present paper develops a real business cycle model along the lines of the home production literature, it precludes the assumption of a countercyclical relation between regular and irregular work effort.

In particular, the model developed in this paper relies to a large extent on that of Busato and Chiarini (2004). The characterization of preferences, however, is its main distinguishing aspect, so that official and unofficial labor are additively separable. Furthermore, leisure time is spent on both irregular work effort and non-market activities. These two features aim not to impose any sort of comovement of regular and irregular production. Given these characteristics, empirical evidence on the elasticity of labor supply in the underground sector is used to calibrate the model. Then, simulations are conducted to examine how the model economy reacts to technology and fiscal policy shocks. The model is able to replicate some stylized facts of business cycle fluctuations fairly well. Moreover, the unofficial sector turns out to be weakly countercyclical, thus reinforcing the relevance of the present approach.

The paper is organized as follows. The next section summarizes the literature on business cycles including an irregular sector and stresses the main distinguishing elements of the approach pursued here. These elements are developed in the third section, while selection and estimation of the model parameters are described in the fourth. Then, the fifth section examines the model's ability to reproduce basic facts about U.S. business cycles. Sensitivity analyses are further applied to consider the potential effects of different tax structures, enforcement rates and tastes for irregular labor on the decision to divert resources underground and on macroeconomic fluctuations. This exercise allows to contrast the moments obtained from the simulations with the estimated correlations featured in Granda-Carvajal (2010), and to make some inferences about the determinants and aggregate implications of the shadow economy. The last section concludes.

2 Background literature

The problem of tax compliance is as old as taxes themselves. Yet the first attempt to address this issue theoretically dates back to Allingham and Sandmo (1972), whose approach to tax evasion relies on the literature on the economics of crime and the economics of risk and uncertainty. In their seminal formulation, what might deter an individual from dodging income taxes is a fixed probability that any taxable income understatement will be detected and subjected to a penalty over and above payment of the true tax liability itself. While higher penalties and audit probabilities clearly discourage cheating in this simple model, the effects of enforcement variables all become ambiguous when the decision about how much income to report is made simultaneously with the decision of how much to work (Andreoni, Erard, & Feinstein, 1998). One alternative way to tackle these two choices is to allow individuals to switch to the underground economy or informal sector. Although the literature on tax evasion has paid most attention to labor supply, there are not many models of the underground economy.

Since its very beginnings, the theoretical literature has been keenly interested in ascertaining whether higher tax rates generate more or less compliance, yielding no unambiguous predictions as yet (Andreoni et al., 1998). In contrast, empirical studies often cite the rise of the burden of taxes and social security contributions as one of the most important causes of the increase of informality. It has been argued that taxes not only affect labor-leisure choices, but also stimulate participation in unofficial economic activities. The greater the difference between the cost of labor in the formal sector and after-tax earnings from work, the greater is the incentive to avoid this difference by diverting resources underground (Schneider, 2005). As this difference depends broadly on the social security system and the overall tax burden, these are key determinants of the existence and rise of the subterranean economy.

Nonetheless, there is no consensus regarding how the unofficial sector interacts with the official one over the business cycle. Several studies have estimated time series of underground output and ascertained their comovements with GDP using a variety of methods (e.g., currency demand, electricity use, MIMIC). Following this approach, Bajada (2003) and Giles (1997) provide evidence of a procyclical relation between the two sectors in Australia during the period 1967-95 and New Zealand in 1968-94, respectively. In contrast, Russo (2008) finds that the cyclical component of the US GDP is negatively correlated with the cyclical component of the hidden output through 1960-2003, suggesting the existence of a 'double business cycle' wherein peaks of the regular economy coincide with troughs of the irregular one and vice versa. This lack of consensus partly reflects the inherent difficulty of obtaining accurate estimates of the extent of the shadow economy at cyclical frequencies.

More focused on stylized facts of the official economy, Ferreira-Tiryaki (2008) presents evidence that countries with a large informal sector tend to undergo increased volatility of major aggregates such as output, investment and consumption over the business cycle. This evidence is confirmed in Granda-Carvajal (2010), which also shows that unemployment is more countercyclical while hours becomes more procyclical the smaller the unofficial economy. The latter paper concludes that much more needs to be done in order to understand the implications of underground activities on macroeconomic performance, given that business cycle models with shadow activities have not addressed the cyclical behavior of some of the mentioned variables as summarized further below. Whereas Ferreira-Tiryaki's findings are explained using a lending-channel argument, my results seem likely to be rationalized within a real business cycle (RBC) framework.

In this regard, there exist a few studies addressing shadow economic activities in business cycles. These studies do not exhibit a uniformity of purposes, as they are all motivated by different concerns. Thus, while Conesa, Díaz-Moreno, and Galdón-Sánchez (2002) address the observed negative relationship between the ratio of employment to population and the standard deviation of GDP, Restrepo Echavarría (2008) shows that relatively high consumption volatility is to be expected in countries with large and poorly measured informal economies. Notwithstanding, these studies share some features in common with the household production literature (see Benhabib et al., 1991; Greenwood & Hercowitz, 1991; McGrattan et al., 1997), with the main difference being that they consider commodities produced in the official and the unofficial sectors as substitutes that are tradeable in the market.

Further, these studies exhibit discrepancies as to the propagation mechanisms involved and the government's role. Most of the models rely to a large extent on the substitution possibilities in labor and consumption allowed for by their two-sector nature. In addition, Conesa et al. (2002) assume that the decision to work in the official or the unofficial sector is mutually excludable and that formal labor is indivisible. That way, the models highlight a process of inter-sectoral reallocation of labor (Conesa et al., 2002; Busato & Chiarini, 2004; Russo, 2008) and consumption goods (Restrepo Echavarría, 2008) in response to aggregate productivity shocks and/or tax disturbances. Regarding the fiscal characterization, some papers introduce an agency that monitors and enforces tax compliance through penalty charges (Busato & Chiarini, 2004; Russo, 2008), whereas in others the government does not play such a role (Restrepo Echavarría, 2008) or plays no role whatsoever (Conesa et al., 2002).

The simulations conducted in these studies overall provide a better account of the business cycle stylized facts than do models without informality. They also show a reallocation of labor and production between the official and the unofficial sectors that rationalizes the multiplicity of concerns motivating the RBC approaches to shadow economic activities. Thus, while Busato and Chiarini (2004) argue that this resource reallocation can resolve some unsatisfactory results concerning the labor market such as the employment variability puzzle,[1] these authors and Restrepo Echavarría (2008) claim that the existence of two income sources –both of them highly volatile– implies

[1]The employment variability puzzle refers to the fact that employment (or total hours worked) is almost as variable as output, and strictly procyclical, something difficult to replicate in a standard neoclassical model.

that individuals are subject to two sources of fluctuations, which gives rise to high consumption volatility. Except for the latter paper, computational experiments assessing the effects of variations in the size of the informal sector on aggregate fluctuations are not conducted systematically.

Out of these theoretical studies, one particular strand deserves special attention. Some models have been built based on the arguable premise that business cycles in the official and unofficial sectors are negatively correlated, the most representative of which being Busato and Chiarini (2004) and Russo (2008). Given the lack of unambiguous evidence on the cyclicality of the shadow economy underlined above, there are no grounds for taking such 'double business cycle' for granted in the development of equilibrium models of aggregate fluctuations. The present paper thus challenges this notion by relaxing two assumptions in Busato and Chiarini's utility function: non-separable preferences in the two types of labor, coupled with the sole use of leisure time for underground activities. That way, the model economy is allowed to deliver its own pattern of comovement between the two sectors, instead of a particular one being imposed.

Given the model setup, empirical evidence on the elasticity of labor supply in the underground sector is used to calibrate the model. Then, simulations are conducted to examine how the economy reacts to technology and fiscal policy shocks. The model is able to replicate some stylized facts of business cycle fluctuations fairly well. Moreover, the unofficial sector turns out to be weakly countercyclical, thus reinforcing the relevance of the present approach. Some sensitivity analyses finally are conducted with the aim to evaluate the influence of different enforcement structures, tax systems and willingnesses to substitute official for unofficial labor on the volatility and comovements of macroeconomic variables. It is noteworthy that none of the studies surveyed above addresses the cyclical consequences of changes in these parameters.

3 The model economy

The present model relies to a large extent on that of Busato and Chiarini (2004). Accordingly, there are two sectors in the economy: official and unofficial. The official or regular sector produces everything that is measured in GDP strictly following all the laws and regulations in place. The unofficial or irregular sector, conversely, comprises the production of legal goods by legally registered firms, but either it is not reported to the tax authority or it is conducted with workers that are not legally registered.

The economy is populated by the government, a large number of identical firms, and a large number of identical households, all of whom are infinitely-lived. The government uses tax revenues to finance a stochastic stream of expenditures and enforces a monitoring system for tax evasion. Firms solve an expected profit maximization problem every period, subject to a technological constraint and to the possibility of being discovered and penalized for producing 'off the books'. Households choose consumption, investment and hours to work on each date and in each sector.

3.1 Firms

Competitive firms in this economy purchase capital and labor services from households to produce a homogeneous good. The representative firm uses two different technologies: one associated with the official sector, and the other with the underground sector.[2] Let y_t^f denote formally-produced output, and y_t^u output produced in the shadows. Technologies are specified as follows:

$$y_t^f = z_t^f k_t^\alpha (l_t^f)^{1-\alpha} \tag{3.1}$$

[2]One could imagine a firm producing in the regular economy in the day, and in the irregular economy by night.

and

$$y_t^u = z_t^u l_t^u. \tag{3.2}$$

Formal output y_t^f is the result of a Cobb-Douglas technology applied to capital, k_t, and regular labor, l_t^f. Given that shadow economic activities are labor intensive, y_t^u is produced using solely underground labor, l_t^u. This amounts to assuming that irregular production has a fixed stock of capital. Finally, z_t^f and z_t^u are sectoral productivity shocks.[3] Total production is defined as $y_t^{tot} = y_t^f + y_t^u$.

Revenues accrued in the official sector, $q_t^f(1-\tau_t^f)y_t^f$, are taxed at the stochastic rate τ_t^f. The representative firm does not pay taxes on revenues accrued unofficially, $q_t^u y_t^u$, where q_t^u is the price of commodities produced off the books. However, the firm may be discovered evading and forced to pay corporate taxes, augmented by a surcharge factor $\varsigma \geq 1$. Note that since the officially-produced and the unofficially-produced goods are identical, they must have the same price in equilibrium $q_t^f = q_t^u \equiv q_t$. For simplicity, this price is imposed along the solution and normalized to unity.

If the firm is discovered, with probability $\phi \in (0,1)$, revenues are $y_t^D = (1-\tau_t^f)y_t^f + (1-\varsigma\tau_t^f)y_t^u$. With probability $1-\phi$, the firm is not discovered, in which case revenues equal $y_t^{ND} = (1-\tau_t^f)y_t^f + y_t^u$. Thus, total expected revenues at time t are

$$E(y_t) = (1-\tau_t^f)y_t^f + (1-\phi\varsigma\tau_t^f)y_t^u. \tag{3.3}$$

The cost of renting capital equals its marginal productivity, r_t. Formal labor cost

[3]Note that the latter shock may be seen as representing several inputs such as managerial skills, creativity, workplace organization, etc. These elements exist in the irregular sector, just as they do in regular economic activities, and are capable of rising the corresponding labor productivity. More precisely, the productivity shock idiosyncratic to the underground sector can be associated, for instance, to the following three arguments: First, since labor in this sector is very flexible, and the worked hours are voluntary, one could argue that the employee's motivation is stronger. Second, a significant component of the irregular labor force is made of immigrants, who try to be as productive as they can in order to be hired as regular workers. Finally, there are young pensioners and unemployeds entering the 'black' labor market who might have a high productivity, but choose to work in the shadows for an additional income while keeping their government benefits at the same time.

is represented by the wage paid for hours worked in the official sector, augmented by the fixed payroll tax rate τ^s. Let $w_t^f \equiv (1+\tau^s)w_t$ define such a cost, where w_t is the pre-tax wage. Since the representative firm does not pay social security contributions on hours hired 'under the table', the cost of unofficial labor equals the pre-tax wage, i.e. $w_t^u = w_t$. As with the corporate tax rate, however, the firm faces the probability ϕ of being detected evading and forced to pay payroll taxes, increased by the surcharge factor ς. If the firm is caught dodging either of the two tax liabilities, then it is penalized on both payments.

With probability ϕ, the firm is discovered and total costs are $C_t^D = w_t^f l_t^f + (1 + \varsigma\tau^s)w_t l_t^u + r_t k_t$. If the firm is not discovered, with probability $1-\phi$, total costs equal $C_t^{ND} = w_t^f l_t^f + w_t^u l_t^u + r_t k_t$. Thus, total expected costs at time t are

$$E(C_t) = w_t^f l_t^f + (1 + \phi\varsigma\tau^s)w_t l_t^u + r_t k_t. \tag{3.4}$$

The representative firm produces so as to maximize expected profits $E(y_t) - E(C_t)$ each period, taking market prices as given. Firm's behavior is characterized by the first order conditions:

$$r_t = (1-\tau_t^f)\alpha z_t^f k_t^{\alpha-1}(l_t^f)^{1-\alpha}, \tag{3.5}$$

$$w_t^f = (1-\tau_t^f)(1-\alpha) z_t^f k_t^{\alpha}(l_t^f)^{-\alpha} \tag{3.6}$$

and

$$w_t = \frac{1-\phi\varsigma\tau_t^f}{1+\phi\varsigma\tau^s} z_t^u. \tag{3.7}$$

These conditions imply that capital and both regular and irregular labor are paid their real marginal products. Setting equations (3.6) and (3.7) equal, and taking account of the definitions of w_t^f and w_t^u above, the following arbitrage condition is obtained:

$$\frac{1-\tau_t^f}{1+\tau^s}(1-\alpha)z_t^f k_t^\alpha (l_t^f)^{-\alpha} = \frac{1-\phi\varsigma\tau_t^f}{1+\phi\varsigma\tau^s} z_t^u. \tag{3.8}$$

This condition underlines that the firm equates the marginal products of labor across the two sectors each period, taking into consideration that the regular sector pays taxes while the irregular sector aims to escape taxation.

