Hybrid Biobjective Evolutionary Algorithms for the Design of a Hospital Waste Management Network

Andrés L. Medaglia [•]
Departamento de Ingeniería Industrial, Centro de Optimización y Probabilidad Aplicada, Universidad de los Andes; Bogotá, Colombia

Juan G. Villegas
Departamento de Ingeniería Industrial, Universidad de Antioquia; Medellín, Colombia

Diana M. Rodríguez-Coca
Facultad de Ingeniería Industrial, Escuela Colombiana de Ingeniería; Bogotá, Colombia

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[•]Corresponding author: Universidad de los Andes, Departamento de Ingeniería Industrial, Calle 19 # 1-37E, Bogotá D.C., Colombia; e-mail: amedagli@uniandes.edu.co, tel.: +57(1)3394949 ext. 2880; url: http://wwwprof.uniandes.edu.co/~amedagli

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Abstract

Colombian environmental authorities are exploring new alternatives for improving the disposal of hospital waste generated in the Department of Boyacá (Colombia). To design this hospital waste management network we propose a biobjective obnoxious facility location problem (BOOFLP) that deals with the existing tradeoff between a low-cost operating network and the negative effect on the population living near the waste management facilities. To solve the BOOFLP we propose a hybrid approach that combines a multiobjective evolutionary algorithm (NSGA II) with a mixed-integer program. The algorithms are compared against the Noninferior Set Estimation (NISE) method and tested on data from Boyacá's hospital waste management network and publicly available instances.

Keywords: facility location, hybrid genetic algorithms, multiobjective evolutionary algorithms, NSGA II, multiobjective optimization

Introduction

The Colombian Department of Boyacá generates 5 tons of hospital waste per day, about 90% of which is disposed in open landfills. Because such disposal poses a public health hazard, Colombian government authorities are exploring new alternatives for properly disposing of Boyacá's hospital waste, including a planned deactivation plant with excess capacity to be located in Bogotá. Boyacá comprises 123 towns generating an average of 40 kg/day of hospital waste per town. As this amount is small, the hospital waste from several towns will have to be collected in small vehicles and consolidated in properly designed cross-docking centers (CDCs). Once in the CDCs, waste will be brought to Bogotá's deactivation plant using larger vehicles. Both the vehicles and the CDCs must meet the specifications and regulations of the Colombian Health and Environment Ministries. Figure 1 depicts the operation of Boyacá's hospital waste management network within this design.

INSERT FIGURE 1

The design of Boyacá's hospital waste management network can be modeled as a facility location problem (FLP); that is, the selection of a set of sites for facilities location that satisfy customer demands for a given good or service. Such facility location decisions are taken both by private firms in the (re)design of distribution and service networks and by government agencies for the deployment of public infrastructure. Public applications of location problems include the location of jails (Marianov and Fresard, 2005), landfills (Eiselt, 2007), disaster recovery centers (Dekle et al., 2005), and perinatal facilities (Galvão et al., 2002). For recent surveys on facility location, the reader is referred to Hale and Moberg (2002), Revelle and Eiselt (2005), and ReVelle et al. (2008).

The FLP literature frequently addresses a family of problems known as FLP with push-pull objectives, in which the push objective captures the behavior of people who want obnoxious facilities located as far away as possible (the so-called NIMBY, or not in my backyard, effect). In contrast, the pull objective captures the benefits derived from the nearness of users to facilities (e.g., shorter times for customer service and lower transportation costs). Krarup, et al. (2002) emphasized that most common facility location models fall into the pull objective class; however, the push objective class of location problems, which has received less attention, is surveyed by Erkut and Neuman (1989).

Because FLPs with push-pull objectives frequently arise in relation to hazardous material (hazmat) transportation (List et al., 1991; Current and Ratick, 1995; Erkut et al., 2007), some authors have considered combining location and routing decisions for better design of hazardous waste management systems (Revelle et al., 1991; Giannikos, 1998; Cappanera et al., 2003). However, it is as yet unclear how to mix location (strategic) and routing (operational) decisions in a single facility location model (Daskin et al., 2005). Therefore, as an alternative, other authors have used a two-phase approach that solves the location (strategic) problem first and the operational routing problem second. This is called by Nagy and Salhi (2006) a sequential method, and was applied successfully, for instance, by Erkut et al. (2000) to the redesign of a service network.

The strategic nature of location decisions calls for consideration of multiple, often conflictive, objectives; for example, designing a distribution network must take into account the tradeoff between cost and service (Nozick and Turnquist, 2001). The various methods by which multiobjective location problems have been solved include the ε–constraint method (Nozcik, 2001), complete enumeration (Erkut and Neumann, 1992), specialized exact methods (Fernandez and Puerto, 2003), multiobjective evolutionary algorithms (Villegas et al., 2006), and the aggregated weighting method (Nozick and Turnquist, 2001; Zhou et al., 2003; Shen and Daskin, 2005). For a review of the models, solution methods, and applications of multiobjective facility location problems, see Current et al. (1990) and the more recent survey of Nickel et al. (2004).

Some researchers, especially those interested in locating undesirable facilities, have proposed multiobjective models (Owen and Daskin, 1998). For example, Erkut and Neumann (1992) developed a multiobjective mixed-integer program to study the tradeoffs between cost, opposition, and equity when locating undesirable facilities to serve a region. These authors identified the efficient set by means of an enumeration algorithm on small instances with up to 30 population centers. Subsequently, Zhang and Melachrinoudis (2001) solved the problem of locating a single point (obnoxious facility) on a general network using two objectives—the maximization of the minimal weighted distance from the point to the vertices (maximin) and the maximization of the sum of weighted distances between the point and the vertices (maxisum). In doing so, they investigated the properties of the biobjective optimization problem in both the decision and objective space to reduce the candidate solution set. Later, Hamacher et al. (2002) presented a multiobjective network location model for locating a single (semi)obnoxious facility (with push-pull objectives) whose solution algorithm was based on concepts from a multiobjective median network location problem. Their method works for piecewise linear objective functions and solves instances of realistic size. More recently, Rakas et al. (2004) proposed and applied a methodology for finding the optimal number of landfill facilities, identifying the best locations, and allocating population to these facilities. Their methodology allows consideration of uncertain waste amounts using fuzzy mathematical programming. Another line of research deals with multiobjective continuous location problems (Nickel et al., 2005; Puerto and Fernandez, 1999; Hamacher and Nickel, 1996); however, continuous location models are out of the scope of this paper, since we model Boyacá’s hospital waste management network using discrete location.