3.2 Households

The representative household chooses consumption and hours to work on each date and in each sector to maximize the present discounted value of utility $E_0 \sum_t \beta^t U(c_t, l_t^f, l_t^u)$. Household behavior is represented by adapting Cho and Cooley's (1994) motivation for modeling family labor supply. According to these authors, households make labor supply decisions along both the intensive (hours worked) and the extensive (employment) margins. Here these two dimensions are reinterpreted as representing households' labor supply in the regular and underground sectors. The momentary utility function is assumed to be separable between consumption and leisure (labor) as follows:

$$U(c_t, l_t^f, l_t^u) = \ln c_t - a\frac{(l_t^f)^{1+\gamma}}{1+\gamma} - b\frac{(l_t^u)^{1+\eta}}{1+\eta}. \tag{3.9}$$

A well-behaved utility function implies that $a, b \geq 0$, $\gamma, \eta > 0$, and that all its components be twice continuously differentiable and increasing. The second term, $a\frac{(l_t^f)^{1+\gamma}}{1+\gamma}$, represents the disutility of working in the formal sector, while the last term, $b\frac{(l_t^u)^{1+\eta}}{1+\eta}$, reflects the idiosyncratic cost of working in the underground economy. This cost may be associated to the lack of any social insurance in this sector. Alternatively, one could interpret the elasticity $1/\eta$ as positively related to the disutility of working in the shadows. Note that implicit in the choice of this functional form is the absence of adjustment costs for moving labor supply (demand) across sectors, so that labor reallocation is almost completely unconstrained.

Households pay a stochastic tax rate τ_t^w on official labor income and receive a lump-sum transfer T_t from the government. Thus, they face the budget constraint:

$$c_t + i_t = (1 - \tau_t^w) w_t l_t^f + w_t l_t^u + r_t k_t + T_t, \tag{3.10}$$

where i_t denotes investment at time t.[4] Investment, in turn, increases the capital stock according to the state equation:

$$i_t = k_t - (1 - \delta) k_{t-1}, \tag{3.11}$$

where δ denotes the depreciation rate.

With the instantaneous utility function defined as in equation (3.9), the value function $V(k_t, K_t, A_t)$ of the representative household satisfies

$$V(k_t, K_t, A_t) = \max_{k_{t+1}, l_t^f, l_t^u} \left\{ U(c_t, l_t^f, l_t^u) + \beta E_t \left[V(k_{t+1}, K_{t+1}, A_{t+1}) \right] \right\},$$

subject to the budget constraint (3.10) and the law of motion for the capital stock (3.11). As specified below, A_t is a vector of technology and fiscal policy shocks. Household decisions are characterized by the intra-temporal conditions for labor supply allocation:

$$a (l_t^f)^\gamma = \frac{1 - \tau_t^w}{c_t} \frac{1 - \tau_t^f}{1 + \tau^s} (1 - \alpha) z_t^f k_t^\alpha (l_t^f)^{-\alpha} \tag{3.12}$$

[4]Note that households are paid the pre-tax wage for working in both sectors. Since they are subject to taxation only on official earnings, this implies that the regular wage is lower than the irregular one. This is not an implausible assumption, as Lemieux, Fortin, and Frechette (1994) show using micro data from a survey conducted in Québec City (Canada). These authors develop a model of time allocation supporting their empirical observations. In the context of the present model, households presumably are willing to receive a higher remuneration for not having social insurance in the underground economy.

and

$$b(l_t^u)^\eta = \frac{1-\phi\varsigma\tau_t^f}{1+\phi\varsigma\tau^s}\frac{z_t^u}{c_t}, \tag{3.13}$$

and by the Euler equation:

$$1 = \beta E_t \left(\frac{c_t}{c_{t+1}}\right)(1 + r_{t+1} - \delta), \tag{3.14}$$

where $r_t \equiv (1-\tau_t^f)\alpha z_t^f K_t^{\alpha-1}(l_t^f)^{1-\alpha}$ from firm profit maximization (see Equation 3.5).

3.3 Government

The government produces non-productive services and makes transfer payments each period by collecting taxes on firms' revenues and labor earnings. Government consumption is assumed to follow a stochastic process given by

$$g_t = z_t^g y_t^{tot}, \tag{3.15}$$

where z_t^g is a random variable and y_t^{tot} is aggregate output as defined above. The flow budget constraint is

$$g_t + T_t = \tau_t^f y_t^f + \phi\varsigma\tau_t^f y_t^u + (\tau^s + \tau_t^w)w_t l_t^f + \phi\varsigma\tau^s w_t l_t^u. \tag{3.16}$$

Note that the lump-sum transfer T_t is treated as a residual that takes on whatever value is necessary to satisfy the government budget constraint at each point in time, given z_t^g, the productivity shocks and the tax disturbances. Likewise, the specification of Equation (3.16) assumes that the government never issues debt.

In order to discourage fiscal evasion, the government enforces a monitoring system whereby firms are inspected each period with a fixed probability ϕ and forced to pay contributions to social insurance and the corporate tax rate on the previously concealed activities. The government is assumed to be always able to perfectly identify the amount of underground production and labor at every inspection or, equivalently, that the cost of verifying the amount of hidden production/labor is zero, so that all the proceeds from the taxation of these activities and the fines are effectively revenue for the government.[5] Given that taxes on irregular production/labor are collected only with a certain probability, the government balances its budget in expectation.

3.4 Sources of aggregate fluctuations

Sectoral productivity and fiscal policy shocks are formalized as a vector of exogenous state variables that follows an autoregressive process around a mean in logs:

$$A_t = PA_{t-1} + \epsilon_t, \tag{3.17}$$

where A_t is a vector $\left[\ln(z_t^f/z_{ss}^f), \ln(z_t^u/z_{ss}^u), \ln(\tau_t^f/\tau_{ss}^f), \ln(\tau_t^w/\tau_{ss}^w), \ln(z_t^g/z_{ss}^g)\right]'$ containing the ratio of the time-t value of each state variable (i.e. productivity shocks, stochastic tax rates, and shock on government expenditures) to their steady state values. Likewise, $P = diag(\rho_i)$, where $i = f, u, \tau^f, \tau^w, g$, is a 5×5 matrix describing the autoregressive components of each of the five shocks. Lastly, the innovation $\epsilon = [\epsilon_f, \epsilon_u, \epsilon_{\tau^f}, \epsilon_{\tau^w}, \epsilon_g]'$ is a vector of random variables with zero mean and covariance matrix

[5]While this assumption greatly simplifies the analysis, it is evidently unrealistic as it ignores enforcement costs.

$$\Omega = \begin{pmatrix} \sigma_f^2 & \sigma_{fu} & \sigma_{f\tau^f} & \sigma_{f\tau^w} & \sigma_{fg} \\ \sigma_{uf} & \sigma_u^2 & \sigma_{u\tau^f} & \sigma_{u\tau^w} & \sigma_{ug} \\ \sigma_{\tau^f f} & \sigma_{\tau^f u} & \sigma_{\tau^f}^2 & \sigma_{\tau^f \tau^w} & \sigma_{\tau^f g} \\ \sigma_{\tau^w f} & \sigma_{\tau^w u} & \sigma_{\tau^w \tau^f} & \sigma_{\tau^w}^2 & \sigma_{\tau^w g} \\ \sigma_{gf} & \sigma_{gu} & \sigma_{g\tau^f} & \sigma_{g\tau^w} & \sigma_g^2 \end{pmatrix},$$

where $\sigma_{ij} = \theta_{ij}\sigma_i\sigma_j$.

3.5 Equilibrium

A recursive competitive equilibrium for this economy consists of a set of prices $\{w_t, r_t\}_{t=0}^{\infty}$, a value function $V(k_t, K_t, A_t)$, decision rules $\left\{c_t, i_t, l_t^f, l_t^u, k_{t+1}, K_{t+1}\right\}_{t=0}^{\infty}$, and policy functions g_t and T_t such that:

- households maximize utility;
- firms maximize profits;
- the government balances its budget;
- individual and aggregate decisions are consistent, i.e. $k_t = K_t$, and
- markets clear.

Note that the market clearing condition implies that the decision rules satisfy the resource constraint

$$c_t + i_t + g_t = y_t^{tot}. \tag{3.18}$$

In addition, the competitive equilibrium is not Pareto optimal due to the distortionary character of taxes in the model. This is why the solution method is applied to the decentralized economy rather than to a fictitiuos planner.

4 Calibration

The model parameters are calibrated to the U.S. economy with the aim to replicate its annual aggregate fluctuations. The main reason for so proceeding is to represent tax shocks as closely as they take place in reality. Tax rates, as Braun (1994) notes, probably vary little over the course of a year, so the strongest comovements are likely to occur at annual frequencies. Furthermore, it is worth noting that the fact that data on the underground economy are difficult to obtain makes calibration at higher frequencies substantially more complicated.

The system of equations used to compute the dynamic equilibria of the model depends on a set of twelve parameters. Five pertain to household preferences (β, a, b, γ, and η), five to the tax structure and the institutional context (the probability of a firm being detected ϕ, the surcharge factor ς, the payroll tax rate τ^s, and the steady state values of labor and corporate income tax rates τ^w_{ss} and τ^f_{ss}), and the remaining two parameters to technology (the capital share α and the depreciation rate δ). In addition to these parameters, one has to characterize the innovations and their interactions.

Given that the calibration is aimed to match fluctuations at annual frequencies, the values of the discount factor and the depreciation rate are 0.95 and 0.1, respectively. Also, the capital income share is set to 0.36. These three parameter values are commonplace in the existing business cycle literature. In contrast, calibrating the utility parameters a, b, γ, and η presents the most difficult problem. Adapting again from Cho and Cooley (1994), the relation between formal and underground labor is obtained using the intratemporal first-order conditions for the household as follows:

$$\ln(l^u_t) = \frac{1}{\eta} \ln\left(\frac{a}{b(1-\tau^w_t)}\right) + \frac{\gamma}{\eta}\ln(l^f_t). \tag{4.1}$$

With γ set to 1, a value often assumed in business cycle studies, this relationship

Table 4.1: Preferences, technology and enforcement parameters

β	δ	α	a	b	γ	η	ϕ	ς
0.95	0.10	0.36	3.7569	2.8814	1	0.2381	0.015	1.2

between regular and irregular labor can be estimated so that the other three parameters fit three empirical observations: First, about one-third of the time endowment is spent in labor market activity, and hence the steady-state fraction of formal hours of work is assumed to equal 0.33. Second, Schneider's (2005) estimates suggest that the steady-state fraction of underground labor in the U.S. is 0.084.[6] Third, an elasticity of unofficial labor with respect to official labor (γ/η) of about 4.2 was derived from a study on labor supply when tax evasion is an option conducted in Norway (Jørgensen, Ognedal, & Strøm, 2005).[7]

It is worthy of note regarding the third observation that there are not many empirical analyses of labor supply in the underground sector. In addition to the study presently used, Frederiksen, Graversen, and Smith (2005) jointly estimate labor supply in the taxed and untaxed sectors for male Danish workers. Extensive searches have yielded no works of this sort for the US at all. Consequently, one shall qualify the procedure pursued here by recognizing that workers in Scandinavian countries face relatively high marginal tax rates on wage income compared to workers in most other OECD countries. Taking Equation (4.1) and the empirical observations as a whole, it follows that $a = 3.7569, b = 2.8814$, and $\eta = 0.2381$.

As for the enforcement parameters, these are taken from Slemrod and Yitzhaki's (2002) survey on tax evasion and administration. They report that the fraction of

[6]These estimates of shadow economic activity cover the period 1989-2003 and were developed using the dynamic multiple input multiple indicator (DYMIMIC) approach. I take an average of them.

[7]This parameter value corresponds to the ratio of the elasticity of hidden labor to the elasticity of formal labor, conditional on individuals actually evading taxes, in 2001. Other values, based on conditional and unconditional elasticities obtained from the same study, are explored in the sensitivity analysis in the upcoming section.

tax returns audited in the U.S. is about 1.5%, whereas the statutory penalty for non-criminal evasion is about 20%. Hence, the values of the probability of detection and the surcharge factor used in the baseline model are 0.015 and 1.2, respectively. The cyclical effects of different values of these two parameters are assessed in the subsequent section. All the parameters mentioned thus far are summarized in Table 4.1.

Moving on to the tax structure, the values for the tax rates are obtained from the OECD Tax Database. The social security contribution rate τ^s is set at 0.0765, which corresponds to the rate in place since 1990. It is worth noting here that the assumption that the payroll tax rate takes a constant value, instead of displaying a stochastic nature, is aimed to consider this particular feature of employer's taxation in the United States. In contrast, the parameters used for corporate and personal income tax rates indicate steady-state levels, as these tax rates are much more likely to exhibit changes over time. The value of the corporate tax rate used, $\tau^f_{ss} = 0.3921$, stands for the combined federal and state statutory corporate income tax rate, while the chosen labor income tax rate, $\tau^w_{ss} = 0.224$, represents the combined federal and sub-national government income tax (plus employee social security contributions) as a percentage of gross wage earnings.[8] For further details, see *OECD Tax Database* (2010).

In the same vein, the steady-state value of the share of government expenditures in total output is estimated by taking the average of the ratio of government consumption expenditures and gross investment to GDP during 1960-2006. Series on these aggregates are featured by the Bureau of Economic Analysis. Furthermore, the steady-state value of official productivity is normalized to unity so that, using the arbitrage condition for labor in the tax evading sector (Equation 3.8), one obtains that the steady-state

[8]These tax rates correspond to the values in place in recent years. However, one could estimate the steady-state values by taking averages over a time period. Moreover, one should ideally use estimates of effective tax rates consistent with the tax distortions faced by a representative agent in a dynamic general equilibrium framework (as argued, for instance, in Mendoza, Razin, & Tesar, 1994). Doing so might lead to slightly different values, without substantially changing the results of the numerical simulations.

value of unofficial productivity is 0.4464. This value points out that productivity in the informal sector is low relative to productivity in the formal sector, a feature extensively documented in the literature (Loayza, 1996; La Porta & Shleifer, 2008).

To maintain symmetry in the model, and since there is no evidence about the persistence of the productivity shocks in the irregular sector (ρ_u), this parameter is assumed equal to ρ_f. The values for these parameters, in turn, are borrowed from the work of Benhabib et al. (1991) on home production, but adjusted to take account of the difference in frequencies studied.[9] Hence, the autocorrelation coefficients for the productivity shocks are $\rho_f = \rho_u = 0.95^4$. Also following Benhabib et al. (1991), the standard deviation of the productivity shocks is fixed to 0.007 and the correlation of the shocks between the two sectors used is 0.66.[10]

Table 4.2: Parameter values for structure of shocks

Parameter	Description	Value	Source
ρ_f, ρ_u	Persistence of sectoral productivity shocks	0.814	BRW (1991)
ρ_{τ^f}	Persistence of corporate tax rate shocks	0.786	Braun (1994)
ρ_{τ^w}	Persistence of labor income tax rate shocks	0.95	Braun (1994)
ρ_g	Persistence of government expenditures shocks	0.702	Braun (1994)
σ_f, σ_u	Standard deviation of sectoral productivity shocks	0.007	BRW (1991)
σ_{τ^f}	Std. dev. of corporate tax rate shocks	0.186	Braun (1994)
σ_{τ^w}	Std. dev. of labor income tax rate shocks	0.049	Braun (1994)
σ_g	Std. dev. of government expenditure shocks	0.036	Braun (1994)
θ_{fu}	Correlation of sectoral productivity shocks	0.66	BRW (1991)
$\theta_{f\tau^f}$	Corr. b/w official TFP and corporate tax shocks	-0.454	Braun (1994)
$\theta_{f\tau^w}$	Corr. b/w official TFP and labor income tax shocks	0.022	Braun (1994)
θ_{fg}	Corr. b/w official TFP and govt. expenditures	-0.533	Braun (1994)
$\theta_{\tau^f\tau^w}$	Corr. b/w corporate and labor income tax shocks	0.122	Braun (1994)
$\theta_{\tau^f g}$	Corr. b/w corporate tax and govt. exp. shocks	0.355	Braun (1994)
$\theta_{\tau^w g}$	Corr. b/w labor income tax and govt. exp. shocks	0.073	Braun (1994)

Note: BRW (1991) refers to Benhabib et al. (1991).

[9] The model by Benhabib et al. (1991) is calibrated to match fluctuations at quarterly frequencies.

[10] I checked the robustness of the benchmark model to changes in the latter parameter using zero (0) and -0.66 as correlations. Neither the impulse response functions nor the obtained moments deviate significantly from the ones of the initial calibration. These results are available upon request.

Finally, the parameters characterizing the distributional properties of fiscal policy shocks (i.e., persistence, standard deviations and correlations between innovations), as well as the interaction of these shocks with formal technology, are calibrated with some estimates obtained by Braun (1994). This author employs historical data to develop a statistical model of the government's feedback rule during the postwar period (1956-1980) using the Generalized Method of Moments. The values of the parameters characterizing the structure of shocks are presented in Table 4.2.

5 Model evaluation

This section compares the performance of the present model with actual data and with selected alternative approaches. After discussing the results, some computational experiments assessing the cyclical implications of changes in the model parameters are conducted. The analyzed parameters pertain to the tax and enforcement structure, as well as households' tastes regarding work effort in the underground sector. Lastly, some inferences derived from the sensitivity analyses are contrasted with the empirical findings reported by Granda-Carvajal (2010), further allowing to evaluate the relevance of this and other related models and to highlight some promising extensions.

5.1 Moments and comparisons

To analyze how well the model accounts for aggregate fluctuations, a number of simulated moments are compared with the stylized facts characterizing the cyclical behavior of the U.S. economy during the period 1960-2006. The data used to obtain the stylized facts were taken from the National Income and Product Accounts calculated by the Bureau of Economic Analysis, in what regards real GDP and the expenditure components (i.e. consumption, investment, government expenditures), and from the International

Economic Database, which features an index of total hours in the manufacturing sector (base year=1992). The dataset was compiled from web-based versions.

Before computing any statistics, both the actual time series and the generated series are logged and detrended using the Hodrick-Prescott filter with a smoothing parameter of 100. Detrending the series in this way is now standard practice, and it is adopted here to facilitate comparison with McGrattan et al. (1997) and Busato and Chiarini (2004). Another advantage of such a procedure is that standard deviations can be interpreted as mean percentage deviations from the trend. After filtering the series, second moments are calculated from each of them. The relative volatility of each variable to output (official and aggregate) is calculated as the ratio of the standard deviation of the two variables, whereas the correlations involve each variable and output or formal hours.

Table 5.1: Relative standard deviations across models

	Data	Model forecast		MRW('97)	BC ('04)
	$\sigma(x)/\sigma(y^f)$	$\sigma(x)/\sigma(y^f)$	$\sigma(x)/\sigma(y^{tot})$	$\sigma(x)/\sigma(y^f)$	$\sigma(x)/\sigma(y^{tot})$
GDP	1.00000	1.00000	1.12169	1.0000	1.86
Total output	–	0.89151	1.00000	–	1.00
Consumption	–	0.30647	0.34377	–	0.80
* Formal	0.89129	0.54997	0.61689	0.8298	–
Investment	4.08135	5.29241	5.93645	2.4628	6.64
Govt expend	1.49125	0.84004	0.94227	2.3085	–
Hours	1.92521	1.16877	1.31100	0.7447	1.10
Productivity	1.20065	1.36146	1.52713	–	2.00

Notes: MRW('97) refers to McGrattan et al. (1997); BC ('04) refers to Busato and Chiarini (2004).

Tables 5.1 and 5.2 display the relative ability of three different models to match the major stylized facts of the business cycle in the U.S. Note that formal consumption is distinguished from total consumption in the present model by making use of the national accounts identity.[11] This model predicts fairly well the volatility and the cyclicality of

[11]In a closed economy, the accounting identity $y^f = c^f + i^f + g$ holds. Since investment only takes place in the formal sector, the present model assumes that $i \equiv i^f$ and hence formal consumption can be obtained as $c^f = y^f - i - g$.

Table 5.2: Correlations across models

	Correl. with y^f			Correl. with y^{tot}	
	Data	Model	MRW('97)	Model	BC ('04)
GDP	1.0000	1.0000	1.00	0.9930	0.95
Consumption	–	0.3450	–	0.2376	0.69
* Formal	0.8795	0.3225	0.91	0.2106	–
Investment	0.8425	0.9233	0.66	0.9582	0.98
Govt expend	0.3542	0.8763	0.40	0.8894	–
Hours	0.8479	0.9613	0.70	0.9837	0.73
Productivity	-0.5268	-0.9572	–	-0.9761	0.08
	Correl. with hours				
	Data	Model	BC ('04)		
Productivity	-0.8973	-0.9935	0.04		

Notes: MRW('97): McGrattan et al. (1997); BC ('04): Busato and Chiarini (2004).

investment, average hours and labor productivity. Indeed, it improves quite significantly on Busato and Chiarini (2004) as far as the comovements of the latter variable with output and total (formal) hours are concerned. This observation cannot be highlighted more, given that the model reproduces recent tendencies in labor market dynamics with considerable accurateness.[12] Yet it fails to replicate the cyclical properties of government consumption, and understates the properties of private consumption as well. This is in contrast to both of the mentioned studies, especially McGrattan et al. (1997), which mimics the corresponding empirical moments somewhat closely. Despite these shortcomings, one should observe that formal consumption is more volatile than total consumption, as it certainly might be taken as an indication of model's success in resembling the data.

It is well known that to create time series displaying cyclical properties more akin

[12]Among other changes in labor market dynamics during the US postwar period, Galí and van Rens (2010) document a sharp drop in the cyclicality of labor productivity dating back to 1984. The correlation of productivity with output, which used to be strongly positive, fell to a level close to zero, while the correlation of productivity with labor input, which was zero or slightly positive in the earlier period, became negative. These changes overall coincided with the reduction in volatility of output and other macroeconomic aggregates –the so-called Great Moderation.

to those from the U.S. economy, a RBC model needs to have more than one shock. Some studies have addressed this stochastic singularity problem by introducing fiscal policy shocks, especially in the form of distortionary taxation (McGrattan et al., 1997). Distorting taxes have been found to enhance the quantitative properties of the model economy, particularly with regard to the labor market. By inducing large intertemporal and intratemporal substitution effects, fluctuations in personal and corporate income tax rates increase the model's predicted relative volatility of hours worked and reduce the predicted correlation between hours and average productivity (Braun, 1994). According to McGrattan (1994), fiscal policy can also potentially increase the variation in consumption in two-sector models. While substitution between formal and underground activities seems to explain the cyclical behavior of labor market variables in the present model, this does not seem to be the case as for private consumption. Admittedly, that the model generates a small consumption volatility relative to the data is not entirely inconsistent with the stochastic growth literature.

One last thing to note deals with the weak countercyclicality of the underground sector. The simulations indeed yield a correlation of unofficial and official output of -0.2720. To some extent, this result is comparable to the correlation found by Busato and Chiarini (2004), about -0.96, and provide further support to the approach pursued in this paper. It reinforces the contention that imposing a negative comovement between both types of production –as implicit in the double business cycle approach– is unnecessary, since a model economy may display this characteristic outcome without taking such *a priori* connection for granted. In effect, regular and irregular work effort barely comove in the model economy (their cyclical correlation is -0.0256), which is to be expected given these two variables are separable in utility. Hence a model wherein labor supply choices across sectors are independent is not incompatible with a particular pattern of cyclicality in the shadow economy. Appendix A contrasts the second moments presented here with those of an economy with similar characteristics, except for the absence of unofficial activities.

5.2 Sensitivity analysis

In general, the mechanisms driving the results of the present model are not substantially different from those of RBC models with home production. Changes in private agents' behavior induced by relative productivity differentials between the formal and the informal sector, as well as by distortionary taxation, explain the response of the economy to exogenous shocks on absolute sectoral productivity and fiscal policy. Yet the possibility that both firms and households evade taxes by diverting resources to another market sector might have some distinguishable cyclical implications, as Granda-Carvajal (2010) points out. One could conjecture that structural characteristics pertaining to tax systems, the strength of institutions and/or individuals' preferences towards underground work effort may lead to differing cyclical properties of macroeconomic aggregates. The sensitivity analyses conducted in the following aim to corroborate this conjecture.

The present examination considers the effect of changes in a number of parameter values on the volatility and comovements of seven variables: formal output and consumption, investment, government expenditures, formal labor input and its productivity, and the real interest rate. While standard deviation stands as the measure of output variability, the relative standard deviation –that is, the ratio of the standard deviation of the variable in question to the standard deviation of output– accounts for the volatility of the remaining variables. The parameters, seven in total, are organized into three main categories: enforcement structure, tax policies, and preference for underground work effort. In each case, the assessment is suplemented by some graphs illustrating the major patterns found, but the quantitative results on which the figures and the analyses are based can be consulted in Appendices B-D.

5.2.1 Sensitivity to enforcement parameters

As related to the model economy, the enforcement structure comprises the probability of a firm being discovered (ϕ) and the penalty surcharge on concealed tax payments (ς). While the detection probability takes a range of values between zero and one, feasible values for the fine are scanned using a sort of bankruptcy constraint on underground production. Figures 1 and 2 show how the volatility and the comovements of a number of macroeconomic variables change as the detection probability increases. Since the moments of all the variables exhibit similar patterns as penalty rates rise, graphs pertaining to this parameter are not included. The chosen values, along with the resulting moments and the steady-state share of irregular labor, are found in Tables B.1 and B.2 on Appendix B.

As firms are discouraged from diverting resources underground, the economy becomes more vulnerable to the disturbances affecting the formal sector. This might lead to a rise in the volatility of hours and government expenditures, as Figure 1 confirms. Note that formal labor fluctuates increasingly within a rather narrow range, which is apparently enough to mitigate the effects of sectoral productivity and fiscal policy shocks on consumption and investment decisions. This is why formal output, consumption and investment exhibit less fluctuations over the business cycle with firms facing higher detection probabilities and/or tougher penalties. Also, this decrease in the variability of output and its private components may reflect that agents are able to assess the proceeds of their activities more accurately once they remain official altogether.