The specific problem of Boyacá’s hospital waste management network design has already been addressed by Rodriguez (2005), who used a monoobjective capacitated p-median problem, with cost as the only objective. As previously mentioned, because the amount of hospital waste produced in each town in Boyacá is small, the collection will be attended using less-than-truckload (LTL) trips; that is, routes departing from (and arriving at) a CDC will collect waste from several towns. Therefore, Rodriguez also proposed a location-routing sequential method to design operational routes for the CDCs. However, her model fails to take into account the undesirable effects of the CDCs on nearby residents. To fill this gap, this work proposes a biobjective obnoxious facility location problem (BOOFLP), which considers the push-pull effect of population exposure and transportation cost. To solve the BOOFLP, we propose two new multiobjective evolutionary algorithms that are specially designed to approximate the nondominated set of network configurations.

The article is organized as follows. Section 1 explains the formulation of the biobjective facility location problem, and Section 2 describes the proposed multiobjective evolutionary algorithms. Section 3 then summarizes the computational results for the Boyacá case study and the performance of our method in other public instances from the literature, together with a comparison against the Noninferior Set Estimation (NISE) method. Section 4 concludes the paper and outlines the model's pragmatic value.

1. Problem formulation

Let $J = \{1,...,j,...,n\}$ be the set of towns in Boyacá and d_j the amount of hospital waste generated by town j. Let $I = \{1,...,i,...,m\}$ be the set of candidate sites for locating CDCs; and s_i and p_i the CDC capacity and population at site i. Let K be the number of CDCs that will be located in the hospital waste management network. Let c_{ij} be the cost of traveling between town j and the CDC located at site i. Let y_i be the binary decision variable that indicates whether the candidate site i is chosen for locating a CDC; and x_{ij} the binary decision variable that indicates whether the (entire) amount of waste generated by town j is collected by CDC i. The biobjective obnoxious facility location problem (BOOFLP) is then formulated as follows:

$$\min \quad z_1 = \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} \tag{1}$$

$$\min \quad z_2 = \sum_{i \in I} p_i y_i \tag{2}$$

subject to,

$$\sum_{i \in I} y_i = K \tag{3}$$

$$\sum_{i \in I} x_{ij} = 1, \quad j \in J \tag{4}$$

$$\sum_{j \in J} d_j x_{ij} \leq s_i y_i, \quad i \in I \tag{5}$$

$$y_i \in \{0,1\}, \quad i \in I \tag{6}$$

$$x_{ij} \in \{0,1\}, \quad i \in I, j \in J \tag{7}$$

The objective function (1) represents the approximated cost of moving hospital waste from the towns to the CDCs, while the second objective (2) represents the total population affected by the CDCs. Constraint (3) indicates the number of CDCs to be opened, constraints (4) guarantee that every customer is assigned to only one facility, constraints (5) enforce the CDC capacities, and constraints (6) and (7) establish the binary nature of the decisions. To assure an even distribution of waste between CDCs, the capacity of every CDC is given by

$$s_i = \frac{\sum_{j \in J} d_j}{K}, \quad i \in I \tag{8}$$

Enforcing an even distribution may pose some difficulties to the solution of the BOOFLP. From a pragmatic point of view, seeking for an even balance may cause the undesirable assignment of a waste generator to a distant CDC. Moreover, it is always possible to avoid such an assignment by negotiating directly with a CDC upon a small deviation from its nominal capacity. On the other hand, from a computational perspective, very tight constraints may cause numerical instability or convergence problems in a branch and bound framework. For these reasons, it is realistic and convenient to treat the CDCs capacities as soft constraints.

Finally, because of the conflictive nature of the objectives (cost vs. population exposure), there may be no single optimal solution to the BOOFLP. Rather, the search aim is a set of solutions in which one objective could not be improved without worsening the other. In the multiobjective optimization literature (Ehrgott, 2000), such a set is termed the *efficient* (or *Pareto optimal*) *set*, whose image in the objective space is called the *nondominated set* (or *efficient frontier*) and denoted by PF. The approximation of the latter is denoted by $\widehat{PF}$.

2. MOEAs for the BOOFLP

Evolutionary algorithms are bio-inspired stochastic search procedures used to solve complex optimization problems by evolving a population of solutions in which the fitter are more likely to survive. These algorithms improve the solution population using selection, recombination, and perturbation mechanisms (such as local search and mutation). This family of metaheuristics includes genetic algorithms (Goldberg, 1989), memetic algorithms (Moscato and Cotta, 2003), and scatter search (Glover, 1998).

Of these, one of the most frequently used metaheuristics for approximating the nondominated set of multiobjective optimization problems (Jones et al. 2002) are the genetic algorithms also known as multiobjective evolutionary algorithms (MOEAs). As Coello (2001) emphasized, MOEAs (with posterior articulation of preferences) can find an approximation of the Pareto optimal front in a single run (see Coello et al., 2002, for a useful review of multiobjective evolutionary algorithms).

One simple but powerful MOEA is the NSGA II (Deb et al., 2002), whose selection mechanism classifies the population into layers or fronts, the first composed of the nondominated solutions in the population; the second front, are the solutions that become nondominated by not considering the solutions belonging to the first front. This procedure continues until every solution has been classified into a front. The NSGA II preserves population diversity by means of a crowding measure based on the average edge of the cuboid enclosing a solution.