Figure 2 depicts how the cyclicality of some macroeconomic variables behaves as enforcement is strengthened. While investment and the interest rate become more procyclical, formal labor turns less so. Note, however, that the correlations of these three variables with formal output vary within a fairly small region, which suggests that higher audit rates affect their comovements negligibly. In contrast, the correlations of

Figure 1: Sensitivity of macroeconomic volatility to detection probability

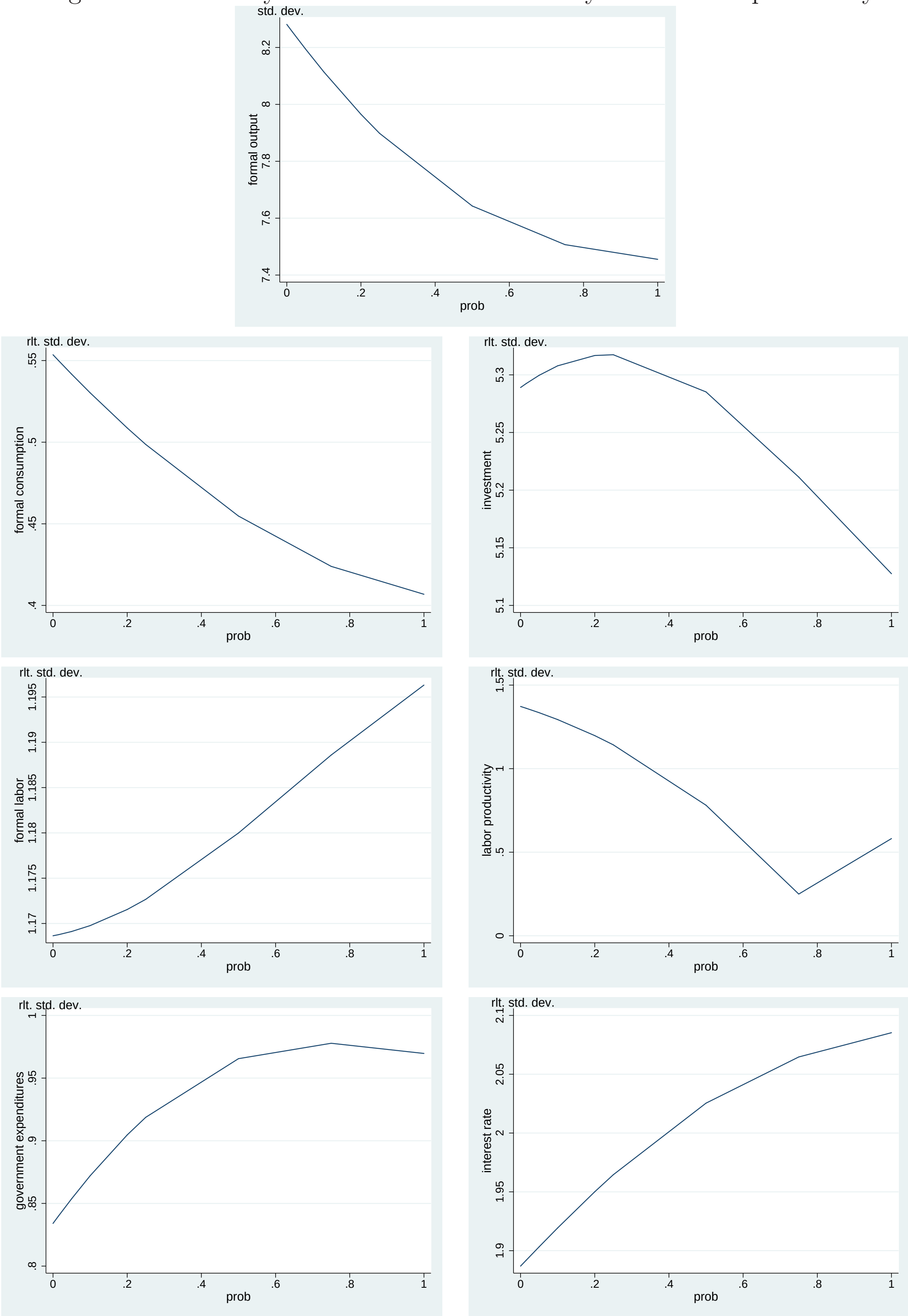
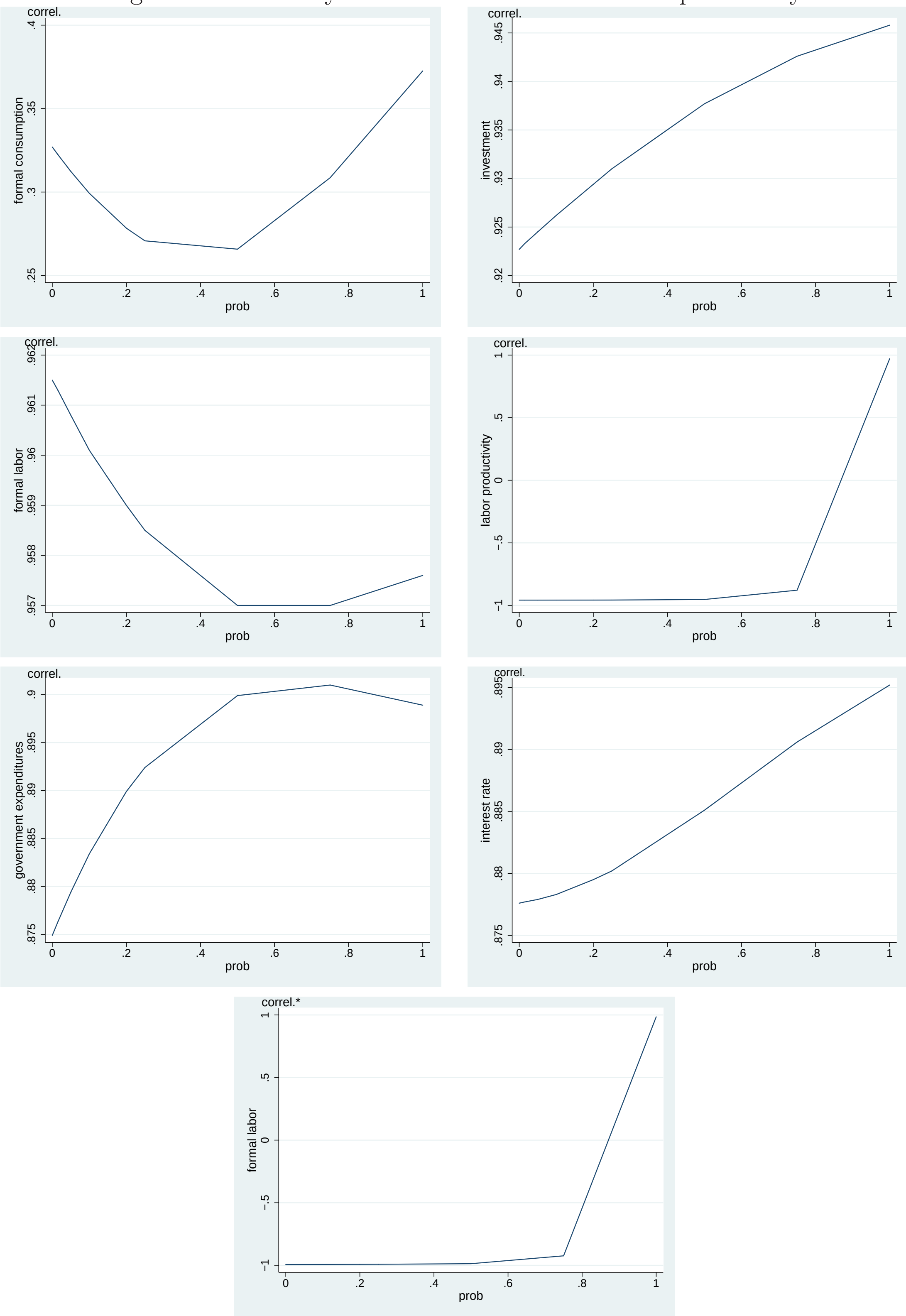


Figure 2: Sensitivity of comovements to detection probability



* Correlation with labor productivity

productivity with output and with labor experience a sharp rise with increases in the detection probability, moving from being strongly countercyclical to highly procyclical. Formal consumption and government expenditures comove more smoothly with output, always being procyclical although they do not exhibit a monotonic pattern.

5.2.2 Sensitivity to tax policies

Tax policies deal with the types of taxes considered individually in the model: payroll tax (τ^s), corporate income tax (τ^f_{ss}) and personal income tax (τ^w_{ss}). Note that these taxes play different roles in the economy since, while the social security contributions rate is modelled as a constant parameter, the other tax rates are made subject to stochastic disturbances. This is why, for the purposes of the present analysis, changes in the latter rates involve modifying their steady-state values. In addition, all three tax rates are jointly adjusted in what is called the 'average tax rate.' The four parameters are varied by percentages with respect to the benchmark values.

One might expect each tax to affect the variability and the comovements of the variables in a somewhat different manner. For comparative purposes, Figures 3 and 4 contrast the impacts of the different tax rates on the cyclical properties of macro aggregates. Changes in the corporate tax rate (long-dashed dotted line) and the average tax rate (solid line) appear to influence macroeconomic fluctuations the most, as the two figures show volatility and cyclicality of each variable changing more prominently with variations in these tax rates. This is unlike the personal income tax rate (dashed line) and the social security contributions rate (dotted line), whose changes barely deviate each variable's moments from the benchmark ones. Hence both tax rates do not exert a substantial effect on business cycle fluctuations. The following analysis, consequently, focuses on the cyclical implications of variations in the average tax rate.

A higher burden of taxation and social security contributions reduces the expected

Figure 3: Sensitivity of macroeconomic volatility to tax rates

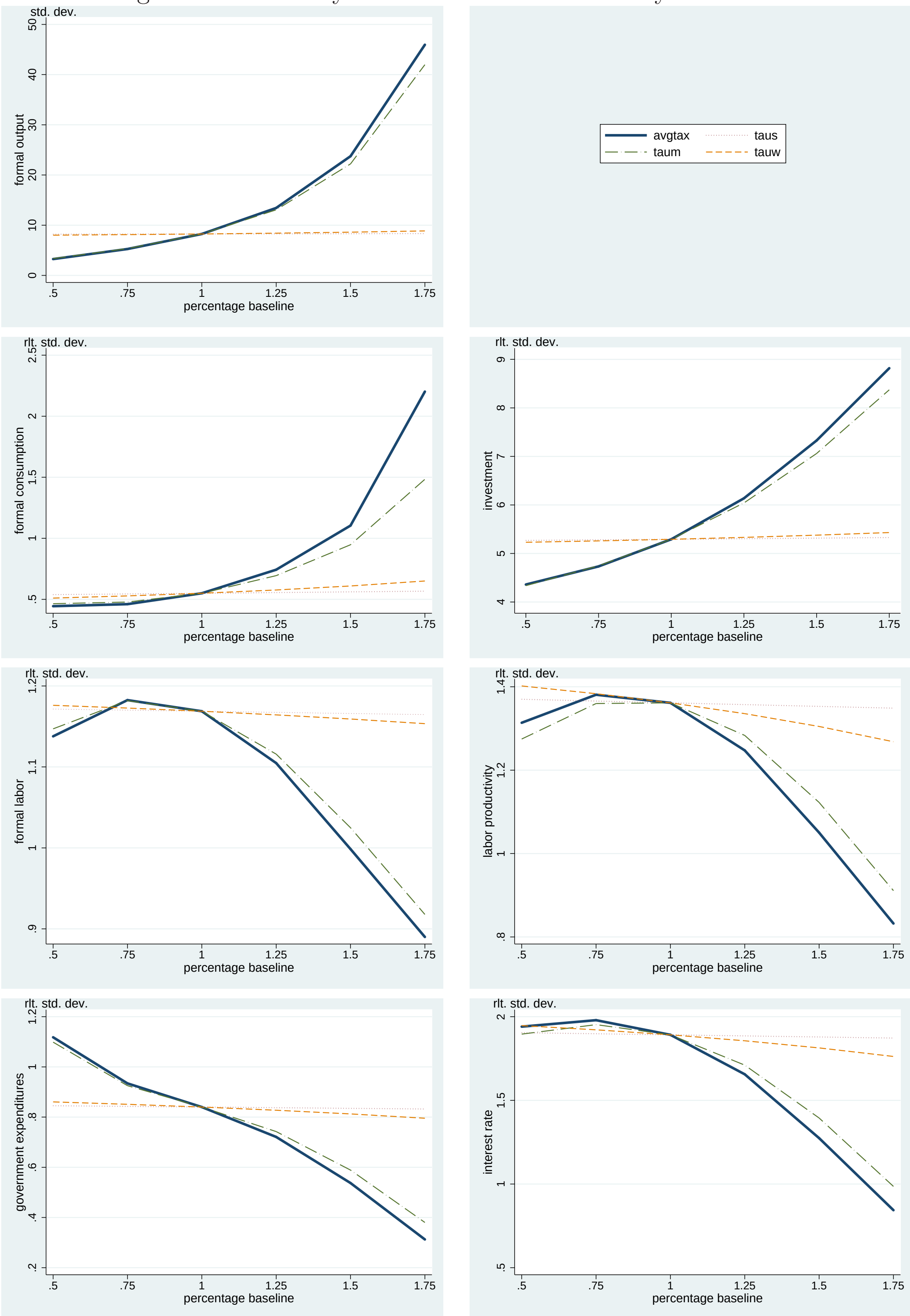
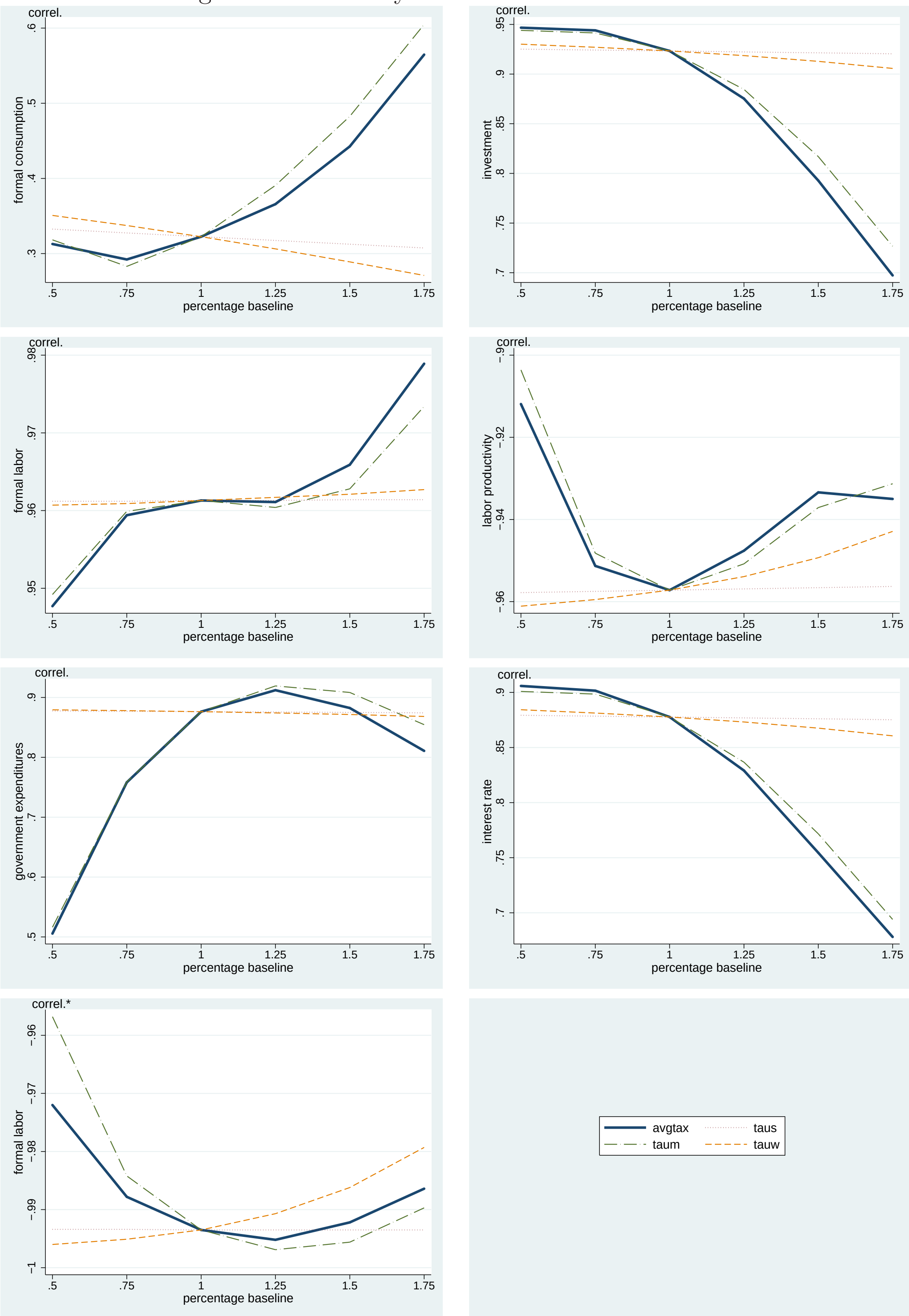


Figure 4: Sensitivity of comovements to tax rates



* Correlation with labor productivity

return on investment and consumption, while increasing their variance. This relation is reflected to a great extent in Figure 3, which shows the standard deviations of output, consumption and investment rise with generalized increments in taxes. Furthermore, government expenditures appears to become less volatile the higher is the tax burden. This pattern is mainly driven by reductions in the variability of work effort, as households attempt to smooth utility in view of increasing cyclical fluctuations in income and consumption. The relative standard deviation of labor productivity follows a similar pattern. Surprisingly, the interest rate tends to fluctuate less over the business cycle.

Moving on to the comovements, investment and the return on capital become less procyclical as the average tax rate increases, while consumption and government expenditures turn more so. These patterns of cyclical behavior are portrayed in Figure 4, which also suggests that the correlation of labor with formal output remains highly positive despite increments in the burden of taxation and social security contributions. In contrast, labor productivity stands as a strongly countercyclical variable, and comoves negatively with labor as well. Some discussion regarding these tendencies of comovement is suggested below. Further, Appendix A compares the cyclical patterns presented here with those featured by an economy without unofficial activities.

5.2.3 Sensitivity to preference parameters

The preference parameters considered in this study mainly refer to the elasticity of underground labor supply, or the elasticity of unofficial labor with respect to official labor ($1/\eta$). To conduct the present analysis, several elasticities are derived based on conditional and unconditional estimates obtained by Jørgensen et al. (2005), as described in the calibration section and displayed on Table D.1 in Appendix D. Increases in the elasticity of underground labor supply reflect a higher responsiveness of irregular work effort to shocks on wages and/or taxes, which in turn might lead to a rise in the volatility of both official and unofficial hours.

Figure 5 shows how macroeconomic volatility behaves as the elasticity of underground labor supply increases. Since the higher variability of work effort contributes to lessen the effects of sectoral productivity and fiscal policy shocks on production and consumption decisions, it is not surprising that formal output, consumption and investment fluctuate less over the business cycle as the elasticity goes up. Note, though, that hours are somewhat more volatile than output and fluctuate increasingly within a rather small region. These two characteristics are indicative of the stronger responsiveness of underground labor and that households' choices between these two types of work effort are independent to some degree, as implicit in their separability in preferences.

Regarding the comovements, Figure 6 shows that, except for the correlation between labor and productivity, the cyclicality of the different variables follows a monotonic tendency. The expenditure components and the interest rate all become more procyclical as the elasticity increases. In contrast, the correlations of hours and productivity with output decrease, meaning that formal labor behaves less procyclically and productivity turns even more countercyclical with shifts in the elasticity of underground labor supply. It is worth noting, however, that the patterns followed by these two comovements take place within a fairly small region in the proximity of 1 and -1, respectively.

5.3 Summary and discussion

Beyond the patterns of volatility and comovement suggested above, some underlying connections between the extent of unofficial activities and aggregate fluctuations might exist. In this regard, Granda-Carvajal (2010) has explicitly pointed out a few empirical associations that are worth being contrasted with the theoretical inferences presented here. Such an endeavor is plausible in light of the effects of the parameter changes on underground labor (and output) addressed before, as quantitatively shown in Appen-

Figure 5: Sensitivity of macro volatility to elasticity of underground labor supply

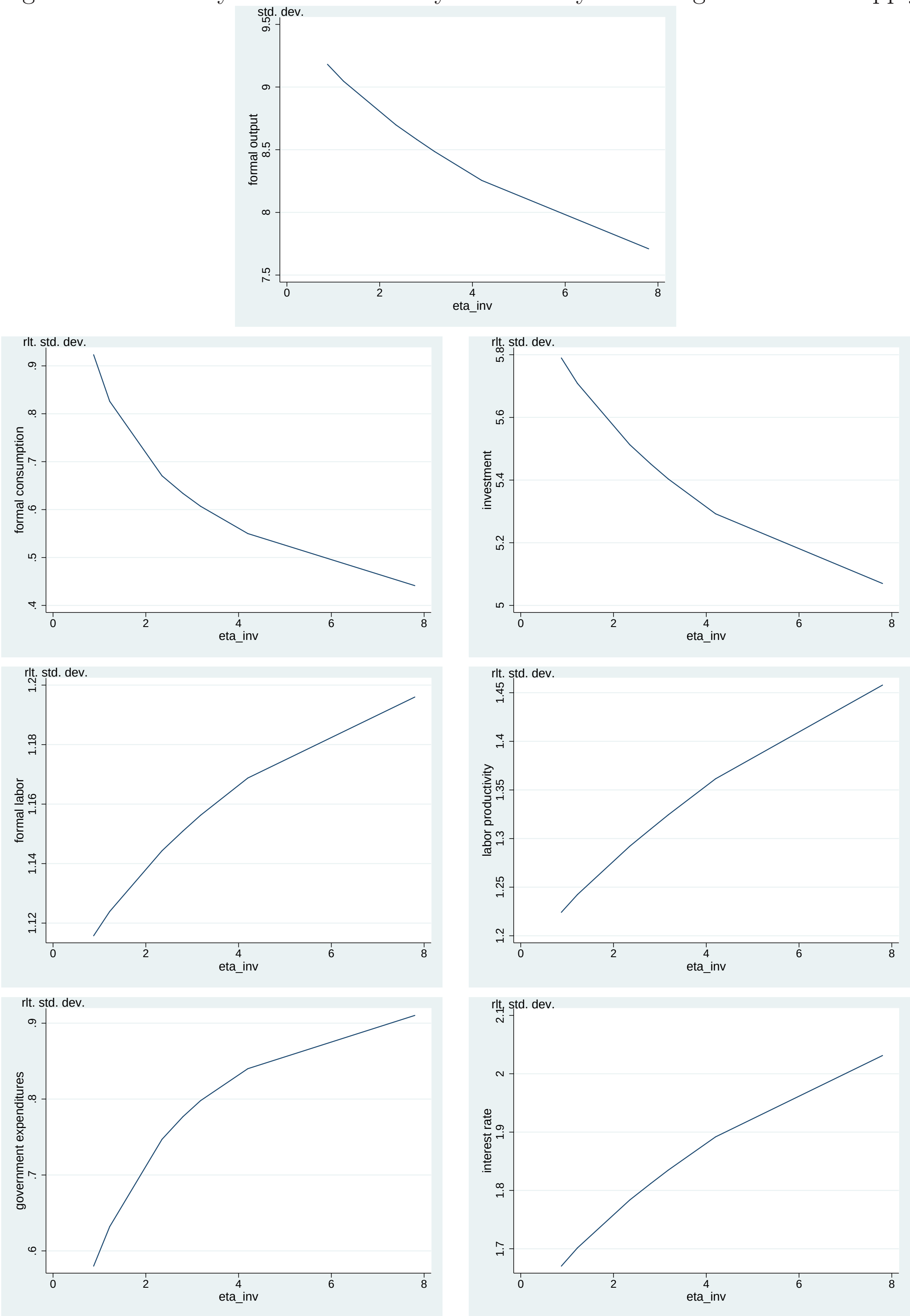
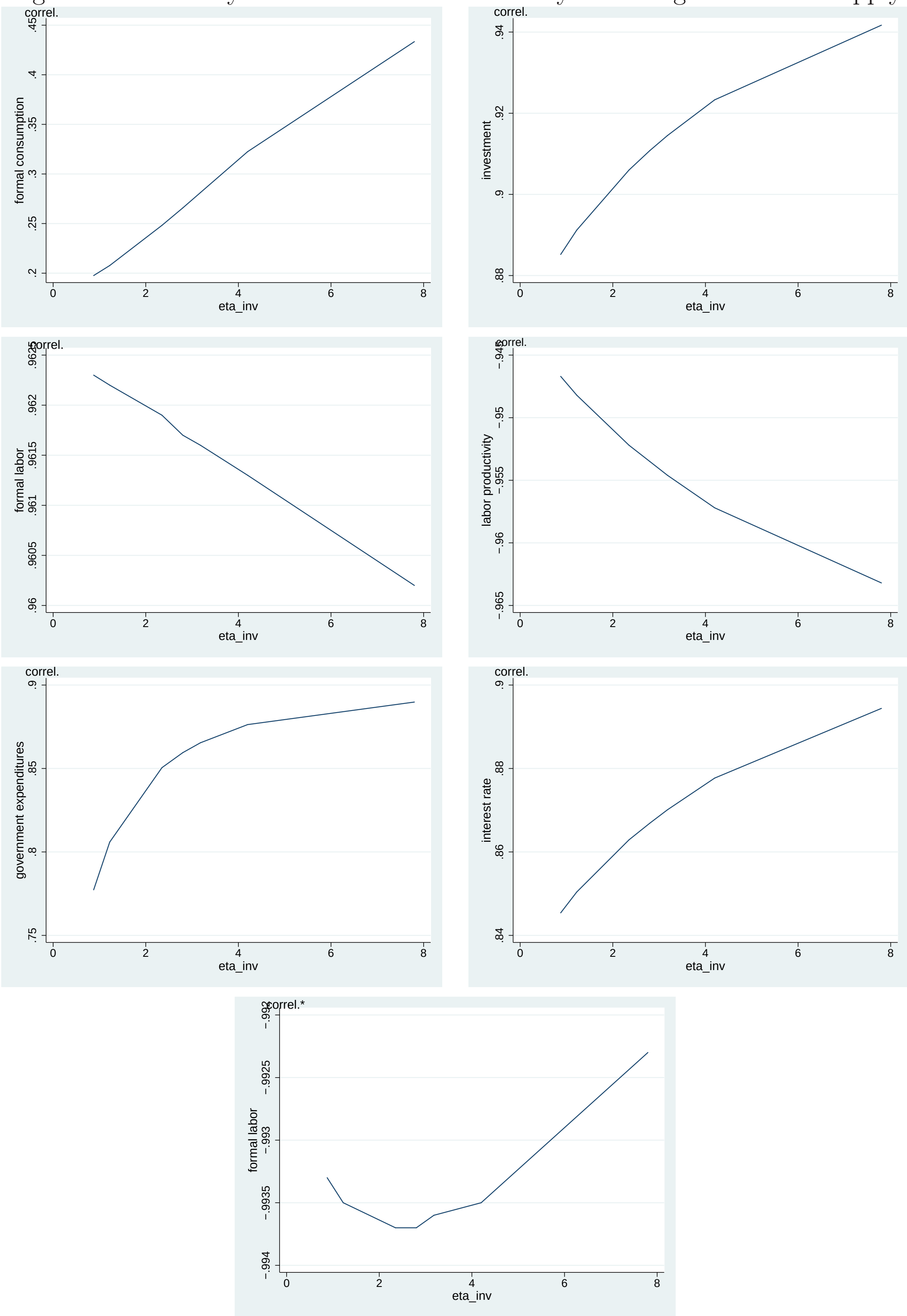


Figure 6: Sensitivity of comovements to elasticity of underground labor supply



* Correlation with labor productivity

dices B-D and commented in the following.

Clearly, the computational experiments show a negative effect of the enforcement parameters on irregular labor, whereas tax rates affect shadow activities positively. This particular feature to some degree stands in contrast to static theoretical models of tax evasion with labor supply, which generate ambiguous predictions about the impact of both enforcement and taxation on compliance (Andreoni et al., 1998). Further, a higher elasticity of underground labor supply intuitively translates into a higher disutility of working off the books, thus discouraging efforts in that direction.

Given these relations, it is feasible to compare the tendencies found by Granda-Carvajal (2010) with the results of the sensitivity analyses underlined above. Tables 5.3 and 5.4 summarize the overall findings and reinterpret them in terms of parameter changes leading to increases in underground labor. A plus (+) sign means increases, while minus (–) refers to decreases in either volatility or cyclicality of macroeconomic aggregates. The word 'no' implies unsubstantial variations in the moments. Lastly, wedgy ($\wedge$) and sharp increasing (Γ) patterns are also indicated.

Taking these conventions into account, one may claim the existence of a positive connection between the size of the unofficial sector and the volatility of output and its private components (i.e. consumption and investment) has been corroborated theoretically. That weak enforcement, high tax rates and low distaste for underground labor each contribute to increase participation in shadow economic activities and to amplify fluctuations in formal output, consumption and, to a lesser degree, investment suggests a possible rationale for those findings.