2.1. MOEA logic

For this design, we chose the NSGA II (Deb et al., 2002) over other MOEAs because of its proven success in other biobjective capacitated (and uncapacitated) location problems

(Villegas et al., 2006; Medaglia et al., 2006) and its ability to find good approximations of nonconvex Pareto fronts (Deb et al., 2002). Except for its second objective, the BOOFLP given by (1)–(7) has the structure of a capacitated p-median problem. Thus, this work borrows the solution encoding used before by Alp et al. (2002) and Bozkaya et al. (2001) and some genetic operator ideas for the uncapacitated p-median problem (Alp et al., 2002).

2.2. Solution encoding

Because the solutions are encoded using a list of size K that represents the selected CDCs, the encoding automatically meets constraint (3). For example, in a problem with 20 potential CDC sites in which K=4, the chromosome {4,11,8,7} represents the solution in which sites 4,7,8 and 11 are selected for CDC location.

2.3. Fitness function

Since the solution encoding only takes into account the location decisions (i.e., y variables), this work presents two heuristics for the assignment of waste generators to CDCs (i.e., x variables).

Greedy assignment heuristic (GAH)

The GAH assigns waste generators to nearby CDCs by taking into account their capacity. In Rodriguez (2005), enforcing constraints (5) generated balanced CDCs but left some waste generators far away from their assigned CDCs, which ultimately made the solution impractical. Therefore, we relax these constraints and use the capacity violation as a measure of the deviation from an even allocation of hospital waste generators to CDCs in the network.

For each waste generator, the maximum distance threshold (Δ_j) is the maximum allowed distance between generator j and its assigned CDC. Mathematically, Δ_j is defined as follows:

$$\Delta_j = \min_{i \in I} c_{ij} + \beta \times (\max_{i \in I} c_{ij} - \min_{i \in I} c_{ij}), \quad j \in J \tag{9}$$

where β is a distance threshold between 0 and 1. It should be noted that when β=0, capacity constraints can be grossly violated if the nearest open CDC is chosen for each generator. In this case, the BOOFLP is basically treated as an uncapacitated problem. On the other hand, when β=1, it is possible to achieve an even distribution of waste among the open CDCs. In this latter case, a town could be assigned to the farthest open CDC.

For the evaluation of a given chromosome, the heuristic tries to assign town j to CDC i^*; that is, the nearest open CDC with enough remaining capacity to serve its entire demand that lies within the maximum distance threshold. Mathematically, i^* is defined as follows:

$$i^* \leftarrow \arg \min_{\{i \in I \mid y_i = 1, c_{ij} \le \Delta_j\}} c_{ij}, \quad j \in J \tag{10}$$

It is noteworthy that by doing so, GAH solves the x variables. If no such CDC exists, the capacity constraint (5) of the nearest open CDC is ignored and the generator j is assigned to it.

Mixed integer programming (MIP) assignment heuristic

This work proposes a second procedure based on mixed-integer programming (MIP) for the assignment of waste generators to CDCs (i.e., x variables). In this procedure, $I' = \{i \in I | y_i = 1\} \subseteq I$ is the set of CDCs opened (i.e., fixed y variables by the solution encoding). Let e_i be a decision variable with the capacity violated at CDC $i \in I'$, $c_{\max}$ is the largest cost in the network $\left(c_{\max} = \max_{\{(i,j) | i \in I'. j \in J\}} c_{ij} \right)$, and α is the penalty for exceeding the aggregated CDC capacities. The proposed MIP can then be represented as follows:

$$\min \quad \sum_{i \in I'} \sum_{j \in J} \frac{c_{ij} x_{ij}}{c_{\max}} + \alpha \sum_{i \in I'} \frac{e_i}{s_i} \tag{11}$$

subject to,

$$\sum_{i \in I'} x_{ij} = 1, \;\; j \in J \tag{12}$$

$$e_i \geq \sum_{j \in J} d_j x_{ij} - s_i, \;\; i \in I' \tag{13}$$

$$e_i \geq 0, \quad i \in I' \tag{14}$$

$$x_{ij} \in \{0,1\}, \quad i \in I', j \in J \tag{15}$$

It should be noted that this model, like the GAH, handles CDC capacities as soft constraints; however, the objective function (11) minimizes cost and penalizes capacity violations. Additionally, constraints (12) guarantee that every customer is assigned to only one CDC; constraints (13) and (14) define the capacity violation variable for every CDC opened and its non-negativity, respectively; and constraints (15) define the binary nature of the assignment decisions.

Fitness function evaluation

The proposed NSGA II-based MOEA uses two objectives: population exposure and penalized cost. The number of residents affected by CDC operations can be calculated by summing the populations of the towns in which the CDCs are located (see (2)). However, to penalize cost by capacity violation, we use the type of procedure suggested by Chu and Beasley (1997):

$$z'_1 = z_1 \times (1 + \rho) \tag{16}$$

where ρ measures the capacity violation as follows:

$$\rho = \sum_{i \in I} \max\left(0, \sum_{j \in J} d_j x_{ij} - s_i y_i \right) \Big/ \sum_{j \in J} d_j \tag{17}$$

It should be noted that $0 \leq \rho \leq 1$. Thus, conveniently, if no capacity violation exists, $\rho=0$ and the cost is not penalized.

This mechanism of evaluation allows for an exploration of the solution space such that the evolutionary algorithms are able to find both supported and non-supported solutions for the BOOFLP. This is due, to a large extent, to the fact that the exploration of the objectives is done independently following a two-phase approach. The solution encoding explicitly deals with the population exposure objective (first phase), while the cost objective is handled (independently) by the assignment of towns to opened CDCs (second phase).