Somewhat similar conclusions can be attained as to the cyclicality of consumption, investment, the real interest rate and hours. As for the former three variables, the behavior displayed by their comovements with formal output in the sensitivity analyses appears to confirm to a fair extent the findings in Granda-Carvajal (2010), which

Table 5.3: Summary of results for volatility

Sensitivity analyses								
Changed parameter	Effect on							
	l^u_{ss}	$\sigma(y^f)$	$\sigma(c^f)/\sigma(y^f)$	$\sigma(i)/\sigma(y^f)$	$\sigma(l^f)/\sigma(y^f)$	$\sigma(g)/\sigma(y^f)$	$\sigma(r)/\sigma(y^f)$	$\sigma(y^f/l^f)/\sigma(y^f)$
$\downarrow \phi$	+	+	+	no	no	−	−	$\wedge$
$\downarrow \varsigma$	+	+	+	no	no	−	−	$\wedge$
$\uparrow \tau^s$	+	no	no	no	no	no	no	no
$\uparrow \tau^f_{ss}$	+	+	+	+	−	−	−	−
$\uparrow \tau^w_{ss}$	+	no	no	no	no	no	no	no
$\uparrow$ avgtax	+	+	+	+	−	−	−	−
$\downarrow 1/\eta$	+	+	+	no	no	−	no	no
Data	l^u_{ss}	$\sigma(y^f)$	$\sigma(c^f)/\sigma(y^f)$	$\sigma(i)/\sigma(y^f)$	$\sigma(l^f)/\sigma(y^f)$	$\sigma(g)/\sigma(y^f)$	$\sigma(r)/\sigma(y^f)$	$\sigma(y^f/l^f)/\sigma(y^f)$
	increase	increase	increase$^{\pm}$	increase$^{\mp}$	uncorrelated	increase$^{\mp}$	increase	uncorrelated

Notes: $^{\pm}$ Robust ; $^{\mp}$ Not robust

Table 5.4: Summary of results for comovements

Sensitivity analyses								
Increased parameter	Effect on l_{ss}^u	$\rho(c^f, y^f)$	$\rho(i, y^f)$	$\rho(l^f, y^f)$	$\rho(g, y^f)$	$\rho(r, y^f)$	$\rho(y^f/l^f, y^f)$	$\rho(l^f, y^f/l^f)$
$\downarrow \phi$	+	−	no	no	no	no	Γ	Γ
$\downarrow \varsigma$	+	−	no	no	no	no	Γ	Γ
$\uparrow \tau^s$	+	no	no	no	no	no	no	no
$\uparrow \tau_{ss}^f$	+	+	−	no	+	−	no	no
$\uparrow \tau_{ss}^w$	+	no	no	no	no	no	no	no
$\uparrow$ avgtax	+	+	−	no	+	−	no	no
$\downarrow 1/\eta$	+	−	−	no	−	−	no	no
Data	l_{ss}^u	$\rho(c^f, y^f)$	$\rho(i, y^f)$	$\rho(l^f, y^f)$	$\rho(g, y^f)$	$\rho(r, y^f)$	$\rho(y^f/l^f, y^f)$	$\rho(l^f, y^f/l^f)$
	increase	decrease$^{\mp}$	decrease$^{\mp}$	decrease$^{\mp}$	uncorrelated	decrease$^{\mp}$	uncorrelated	uncorrelated

Note: $^{\mp}$ Not significant

highlight a negative but non-significant relation between the size of unofficial activities and the correlation of these macroeconomic aggregates with GDP. In this regard, the comovement of consumption is noteworthy because of the various patterns it follows across parameters, thus supporting the lack of significance just mentioned.

Regarding hours, the evolution of its cyclical behavior across parameters apparently validates the results in Granda-Carvajal (2010). By pointing out that stronger enforcement, lower tax rates and higher distaste for underground labor do not lead to substantial reductions in the correlation of formal hours with output, the present model confirms my findings of a negative but non-significant relation between the comovement of labor input and the magnitude of shadow activities. Note in this respect that hours remains highly procyclical across all parameter changes. Thus, it can be said that both the theoretical and the empirical results just mentioned coincide.

Furthermore, the lack of significance of these results and the empirical uncorrelatedness of labor volatility with the extent of unofficial activities –also corroborated theoretically in the sensitivity analyses– challenge Busato and Chiarini's (2004) argument that opportunities for intratemporal substitution induced by the existence of an irregular sector explain the so-called employment volatility puzzle. Those opportunities, rather, can be very well induced by fiscal policy shocks as discussed further above, regardless of the presence of a second sector and its immanent characteristics.

Moreover, the heterogeneous patterns followed by the cyclical features of labor productivity across the computational experiments match my results of an absence of correlation of these features with the extent of unrecorded activities. This correspondence between theoretical inferences and empirical findings lends further support for the role of distortionary taxation and highlights my contention that the multi-sector framework underpinning household-production based RBC models –rather than the inherent characteristics of the underground sector– rationalizes the cyclical behavior of productivity found in that strand of literature.

Despite these apparent similarities and correspondences, the sensitivity analyses yield some patterns that do not corroborate the aforementioned empirical findings. This is the case of the volatility of the interest rate. By pointing out that weaker enforcement, higher tax rates and lower distaste for underground labor each lead to increased unofficial work effort and to reduced fluctuations in the return on capital, the present model challenges the empirical evidence of a positive correlation between the relative standard deviation of the latter variable and the size of shadow economic activities. Further research on figuring out this discrepancy might be worthwhile.

Finally, one alternative way wherein the comparisons and discussion just highlighted can be used involves re-calibration of the model. Allowing for different combinations of parameter values enables one to validate, for instance, that countries with high tax rates and weak enforcement tend to exhibit large informal sectors as well as high volatility of output, consumption and investment. Furthermore, one can follow this avenue to demonstrate Restrepo Echavarría's (2008) results are somehow to be expected, given the absence of enforcement characterizing her model setup. Extensions of the present model to characterize macroeconomic fluctuations in economies other than the U.S.'s thus are feasible, not to mention desirable.

6 Concluding notes

This paper explores the macroeconomic implications of the existence of an underground sector. Focused on short-term fluctuations, it develops a real business cycle model featuring sectoral productivity and fiscal policy shocks. These shocks affect agents' responses to productivity differentials and tax changes to the extent that they are willing to substitute irregular for regular activities. That way, the unofficial economy can have substantial effects on the behavior of some aggregate variables. The implementation of a few computational experiments further confirms this consideration, allowing to make inferences regarding the interaction between the determinants of shadow economic activity and business cycle fluctuations.

The present model differs from that of Busato and Chiarini (2004) mainly in the structure of preferences. Even though both representations of household behavior are adapted from Cho and Cooley's (1994) family labor supply model, the characterization adopted here makes official and unofficial labor separable in utility. Furthermore, non-market time is divided between leisure and irregular work effort, rather than entirely devoted to shadow activities. These two features aim not to impose any sort of comovement of regular and irregular production, as opposed to the assumptions introduced by the former authors. They enable, in addition, to take account of some empirical findings on the elasticity of underground labor supply (Jørgensen et al., 2005).

Nevertheless, the functional form employed in the model does not preclude alternative specifications of preferences. In an attempt to reconcile models of macroeconomic fluctuations with microeconomic studies on the irregular sector, both theoretical (see, e.g., Cowell, 1985) and empirical (Lacroix & Fortin, 1992), further extensions should consider utility functions that are not additively separable. Moreover, the very assumption of a homogeneous commodity could be defied, thus allowing for imperfect substitution in consumption between officially- and unofficially-produced goods. Implementing these suggestions might provide a more realistic portrait of the intricacies associated to underground activities.

Other differences with respect to Busato and Chiarini (2004) deal with the characterization of fiscal policy and tax enforcement. Firms in the present model are audited in order to discourage evasion pertaining to corporate taxes and social security contributions. This is unlike the mentioned authors, who solely consider monitoring in regard to corporate revenue taxation. Furthermore, the payroll tax rate is allowed to assume here a constant value, rather than a stochastic nature. Once these characteristics are accounted for, the model is calibrated using actual estimations of the distributional properties of government spending and tax disturbances (see Braun, 1994). Doing so attempts to better reflect the reality of tax collection in the United States.

The model is able to replicate the cyclical properties of average hours and labor productivity fairly well. It indeed improves substantially on Busato and Chiarini (2004) as far as the comovements of productivity are concerned. Furthermore, the patterns followed by the volatility and the cyclicality of labor market variables across the computational experiments confirm the absence of an empirical correlation between these variables and the extent of unofficial activities found by Granda-Carvajal (2010). These results as a whole challenge the argument of the double business cycle approach that opportunities for intratemporal substitution induced by the existence of an irregular sector explain the so-called employment volatility puzzle.

Even though the model understates the volatility of consumption, the computational experiments corroborate the existence of a positive connection between the size of the underground sector and the (relative) standard deviations of output and its private components. These findings support Ferreira-Tiryaki's (2008) and Restrepo Echavarría's (2008) results. Also, the experiments validate the uncorrelatedness of irregular activities with the comovements of consumption and investment found by Granda-Carvajal (2010). Hence the sensitivity analyses contribute to clarify which cyclical features are actually associated to the extent of the unofficial economy.

Having said this, it is worth noting that the results presented here emphasize the underground sector and the presumption of the double-business cycle approach as unnecessary when it comes to explaining certain features of macroeconomic fluctuations. In particular, they allow to confirm that these elements *per se* do not explain the cyclical behavior of labor market variables, but the role of tax disturbances (in a two-sector framework) does. By considering how changes in the determinants of informality affect a broad set of moments and aggregates, the analyses pursued in this paper offer a more comprehensive view of the cyclical implications of the shadow economy.

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A A model without an underground economy

This appendix presents a simple model wherein only a formal sector exists. Except for the absence of underground activities, the economy borrows its main features and parameter values from the model developed in the text. After briefly describing the setup, simulations and computational experiments allow to compare the resulting moments with those featured by the model with an unofficial sector. Doing so contributes to disentangle the cyclical implications of the existence of a shadow economy.

A.1 Model setup

The economy is populated by a large number of infinitely-lived agents. Firms solve a profit maximization problem every period, while households maximize utility over choices of consumption, investment and hours of work. The government uses tax revenues to finance a stochastic stream of expenditures.

Competitive firms purchase capital and labor services from households to produce output using a Cobb-Douglas technology

$$y_t^f = z_t^f k_t^\alpha (l_t^f)^{1-\alpha}, \tag{A.1}$$

where z_t^f is a total factor productivity shock.

Furthermore, revenues accrued $(1-\tau_t^f)y_t^f$ are taxed at the stochastic rate τ_t^f. On the other hand, the cost of renting capital equals its marginal productivity r_t. Labor cost is represented by the wage paid for hours worked, augmented by the fixed payroll tax rate τ^s. Let $w_t^f \equiv (1+\tau^s)w_t$ define such a cost, where w_t is the pre-tax wage. As factors are paid their real marginal products, firm's behavior is characterized by the first order conditions:

$$r_t = (1-\tau_t^f)\alpha z_t^f k_t^{\alpha-1}(l_t^f)^{1-\alpha}, \tag{A.2}$$

and

$$w_t^f = (1-\tau_t^f)(1-\alpha)z_t^f k_t^\alpha (l_t^f)^{-\alpha}. \tag{A.3}$$

Households choose consumption and hours to work to maximize the present discounted value of utility $E_0 \sum_t \beta^t U(c_t, l_t^f)$. Note that they make labor supply decisions along the intensive (hours worked) margin only. The per-period utility function is assumed to be separable between consumption and leisure (labor) as follows:

$$U(c_t, l_t^f) = \ln c_t - a\frac{(l_t^f)^{1+\gamma}}{1+\gamma}. \tag{A.4}$$

A well-behaved utility function implies that $a \geq 0$, $\gamma > 0$, and that all its components be twice continuously differentiable and increasing. The second term $a\frac{(l_t^f)^{1+\gamma}}{1+\gamma}$ represents the disutility of working. Further, households pay a stochastic tax rate τ_t^w on labor income and receive a lump-sum transfer T_t from the government, hence facing the budget constraint:

$$c_t + i_t = (1-\tau_t^w)w_t l_t^f + r_t k_t + T_t, \tag{A.5}$$

where i_t denotes investment at time t. Investment, in turn, increases the capital stock according to the law of motion:

$$i_t = k_t - (1-\delta)k_{t-1}, \tag{A.6}$$

where δ denotes the depreciation rate.

Let A_t be a vector of technology and fiscal policy shocks. With the instantaneous utility function defined as in equation (A.4), the value function $V(k_t, K_t, A_t)$ of the representative household satisfies

$$V(k_t, K_t, A_t) = \max_{k_{t+1}, l_t^f} \left\{ U(c_t, l_t^f) + \beta E_t \left[V(k_{t+1}, K_{t+1}, A_{t+1}) \right] \right\},$$

subject to the budget constraint (A.5) and the law of motion for the capital stock (A.6). Thus, household decisions are characterized by the intra-temporal condition for labor supply allocation:

$$a(l_t^f)^\gamma = \frac{1-\tau_t^w}{c_t} \frac{1-\tau_t^f}{1+\tau^s} (1-\alpha) z_t^f k_t^\alpha (l_t^f)^{-\alpha}, \tag{A.7}$$

and by the Euler equation:

$$1 = \beta \left(\frac{c_t}{c_{t+1}} \right) (1 + r_{t+1} - \delta), \tag{A.8}$$

where $r_t \equiv (1-\tau_t^f) \alpha z_t^f K_t^{\alpha-1} (l_t^f)^{1-\alpha}$ from firm profit maximization (see Equation A.2).

The government produces non-productive services and makes transfer payments each period by collecting taxes on firms' revenues and personal income. Government consumption is assumed to follow a stochastic process given by

$$g_t = z_t^g y_t^f, \tag{A.9}$$

where z_t^g is a random variable. Thus, the flow budget constraint is

$$g_t + T_t = \tau_t^f y_t^f + (\tau^s + \tau_t^w) w_t l_t^f. \tag{A.10}$$

Note that the lump-sum transfer T_t is treated as a residual that takes on whatever value is necessary to satisfy the government budget constraint at each point in time, given the productivity and fiscal policy shocks.

A recursive competitive equilibrium for this economy consists of a set of prices $\{w_t, r_t\}_{t=0}^\infty$, a value function $V(k_t, K_t, A_t)$, decision rules $\left\{ c_t, i_t, l_t^f, k_{t+1}, K_{t+1} \right\}_{t=0}^\infty$, and policy functions g_t and T_t such that:

- households maximize utility;
- firms maximize profits;
- the government balances its budget;
- individual and aggregate decisions are consistent, i.e. $k_t = K_t$, and
- markets clear.

Note that the market clearing condition implies that the decision rules satisfy the resource constraint

$$c_t + i_t + g_t = y_t^f. \qquad \text{(A.11)}$$

A.2 Comparison with model featuring a shadow economy

A.2.1 Moments

By assigning the same parameter values, the present model without an underground sector and its counterpart in the text are made comparable. Tables A.1 and A.2 display the relative standard deviations and the correlations generated by the two model economies. Note that these second moments pertain to the formal sector, which allows contrast with the stylized facts of the business cycle in the U.S.

Table A.1: Comparing volatility across models

Rlt. Std. dev.	Data	Model forecast	
		w/ under	w/o under
$\sigma(y^f)$	0.01901	0.08255	0.07462
$\sigma(c^f)/\sigma(y^f)$	0.89129	0.54997	0.39877
$\sigma(i)/\sigma(y^f)$	4.08135	5.29241	4.99089
$\sigma(g)/\sigma(y^f)$	1.49125	0.84004	0.93680
$\sigma(l^f)/\sigma(y^f)$	1.92521	1.16877	1.20701
$\sigma(y^f/l^f)/\sigma(y^f)$	1.20065	1.36146	0.37524

Table A.2: Comparing comovements across models

Correlation	Data	Model forecast	
		w/ under	w/o under
$\rho(c^f, y^f)$	0.8795	0.3225	0.4858
$\rho(i, y^f)$	0.8425	0.9233	0.9488
$\rho(g, y^f)$	0.3542	0.8763	0.8922
$\rho(l^f, y^f)$	0.8479	0.9613	0.9594
$\rho(y^f/l^f, y^f)$	-0.5268	-0.9572	-0.4211
$\rho(l^f, y^f/l^f)$	-0.8973	-0.9935	-0.6598

The tables show that consumption and labor productivity are more volatile in the model with an underground economy. Furthermore, consumption tends to be more procyclical and productivity less countercyclical in the one-sector model. The cyclical behavior of consumption can be rationalized by the absence of unofficially-produced goods available to households in the latter specification, so their consumption choices are relatively tied to fluctuations in official output. It is also worth noting that consumption volatility is small in the two model economies compared to the data, a feature often found in the stochastic growth literature.

As for labor productivity, its cyclical behavior can be similarly explained by households only working in one sector. This makes productivity fluctuate more in tandem with output variations over the business cycle. Moving on to the cyclicality of productivity with hours, that these two variables comove in opposite directions in both model specifications highlights the role of distortionary taxation in inducing substitution effects. The existence of a second sector appears to reinforce this role by providing further opportunities for intratemporal substitution, as the highly negative correlation between labor and productivity in the model with an underground economy suggests.

A.2.2 Sensitivity analysis

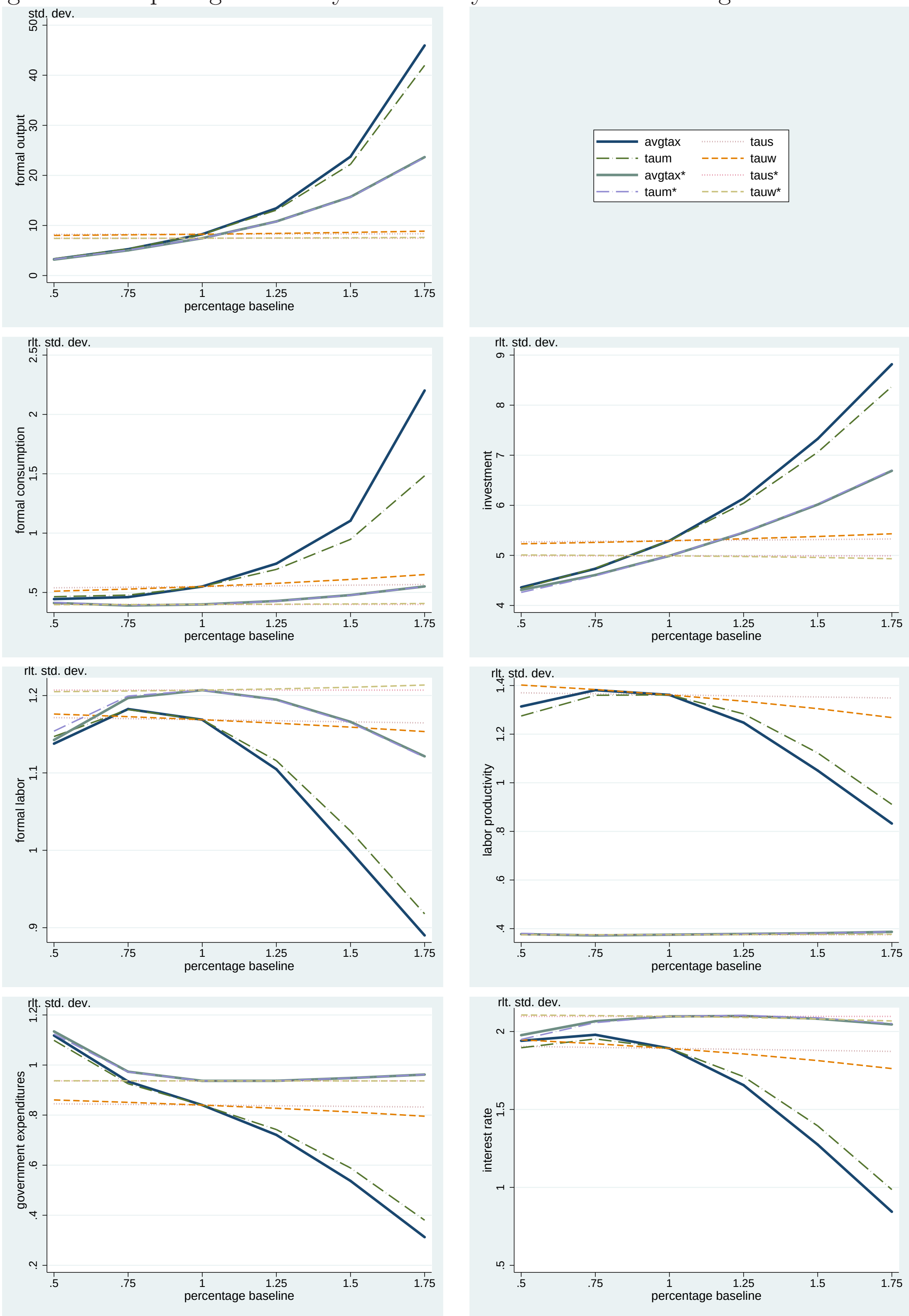
Provided that both the present model and the one in the text are subject to the same tax disturbances, computational experiments allowing for changes in tax rates are conducted. The purpose of these experiments is to compare how distortionary taxation affects the business cycle properties of the two economies, thereby disentangling the cyclical implications of taxes in the model with a tax-evading sector.

Figures 7 and 8 display the sensitivity of the volatility and comovements of the different variables to changes in the four tax parameters described in the text. Clearly, shifts in the social security contributions rate and the labor income tax rate do not have a substantial effect on the cyclical properties of any of the model economies. This is in stark contrast to impacts of variations in the average tax rate, which in turn are driven by changes in the corporate tax rate. These two tax rates significantly influence the standard deviations and the correlations of both economies, but especially of the model with an underground sector.

Regarding volatility, these observations mean that increases in the corporate tax rate or generalized increments in taxes are associated to higher variability of consumption in the model featuring an underground economy. These increases also are related to reductions in the relative standard deviation of hours, labor productivity, government expenditures and the interest rate, as Figure 7 suggests for the same model. One could argue that these patterns of cyclical fluctuations are connected to rises in shadow economic activities attributable to burdensome tax structures. Furthermore, they provide additional support to the analyses and comparisons conducted in the text.

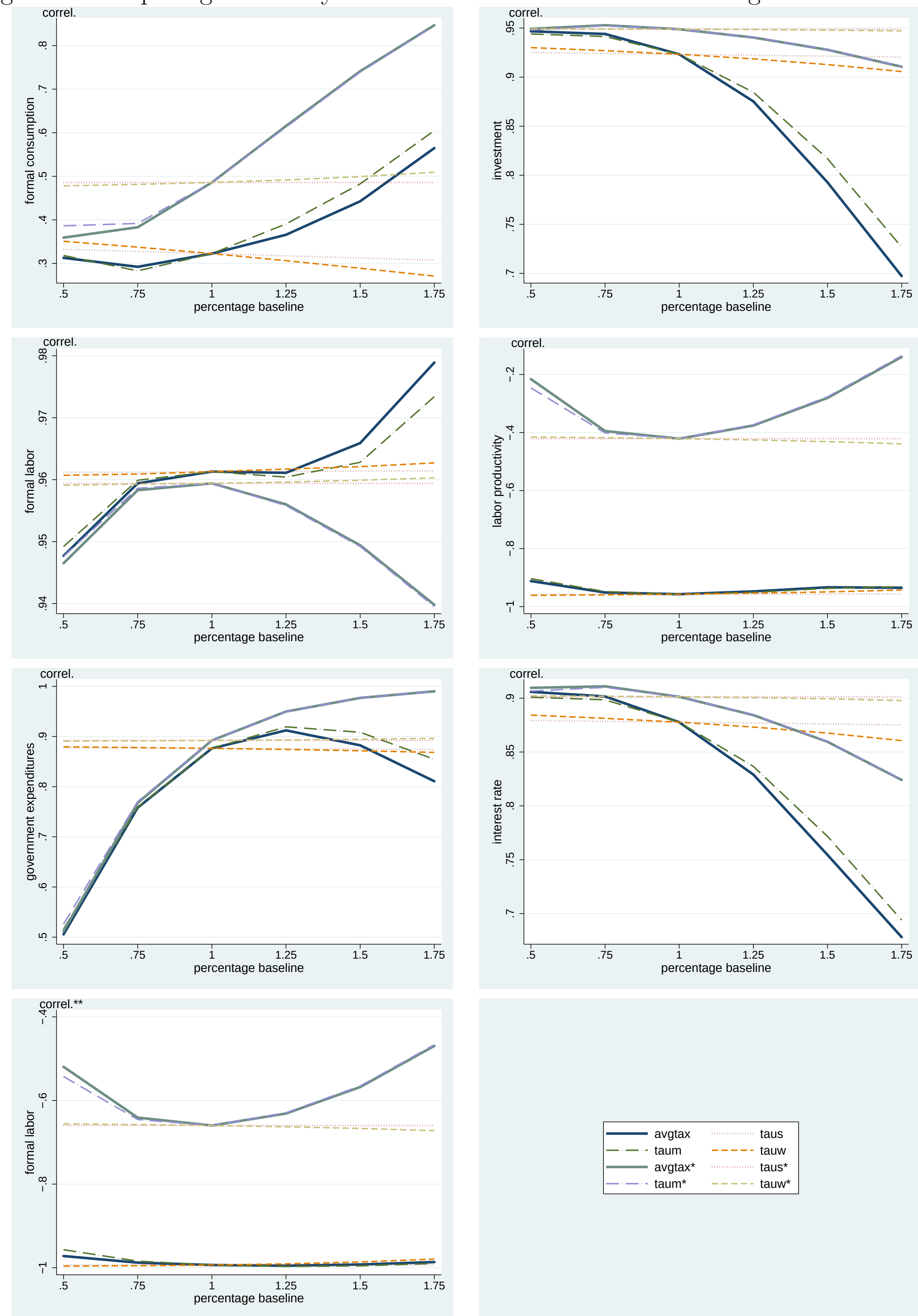
Moving on to the comovements, a higher burden of taxes and social security contributions is related to increases in the procyclicality of both consumption and government expenditures in the model without an undergound sector. That taxation further ties private and government spending to fluctuations in economic activity seems to explain

Figure 7: Comparing sensitivity of volatility with model featuring shadow economy



Note: * Model without shadow economy

Figure 8: Comparing sensitivity of comovements with model featuring shadow economy



Notes: * Model without shadow economy; ** Correlation with labor productivity these two patterns of correlation with output. In contrast, generalized increments in tax rates do not substantially affect the cyclicality of hours and labor productivity in any of the models. While labor remains highly procyclical in the two specifications, shifts in the corporate/average tax rate do not deviate the correlations of productivity with output and hours worked significantly away from their values in the baseline economies.

Overall, these findings provide support to the analyses in the text. They, for instance, contribute to corroborate that increases in the size of the shadow economy lead to a rise in the volatility of consumption and to a relative decline in its cyclicality. Likewise, that hours worked and productivity are slightly less volatile as the tax structure becomes more burdensome sort of validates the uncorrelatedness between the standard deviation of these two variables and the extent of unofficial activities found by Granda-Carvajal (2010). A similar conclusion can be more firmly reached regarding the comovements of labor productivity, given the negligible influence that changes in the corporate/average tax rate exert on them. The many patterns of cyclical behavior that can be confirmed or contested using these comparisons warrant their further implementation.