2.4.Genetic operators

The proposed crossover operator is based on Alp et al. (2002), in which P^1 and P^2 are two parents and C^1 and C^2 are their children after crossover. The underlying purpose of the operator is to retain the CDCs common to both parents and randomly exchange some of the unshared CDCs between parents. For the sake of clarity, we treat the chromosomes as sets. $L^1=P^1\backslash P^2$ ($L^2=P^2\backslash P^1$) represents the set of open CDCs in parent 1 (in parent 2), but not in parent 2 (not in parent 1). The C^1 (C^2) set for the children is created by randomly selecting u CDCs from L^2 (L^1) to replace u CDCs from P^1(P^2) that are not in $P^1 \cap P^2$. The parameter u is drawn from a discrete uniform distribution over the interval $1 \le u \le |L^1| = |L^2|$. To illustrate, we consider a problem with 20 candidate sites where K= 4. Letting the two parents be P^1={5, 15, 14, 13} and P^2={3, 15, 13, 9}, with L^1={5,14} and L^2={3,9}, if u randomly takes the value of 1, the parents exchange one CDC (i.e., 5 for P^1 and 3 for P^2), resulting in C^1={3,15,14,13} and C^2={5,15,13,9} for the children.

For the mutation operator, if a chromosome mutates (which happens with a small probability) one of the selected sites for locating a CDC is chosen randomly and replaced with one site not present in the current chromosome.

3. Computational testing

This section shows the computational results obtained with the proposed multiobjective evolutionary algorithms based on NSGA II and NISE. Henceforth, because of the use of the greedy and MIP assignment heuristics in the GAs, we refer to these algorithms as GA-GAH and GA-MIP, respectively. Unless specified, all the experiments reported in this section were performed on a Dell Precision workstation with an Intel Core2 CPU 6700 at 2.66GHz, 4.096 GB of RAM, running under Windows Vista Business. To solve the MIP assignment procedure, we used the Xpress-MP Optimizer Version 18.00.01 from Dash Optimization.

3.1.Implementation

GA-GAH and GA-MIP were coded on MO-JGA (Medaglia, Gutiérrez and Villegas, 2006), a publicly available Java-based object-oriented framework for solving multiobjective optimization problems using evolutionary algorithms[1]. MO-JGA allows the user to focus on the application's logic by reusing a set of built-in components. Implementing GA-GAH and GA-MIP required only that the framework be extended by coding the chromosome's genotype (`PFLPGenotype`); the fitness function evaluators `FitnessMOOFLP` and `FitnessMIPAssignment` for the GAH and MIP assignment heuristics, respectively; and the crossover (`PFLPExchangeCrossover`) and mutation (`IndividualPFLPMutation`) operators. The logic embedded in NSGA II had already been implemented in the middle tier (see Figure 2) built by extending the Java Genetic Framework (JGA) developed by Medaglia and Gutiérrez (2006). The NSGAII implementation included in MO-JGA differs from that of Deb et al. (2002) algorithm in the selection of parents for the mating pool. MO-JGA's version uses simple random selection rather than the binary tournament proposed in the original paper.

[1] Available at http://copa.uniandes.edu.co/soft-evol-jga.html

INSERT FIGURE 2

3.2. Performance metrics

Size of the Space Covered (SSC) metric

A single run of GA-GAH or GA-MIP provides an approximation of the nondominated set. To measure how good this approximation is, we use the *size of the space covered* metric (SSC) (Zitzler and Thiele, 1998), which estimates the quality of the approximate nondominated set by measuring the size of the space enclosed by the set and a reference point. An approximation with a larger space is considered better.

To obtain a dimensionless SSC metric ranging from 0 to 1, the criteria of the nondominated set are normalized to fall between 0 and 1. First, the cost in Colombian peso, (COP$), is normalized as follows:

$$\bar{z}_1(p) = \underline{z}_1 / z_1(p) \qquad (18)$$

where $z_1(p)$ and $\bar{z}_1(p)$ are the original and normalized costs for the *p*-th solution of the nondominated set, and $\underline{z}_1$ is a lower bound for the cost in any configuration with four CDCs. The value of $\underline{z}_1$ has already been obtained by solving a p-median problem. Second, population exposure, measured in inhabitants, is normalized as follows:

$$\bar{z}_2(p) = \underline{z}_2 / z_2(p) \qquad (19)$$

where $z_2(p)$ and $\bar{z}_2(p)$ are the original and normalized populations for the *p*-th solution of the nondominated set, and $\underline{z}_2$ is a lower bound for the exposed population. The value of $\underline{z}_2$ has already been obtained by adding up the population of the four smaller towns in Boyacá. Finally, although algorithmically, capacity violation is treated more as a soft constraint than an explicit objective, this aspect must still be considered when designing the hospital waste management network. Therefore, we include *balance* as a third objective in the calculation of the SSC metric, even though it has not been explicitly sought by the NSGA II selection procedure. The normalized balance expression is then as follows:

$$\bar{z}_3(p) = 1 - \rho(p) \qquad (20)$$

where $\rho(p)$ and $\bar{z}_3(p)$ are the capacity violation (see 17) and normalized balance for the *p*-th solution of the nondominated set, respectively.

In sum, all criteria are normalized to compute a dimensionless SSC ranging from 0 to 1. The theoretical value of 1 is achieved only if the ideal solution is part of the unveiled nondominated set (i.e., an ideal network with the lowest cost solution, minimum population exposure, and perfect balance without any capacity violation). On the other hand, in the absence of a solution, the value of SSC takes the value of 0. In this latter case, there is no space covered by the nondominated set.

Relative and absolute quality

As in Medaglia et al. (2007), to compare the quality of the nondominated front obtained with the proposed evolutionary algorithm against that obtained with an alternative method, we report the fraction of solutions provided by each algorithm in the aggregated front obtained by combining the solutions of both algorithms. In the remaining part of this document, we call this fraction *absolute quality*. In addition, we report how many of the nondominated solutions generated by each algorithm, were truly nondominated (i.e., they appear in the aggregated front). We call this fraction *relative quality*. For more information on the latter and other metrics in multiobjective optimization, the reader is referred to Jaszkiewicz (2004).