B Sensitivity to changes in enforcement parameters

Table B.1: Moments vs. Probability of detection

	Probability of detection (ϕ)						
Moment	0	0.015	0.1	0.25	0.5	0.75	1
l^u_{ss}	0.0865	0.0840	0.0709	0.0511	0.0272	0.0128	0.0052
$\sigma(y^f)$	8.2813	8.2552	8.1141	7.8984	7.6428	7.507	7.4553
$\sigma(c^f)/\sigma(y^f)$	0.5536	0.5500	0.5303	0.4986	0.4548	0.4239	0.4068
$\sigma(i)/\sigma(y^f)$	5.2890	5.2924	5.3078	5.3175	5.2852	5.2113	5.1275
$\sigma(g)/\sigma(y^f)$	0.8340	0.8400	0.8720	0.9187	0.9655	0.9778	0.9697
$\sigma(l^f)/\sigma(y^f)$	1.1686	1.1688	1.1698	1.1727	1.1800	1.1886	1.1963
$\sigma(y^f/l^f)/\sigma(y^f)$	1.3722	1.3615	1.2937	1.1425	0.7812	0.2496	0.5816
$\sigma(r)/\sigma(y^f)$	1.8869	1.8918	1.9193	1.9645	2.0254	2.0647	2.0853
$\rho(c^f, y^f)$	0.3270	0.3225	0.2993	0.2708	0.2658	0.3086	0.3726
$\rho(i, y^f)$	0.9227	0.9233	0.9262	0.9310	0.9377	0.9426	0.9458
$\rho(g, y^f)$	0.8749	0.8763	0.8834	0.8924	0.8999	0.9010	0.8989
$\rho(l^f, y^f)$	0.9615	0.9613	0.9601	0.9585	0.9570	0.9570	0.9576
$\rho(y^f/l^f, y^f)$	-0.9572	-0.9572	-0.9571	-0.9567	-0.9523	-0.8781	0.9715
$\rho(r, y^f)$	0.8776	0.8777	0.8783	0.8802	0.8851	0.8906	0.8952
$\rho(l^f, y^f/l^f)$	-0.9935	-0.9935	-0.9929	-0.9916	-0.9868	-0.9241	0.9851

Table B.2: Moments vs. Fines

	Penalty surcharge (ς)						
Moment	1	1.2	5	10	25	50	75
l^u_{ss}	0.0844	0.0840	0.0765	0.0673	0.0441	0.0190	0.0066
$\sigma(y^f)$	8.2595	8.2552	8.1748	8.0751	7.8222	7.5618	7.4627
$\sigma(c^f)/\sigma(y^f)$	0.5506	0.5500	0.5388	0.5247	0.4866	0.4376	0.4099
$\sigma(i)/\sigma(y^f)$	5.2919	5.2924	5.3019	5.3110	5.3147	5.2515	5.1477
$\sigma(g)/\sigma(y^f)$	0.8390	0.8400	0.8584	0.8807	0.9341	0.9752	0.9727
$\sigma(l^f)/\sigma(y^f)$	1.1688	1.1688	1.1693	1.1702	1.1742	1.1843	1.1946
$\sigma(y^f/l^f)/\sigma(y^f)$	1.3633	1.3615	1.3250	1.2714	1.0661	0.5385	0.3514
$\sigma(r)/\sigma(y^f)$	1.8910	1.8918	1.9073	1.9272	1.9817	2.0477	2.0815
$\rho(c^f, y^f)$	0.3233	0.3225	0.3090	0.2934	0.2644	0.2825	0.3564
$\rho(i, y^f)$	0.9232	0.9233	0.9249	0.9270	0.9328	0.9404	0.9451
$\rho(g, y^f)$	0.8761	0.8763	0.8805	0.8852	0.8951	0.9010	0.8996
$\rho(l^f, y^f)$	0.9613	0.9613	0.9606	0.9598	0.9580	0.9568	0.9574
$\rho(y^f/l^f, y^f)$	-0.9572	-0.9572	-0.9572	-0.9571	-0.9562	-0.9422	0.9592
$\rho(r, y^f)$	0.8777	0.8777	0.8780	0.8786	0.8813	0.8879	0.8942
$\rho(l^f, y^f/l^f)$	-0.9935	-0.9935	-0.9932	-0.9927	-0.9909	-0.9785	0.9663

C Sensitivity to changes in tax parameters

Table C.1: Moments vs. Social security contributions

	Payroll tax rate (τ^s)					
Moment	0.0382	0.0574	0.0765	0.0956	0.1148	0.1339
l^u_{ss}	0.0794	0.0817	0.0840	0.08632	0.0886	0.0909
$\sigma(y^f)$	8.2037	8.2294	8.2552	8.2811	8.3071	8.3332
$\sigma(c^f)/\sigma(y^f)$	0.5389	0.5444	0.5500	0.5556	0.5613	0.5671
$\sigma(i)/\sigma(y^f)$	5.2694	5.2808	5.2924	5.3042	5.3161	5.3281
$\sigma(g)/\sigma(y^f)$	0.8453	0.8427	0.8400	0.8374	0.8348	0.8323
$\sigma(l^f)/\sigma(y^f)$	1.1715	1.1701	1.1688	1.1674	1.1660	1.1646
$\sigma(y^f/l^f)/\sigma(y^f)$	1.3700	1.3657	1.3615	1.3572	1.3530	1.3487
$\sigma(r)/\sigma(y^f)$	1.9046	1.8982	1.8918	1.8854	1.8789	1.8725
$\rho(c^f, y^f)$	-0.9883	-0.9886	-0.9889	-0.9892	-0.9895	-0.9898
$\rho(i, y^f)$	0.0678	0.0753	0.0828	0.0903	0.0978	0.1052
$\rho(g, y^f)$	-0.1049	-0.0971	-0.0894	-0.0817	-0.0739	-0.0662
$\rho(l^f, y^f)$	-0.0351	-0.0303	-0.0256	-0.0208	-0.0161	-0.0114
$\rho(y^f/l^f, y^f)$	0.0612	0.0551	0.0489	0.0428	0.0367	0.0307
$\rho(r, y^f)$	0.1836	0.1901	0.1966	0.2031	0.2095	0.216
$\rho(l^f, y^f/l^f)$	-0.9934	-0.9934	-0.9935	-0.9935	-0.9935	-0.9935

Table C.2: Moments vs. Corporate income tax rate

	Corporate tax rate (τ^f_{ss})					
Moment	0.1961	0.29408	0.3921	0.49012	0.5882	0.6862
l^u_{ss}	0.0338	0.0522	0.0840	0.1370	0.2166	0.3168
$\sigma(y^f)$	3.3564	5.3418	8.2552	13.0626	22.2140	41.9750
$\sigma(c^f)/\sigma(y^f)$	0.4643	0.4778	0.5500	0.6943	0.9473	1.4832
$\sigma(i)/\sigma(y^f)$	4.3371	4.7463	5.2924	6.0411	7.0601	8.3727
$\sigma(g)/\sigma(y^f)$	1.0986	0.9253	0.8400	0.7424	0.5889	0.3802
$\sigma(l^f)/\sigma(y^f)$	1.1470	1.1818	1.1688	1.1158	1.0249	0.9178
$\sigma(y^f/l^f)/\sigma(y^f)$	1.2747	1.3598	1.3615	1.2833	1.1226	0.9111
$\sigma(r)/\sigma(y^f)$	1.8959	1.9526	1.8918	1.7106	1.3947	0.9859
$\rho(c^f, y^f)$	0.3183	0.283	0.3225	0.3907	0.4826	0.6049
$\rho(i, y^f)$	0.9440	0.9414	0.9233	0.8846	0.8169	0.7267
$\rho(g, y^f)$	0.5162	0.7588	0.8763	0.9193	0.9084	0.8546
$\rho(l^f, y^f)$	0.9492	0.9599	0.9613	0.9604	0.9628	0.9734
$\rho(y^f/l^f, y^f)$	-0.9036	-0.9482	-0.9572	-0.9508	-0.9371	-0.9313
$\rho(r, y^f)$	0.9009	0.8985	0.8777	0.8367	0.7717	0.694
$\rho(l^f, y^f/l^f)$	-0.9568	-0.9842	-0.9935	-0.9969	-0.9956	-0.9897

Table C.3: Moments vs. Personal income tax rate

	Labor income tax rate (τ_{ss}^{w})					
Moment	0.112	0.168	0.224	0.280	0.336	0.392
l_{ss}^{u}	0.0670	0.0749	0.0840	0.0946	0.1070	0.1214
$\sigma(y^f)$	8.0168	8.1244	8.2552	8.4154	8.6138	8.8626
$\sigma(c^f)/\sigma(y^f)$	0.5103	0.5282	0.5500	0.5767	0.6095	0.6503
$\sigma(i)/\sigma(y^f)$	5.2310	5.2592	5.2924	5.3315	5.3776	5.4314
$\sigma(g)/\sigma(y^f)$	0.8606	0.8511	0.8400	0.8274	0.8127	0.7955
$\sigma(l^f)/\sigma(y^f)$	1.1761	1.1727	1.1688	1.1643	1.1592	1.1534
$\sigma(y^f/l^f)/\sigma(y^f)$	1.4019	1.3834	1.3615	1.3355	1.3048	1.2681
$\sigma(r)/\sigma(y^f)$	1.9468	1.9217	1.8918	1.8562	1.8135	1.7624
$\rho(c^f, y^f)$	0.3508	0.3374	0.3225	0.3063	0.2890	0.2710
$\rho(i, y^f)$	0.9301	0.9270	0.9233	0.9186	0.9128	0.9057
$\rho(g, y^f)$	0.8794	0.8780	0.8763	0.8742	0.8716	0.8683
$\rho(l^f, y^f)$	0.9607	0.9609	0.9613	0.9617	0.9621	0.9627
$\rho(y^f/l^f, y^f)$	-0.9611	-0.9595	-0.9572	-0.9539	-0.9493	-0.9429
$\rho(r, y^f)$	0.8843	0.8813	0.8777	0.8732	0.8676	0.8606
$\rho(l^f, y^f/l^f)$	-0.9960	-0.9951	-0.9935	-0.9907	-0.9862	-0.9793

Table C.4: Moments vs. Average tax rate

	Percentage of (benchmark) average tax rate					
Moment	0.5	0.75	1	1.25	1.5	1.75
l_{ss}^{u}	0.0242	0.0445	0.0840	0.1538	0.2537	0.3618
$\sigma(y^f)$	3.2567	5.2601	8.2552	13.4388	23.7452	45.9485
$\sigma(c^f)/\sigma(y^f)$	0.4440	0.4606	0.5500	0.7428	1.1043	2.2016
$\sigma(i)/\sigma(y^f)$	4.3602	4.7334	5.2924	6.1365	7.3278	8.8177
$\sigma(g)/\sigma(y^f)$	1.1180	0.9343	0.8400	0.7210	0.5374	0.3126
$\sigma(l^f)/\sigma(y^f)$	1.1378	1.1825	1.1688	1.1048	0.9986	0.8900
$\sigma(y^f/l^f)/\sigma(y^f)$	1.3137	1.3809	1.3615	1.2474	1.0502	0.8323
$\sigma(r)/\sigma(y^f)$	1.9406	1.9792	1.8918	1.6559	1.2745	0.8438
$\rho(c^f, y^f)$	0.3128	0.2922	0.3225	0.3658	0.4426	0.5647
$\rho(i, y^f)$	0.9467	0.9440	0.9233	0.8753	0.7928	0.6973
$\rho(g, y^f)$	0.5056	0.7582	0.8763	0.9122	0.8824	0.8108
$\rho(l^f, y^f)$	0.9477	0.9594	0.9613	0.9611	0.9659	0.9789
$\rho(y^f/l^f, y^f)$	-0.9119	-0.9513	-0.9572	-0.9476	-0.9334	-0.9350
$\rho(r, y^f)$	0.9059	0.9016	0.8777	0.8291	0.7545	0.6781
$\rho(l^f, y^f/l^f)$	-0.9720	-0.9878	-0.9935	-0.9952	-0.9922	-0.9864

D Sensitivity to changes in elasticity of underground labor supply

Table D.1: Moments vs. Preference parameters

	Elasticity of underground labor supply ($1/\eta$)						
Moment	0.875	1.222	2.35	2.80	3.18	4.2	7.8
l^u_{ss}	0.4794	0.3849	0.2068	0.1656	0.1373	0.0840	0.0143
$\sigma(y^f)$	9.1814	9.0466	8.6984	8.5816	8.4864	8.2552	7.7094
$\sigma(c^f)/\sigma(y^f)$	0.9232	0.8260	0.6705	0.6338	0.6070	0.5500	0.4413
$\sigma(i)/\sigma(y^f)$	5.7897	5.7087	5.5131	5.4518	5.4035	5.2924	5.0700
$\sigma(g)/\sigma(y^f)$	0.5800	0.6320	0.7470	0.7768	0.7979	0.8400	0.9102
$\sigma(l^f)/\sigma(y^f)$	1.1157	1.1239	1.1443	1.1509	1.1562	1.1688	1.1960
$\sigma(y^f/l^f)/\sigma(y^f)$	1.2241	1.2424	1.2921	1.3097	1.3244	1.3615	1.4578
$\sigma(r)/\sigma(y^f)$	1.6703	1.7014	1.7836	1.8117	1.8348	1.8918	2.0313
$\rho(c^f, y^f)$	0.1976	0.2077	0.2482	0.2658	0.2812	0.3225	0.4334
$\rho(i, y^f)$	0.8852	0.8912	0.9060	0.9108	0.9145	0.9233	0.9417
$\rho(g, y^f)$	0.7773	0.8059	0.8505	0.8595	0.8654	0.8763	0.8898
$\rho(l^f, y^f)$	0.9623	0.9622	0.9619	0.9617	0.9616	0.9613	0.9602
$\rho(y^f/l^f, y^f)$	-0.9467	-0.9482	-0.9522	-0.9535	-0.9546	-0.9572	-0.9632
$\rho(r, y^f)$	0.8454	0.8504	0.8629	0.8669	0.8701	0.8777	0.8944
$\rho(l^f, y^f/l^f)$	-0.9933	-0.9935	-0.9937	-0.9937	-0.9936	-0.9935	-0.9923