Convex hull of the approximated efficient frontier

In the case of non convex solution spaces, the Pareto optimal set may contain two types of solutions: *supported* and *non-supported*. Supported solutions are optimal for a weighted objective function with non negative weights; and can be found using methods that use such an aggregated function. On the contrary, non-supported solutions cannot be found using an aggregated function, because there is no combination of weights for which the solutions are optimal. Since the BOOFLP is a biobjective combinatorial optimization problem, its efficient frontier may contain non-supported efficient solutions.

To estimate the potential number of non-supported solutions found by the proposed algorithms, we count the points that are not in the boundary of the convex hull of the approximate efficient frontier. These points could be efficient non-supported solutions. We denote this metric by $\left|\mathrm{int}(\widehat{PF})\right|$.

3.3. Parameter tuning

To tune the algorithms' common parameters, we experiment with a set of 144 runs on the GA-GAH to explore the impact on quality (SSC metric) and time (in ms) of different levels of P, population size *($P=20,50,100$)*; N, maximum number of generations *($N=20,50,100,200$)*; p_c, crossover rate *($p_c=0.5,0.7,0.9$)*; and p_m, mutation rate *($p_m=0.01,0.02,0.05,0.10$)*. The parameter β for GA-GAH is set at *0.5*, its central value. The results, given in Figure 3, show the compromise in terms of quality and time among the parameters. Specifically, Figure 3(a) shows the SSC for every combination of parameters, while Figure 3(b) shows the corresponding computing times. Based on these results, in which white zones are preferable, we fix p_c=0.7, p_m=0.05, $P=50$, and $N=100$ as the parameter combination that will give good results with acceptable computing times. This experiment was conducted on a Dell Optiplex GX280 with 1GB of RAM and an Intel Pentium IV processor running at 3GHz under Windows XP Professional.

INSERT FIGURE 3

The results for a second experiment to tune the GA-MIP's penalty parameter α (see Figure 4) show how α affects the quality (deviation from the optimal cost assignment) and the CPU time of the MIP (11)– (15) embedded in the assignment heuristic. Because the MIP is solved repeatedly in GA-MIP, it is especially important to save computing time by tuning α correctly. A careful appraisal of Figure 4 suggests that there is no gain in setting α at a value greater than 100: in fact, values around 100 enable assignments with perfect balance (no capacity violations).

INSERT FIGURE 4

Nonetheless, even though GA-MIP is good at achieving balanced configurations with little sacrifice in cost (see Figure 4), this attainment comes at a high computational price. Therefore, to reduce computing time, we run a further GA-MIP experiment to gage the sensitivity of SSC to changes in the penalty α (for values below 100). After fixing *P=10, N=10,* $p_c=0.7$, and $p_m=0.05$, we conduct ten independent runs for α*=1, 2, 5, 10,* and *100*. This third experiment, summarized in Table 1, reveals that for values of α>2, GA-MIP takes significantly more time with no improvement to the *SSC* metric.

INSERT TABLE 1

To increase processing speed, GA-MIP implements a map structure that stores the results of the MIP assignment heuristic for every chromosome evaluated. Specifically, before running the MIP assignment procedure naively, GA-MIP checks whether the MIP has been run before; if so, it retrieves the value of the objective function from the map instead of running the expensive MIP again. This enhancement saves about 30% of computing time compared to a GA-MIP without such a map.

3.4.NISE

As an alternative method, we implemented the Noninferior Set Estimation (NISE) method (Cohon et al., 1979; Cohon, 2003) for biobjective optimization, which has been applied recently to explore tradeoffs in a supply chain design problem (Shen and Daskin, 2005). NISE was designed to generate quickly a good approximation of the efficient frontier based on supported nondominated solutions. The method solves a sequence of optimization problems using a weighted objective function. The method begins by optimizing each objective individually, thus obtaining the two supported solutions (z^1 and z^2) on the extremes of the efficient frontier as shown in Figure 5 (a). The height of the triangle formed by the extreme points and the ideal, denoted by $\psi_{1,2}$, is an upper bound to the distance of the line connecting z^1 and z^2 and a potential unexplored efficient point. In each intermediate step (see Figure 5 (b)), based on the geometric upper bound denoted by $\psi_{n,n+1}$, the method finds an unexplored segment, which could lead to a new supported solution on that boundary of the frontier. The unexplored segment, formed by two adjacent supported solutions, is used to compute the weights for the aggregate objective function. By choosing in every step the largest geometric bound, the method guarantees an even approximation of the efficient frontier, even if the method is stopped prematurely without exploring all segments of the frontier. To stop the method, the bound $\psi_{n,n+1}$ is compared against $T = \gamma \cdot \psi_{1,2}$, where γ is a parameter chosen by the user ($0 \leq \gamma \leq 1$). Small values of γ achieve a very good approximation of the efficient frontier because the algorithm stops when all $\psi_{n,n+1}$ are no larger than *T*.

INSERT FIGURE 5

To find an approximate efficient frontier for the BOOFLP with NISE, we solve the following problem repeatedly:

$$\min \quad w_1 \cdot z_1 + w_2 \cdot z_2 \tag{21}$$

subject to,

(3), (4), (6) and (7)

$$\sum_{j\in J} d_j x_{ij} - e_i \le s_i y_i, \quad i \in I \tag{22}$$

$$\frac{\sum_{i\in I} e_i}{\sum_{j\in J} d_j} \le \rho \tag{23}$$

$$e_i \ge 0, \quad i \in I \tag{24}$$

The objective function (21) represents the compromise between the cost objective z_1, defined in (1), weighted by w_1; and the population exposure, defined in (2), weighted by w_2. The constraints (22) include the term e_i which accounts for the violation of the soft capacity constraint of the i-th CDC. Constraints (23) impose a bound on the maximum allowable aggregated capacity violation for the CDCs (ρ). Finally, the relations (24) define the capacity violation variables, one for each CDC.

3.5. Boyacá's hospital waste management network design

The data used in this section are taken from Rodriguez's (2005) description of Boyacá's hospital waste management network, in which 120 hospital waste generators are connected through Boyacá's road network. The population at every potential CDC and the daily hospital waste generated by each town are known. The distances between towns were calculated by applying Dijkstra's algorithm to the road network. These distance data were then used as proxy for transportation cost. According to Rodríguez (2005), it is advisable from an economic and operational perspective to set up a hospital waste management network with four CDCs.

Table 2 compares the performance of GA-GAH and GA-MIP on Boyacá's hospital waste management network by outlining the average and maximum SSC calculated with the final population of 10 independent executions of the algorithms. For GA-GAH, the parameter β is set at five different levels. The column labeled "All" reports the SSC metric on the frontier obtained by merging all the final GA-GAH populations. From these results, it is clear that the hybrid GA-MIP outperforms the GA-GAH in terms of SSC. The size of the dominated space obtained with all the GA-GAH runs is just as large as the average dominated space obtained with GA-MIP. Thus, even though the GA-MIP is slower than the GA-GAH, it seems a reasonable price to pay for better configurations of the hospital waste network, especially considering the strategic nature and long-term effects of these decisions.

INSERT TABLE 2

Figure 6 shows a projection of the approximate efficient frontier obtained with GA-GAH and GA-MIP. It is worth mentioning that the seemingly dominated solutions obtain better balance on the (soft) capacity constraints. Moreover, GA-MIP solutions are generally cheaper and affect fewer of the population. Figure 6 also illustrates the existing tradeoff between population exposure and transportation cost. If cost is taken as the sole objective, the lowest cost solution (black triangle) will impact twice the population affected by a solution that is only 9.89% more expensive. Likewise, another solution that is 16.8% more expensive will affect only a quarter of the population of the lowest cost solution. Therefore, these are prices that society may be willing to pay in order to avoid the location of obnoxious CDCs.

INSERT FIGURE 6

It is important to note that, unlike the single objective uncapacitated solution (black triangle in Figure 6) found by Rodriguez (2005), the GA-MIP balances the assignment of hospital waste to CDCs, a balance that is even better illustrated in the approximate efficient frontier shown in Figure 7. Moreover, all the solutions found by GA-MIP with α=2 are no more than 6% off from the ideal (completely even) assignment. The values for the cost, population exposure, and the capacity constraint violation for every solution shown in Figure 7 is given in Table 3. Among the best of these is solution 1, which affects very few of the nearby residents (4,377) and has only a 0.3% capacity violation; and solution 41, that compared to the lowest-cost completely-balanced solution obtained with the capacitated FLP, is only 4.7% more expensive, achieves a reasonable balance (2.1% of capacity violation), and affects just half of the exposed population.

INSERT FIGURE 7

INSERT TABLE 3

Finally, Figure 8 compares two hospital waste management network configurations for Boyacá. The colored dots are waste generators (towns), while the dots bounded by boxes are the sited CDCs. Figure 8(a) shows the lowest cost solution obtained by Rodríguez (2005) using the uncapacitated facility location problem. It is worth noting that this solution, which optimizes transportation costs, tries to locate CDCs at the centroid of their service area. Figure 8(b) shows the solution with the least population exposure (Solution 1 in Table 3), which is a configuration at the efficient frontier obtained by a GA-MIP with α=2. However, this solution tries to locate CDCs in off-center small towns, which leads to larger transportation costs.

INSERT FIGURE 8

3.6.GA-MIP vs. NISE

We tested the robustness of GA-MIP with a larger set of benchmark problems adapted from the literature. For this experiment, GA-MIP was selected over GA-GAH because of its better results in terms of the size of the space covered metric (SSC). The selected test problems have similar size to that of Boyacá's and it is possible to resemble the structure of the BOOFLP using their data. The first set is comprised of instances proposed by Alp et al. (2002) for the uncapacitated *p*-median problem with data from the Province of Alberta (Canada). Problems from this set are labeled in this section with the prefix *Alberta*. The second set of capacitated *p*-median instances, proposed by Lorena and Senne (2003), is composed of real data collected using a Geographical Information System for the city of São José dos Campos (Brazil). Instances from this set are labeled *SJC*. To convert Alberta's instances into the BOOFLP format, we used the *weight-of-node* as demand d_j and population p_i; and calculated s_i as shown in equation (8). For SJC's instances, we use the demand, represented by the number of houses (apartments) at each block, as a proxy for population p_i. Henceforth, these test problems are labeled *name-nodes-K*, where *name* stands for the problem set; *nodes* is the number of candidate facilities or customers (i.e., $|I| = |J|$); and K is the total number of facilities to be opened.

Table 4 summarizes the results for GA-MIP. For each instance, ten independent runs were executed. Then, the final populations of each run were aggregated to construct the approximate efficient frontier $\widehat{PF}$. The parameters for GA-MIP were set according to those found in Section 3.3, that is, p_c=0.7, p_m=0.05, *P=50, N=100,* and $\alpha = 2$. To speed up the convergence on instance Alberta-316-10, we tried a variant of GA-MIP powered by GAH. First, the algorithm evaluates the chromosome with the GAH, and provided the result is promising, it evaluates the MIP with $\alpha = 2$. This is supported by a positive correlation between the cost assignment of GAH and MIP on a random sample of chromosomes.

INSERT Table 4

Table 5 shows the summary results for NISE. To make GA-MIP's frontiers comparable to those obtained with NISE, we use as input for the latter the maximum capacity violation in the frontier $\widehat{PF}$ (for GA-MIP) as the value for ρ in constraint (23). The values for γ were set as follows: for Boyacá-120-4 and SJC-100-10, γ=0%; for SJC-200-15, γ=0.01%; and for Alberta-316-5 and Alberta-316-10, γ=0.5%. Because of the high computational burden on Alberta-316-5 and Alberta-316-10, we ran these experiments on a PowerEdge SC1430 with two Intel(R) Xeon(R) CPU 5120 processors running at 1.86GHz with 4 GB of RAM. To illustrate how difficult is to solve the underlying MIPs, for Alberta-316-5 the minimum cost nondominated solution for NISE took 1405.5 seconds (about 23 minutes).

INSERT TABLE 5

Solution times, shown in the second column of Table 4 and Table 5, vary widely depending on the instance. For GA-MIP, it ranges from 6 to 77 minutes; while for NISE, time ranges from 2.1 to 132 minutes. NISE and GA-MIP show similar solution times for SJC-100-10, but Boyacá-120-4 and SJC-200-15 seem slightly easier for NISE. However, for the larger Alberta instances, NISE do not scale as well as GA-MIP. For instance, GA-MIP found an acceptable approximation of the frontier in only 25% of the time it took NISE (on a better machine) for Alberta-316-5; likewise, there is a 42% time reduction on Alberta-316-10 when using GA-MIP over NISE. Nevertheless, Alberta-316-10 seems most difficult than any other problem for both methods. Finally, note that these times are completely reasonable for strategic location problems like the BOOFLP, and particularly for Boyacá's hospital waste management network design.

In terms of the quality of the efficient frontiers, the metrics shown in columns 3 to 6 of Table 4 and Table 5 include capacity balance as a third objective in addition to cost and population exposure. The relative quality measure shows that an important fraction of the solutions generated by GA-MIP (88.5% in average) remain nondominated in the aggregated efficient frontier. This is due mainly because GA-MIP solutions have better balance than those of NISE. Also, in terms of quantity (column 5), GA-MIP provides in average 65.5% of the solutions of the aggregated efficient frontier. This is a good result since GA-MIP provides the decision maker with a rich set of solutions and wider range of options than NISE.

In terms of the size of the space covered (SSC), the frontiers generated by both methods in Boyacá-120-4 and SJC-100-10, dominate spaces of similar sizes; while in SJC-200-15, the GA-MIP covers a smaller space than NISE. These results are better illustrated in Figure 9 and Figure 10, where the size of the bubble is proportional to the capacity violation. Figure 9 shows that while in SJC-100-10 GA-MIP is able to find widespread solutions along the efficient frontier, for SJC-200-15 the GA-MIP fails to find low-cost solutions (see Figure 10).

INSERT FIGURE 9

INSERT FIGURE 10

The last column of Table 4 shows the number of solutions in the efficient frontier found by GA-MIP that could be non-supported. For all instances more than half of the solutions found by GA-MIP lie in the interior of the convex hull of the frontier. This result shows how GA-MIP also finds non-supported solutions, which is an advantage of GA-MIP over NISE.

4. Conclusions

This formulation of a biobjective obnoxious facility location problem was motivated by the design of a hospital waste management network for Boyacá (Colombia). This work proposes two multiobjective evolutionary algorithms capable of showing the tradeoff between transportation cost and exposed population. The first algorithm uses a fast greedy fitness assignment heuristic (GA-GAH), while the second uses a fitness assignment approach based on mixed-integer programming (GA-MIP). Experiments on data from Boyacá's hospital waste management network show that the hybrid GA-MIP obtains better solutions than the GA-GAH in terms of the SSC metric. However, these solutions come at the price of slightly greater, albeit reasonable, computational time.

Even though the proposed evolutionary algorithms were designed for the case of Boyacá's network, we tested the robustness of GA-MIP on a larger set of benchmark problems adapted from the literature. GA-MIP was compared against the Noninferior Set Estimation (NISE) method, a biobjective optimization approach able to find supported nondominated solutions. In terms of speed, GA-MIP and NISE obtained comparable results, but the proposed evolutionary algorithm scaled better than NISE on large instances. GA-MIP was able to find efficient frontiers of similar quality for most instances, but it failed to find low-cost solutions in certain cases, thus, affecting the size of the space covered metric. A unique advantage of GA-MIP is its ability to find non-supported solutions, contrary to NISE, which by design only finds supported solutions.

This obnoxious facility location problem is of public concern, and has been shown to be a valuable tool for political discussion. Specifically, it allows public decision makers to analyze several alternative solutions with different compromises between criteria. Because of current legislation, this approach might be useful for designing other hospital waste management networks for other Colombian departments.

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Tables

α	***1***	***2***	***5***	***10***	***100***
SSC (avg.)	0.313	0.334	0.327	0.344	0.309
Avg. time (s)	8.2	12.0	22.0	34.1	254.4

Table 1. Sensitivity of SSC to changes in α

	GA-GAH						GA-MIP	
	β=0.00	*β=0.25*	*β=0.50*	*β=0.75*	*β=1.00*	*All*	*α=1*	*α=2*
Average SSC (%)	54.3	56.3	56.9	50.9	48.9	-	70.1	71.1
Maximum SSC (%)	60.3	64.1	62.8	56.7	53.9	71.1	73.7	76.6
Average time (s)	0.74	0.69	0.68	0.71	0.72	-	76.0	124.3

Table 2. SSC for the GA-GAH and GA-MIP in Boyacá's hospital waste management network design

Solution	*Cost(COP$)*	*Population (inhabitants)*	*Capacity violation(%)*	*Solution*	*Cost (COP$)*	*Population (inhabitants)*	*Capacity violation(%)*
1	8,351,308	4,737	0.3	22	5,221,875	9,256	1.6
2	7,524,252	5,111	1.7	23	5,207,996	9,780	2.3
3	7,065,272	5,408	3.3	24	5,151,488	9,949	2.9
4	6,610,810	5,324	5.8	25	5,067,361	10,100	1.0
5	6,464,680	5,729	0.7	26	5,044,307	10,474	1.0
6	6,149,095	6,778	0.9	27	5,021,994	10,133	2.3
7	6,128,210	6,860	1.0	28	5,001,500	9,765	3.7
8	5,999,550	5,839	3.4	29	4,980,880	10,562	1.0
9	5,697,271	7,656	0.7	30	4,978,624	11,860	1.6
10	5,589,548	8,048	0.7	31	4,927,185	10,970	1.9
11	5,580,027	6,962	0.8	32	4,912,114	22,845	0.8
12	5,554,095	6,670	2.5	33	4,899,266	12,077	0.9
13	5,501,817	9,391	0.7	34	4,863,200	14,690	1.0
14	5,497,822	9,462	0.4	35	4,777,126	12,762	2.1
15	5,394,093	9,783	0.7	36	4,767,409	14,818	1.1
16	5,384,500	8,697	0.8	37	4,756,794	16,183	1.0
17	5,357,087	7,444	1.0	38	4,753,476	14,060	2.1
18	5,314,110	8,171	0.9	39	4,704,455	17,816	1.0
19	5,309,013	7,953	1.6	40	4,700,762	19,114	2.1
20	5,262,847	8,011	1.0	41	4,684,090	21,638	2.1
21	5,238,152	8,853	0.9	Best	4,684,090	4,737	0.3

Table 3. Solutions for the approximate efficient frontier obtained using a GA-MIP with α=2

Problem	***Time (s)***	$\left\|\widehat{PF}\right\|$	***Relative Quality***	***Absolute Quality***	***SSC***	$\left\|int(\widehat{PF})\right\|$
Boyacá-120-4	480	41	97.6%	76.9%	0.79	34
SJC-100-10	2035	43	97.7%	73.7%	0.62	17
SJC-200-15	3904	32	65.6%	44.7%	0.36	16
Alberta-316-5	1184	41	90.2%	77.1%	0.49	25
Alberta-316-10	4595	23	91.3%	55.3%	0.30	21

Table 4. Summary results for GA-MIP

Problem	***Time (s)***	$\lvert\widehat{PF}\rvert$	***Relative Quality***	***Absolute Quality***	***SSC***
Boyacá-120-4	127	26	100.0%	23.1%	0.81
SJC-100-10	2616	15	100.0%	26.3%	0.74
SJC-200-15	1694	28	100.0%	55.3%	0.81
Alberta-316-5	4760	11	100.0%	22.9%	0.62
Alberta-316-10	7934	17	100.0%	31.5%	0.45

Table 5. Summary results for NISE

Figures

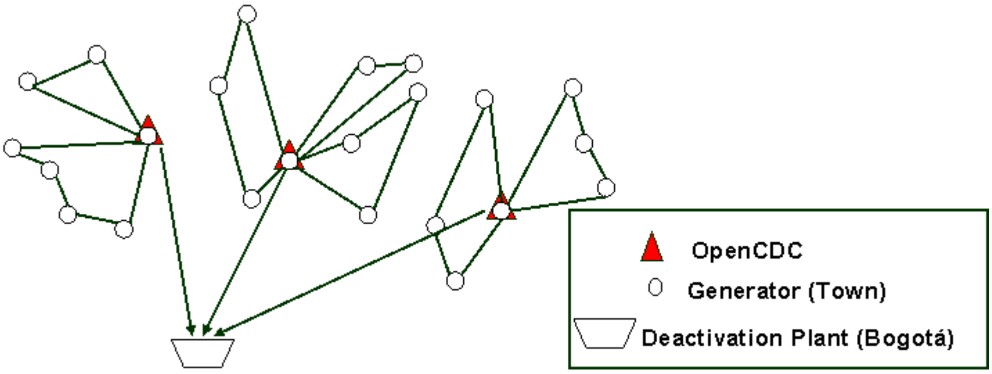


Figure 1: Boyacá's proposed hospital waste management network

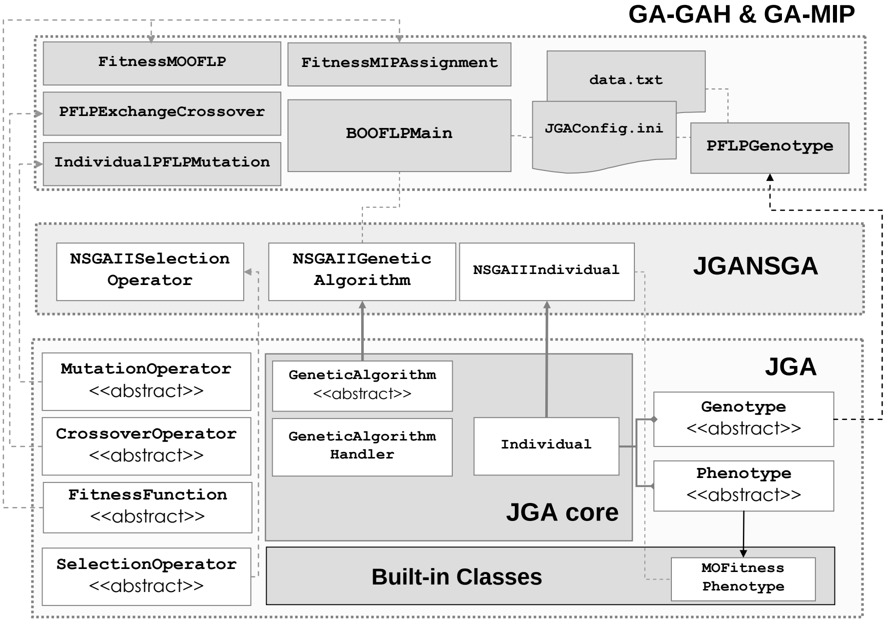
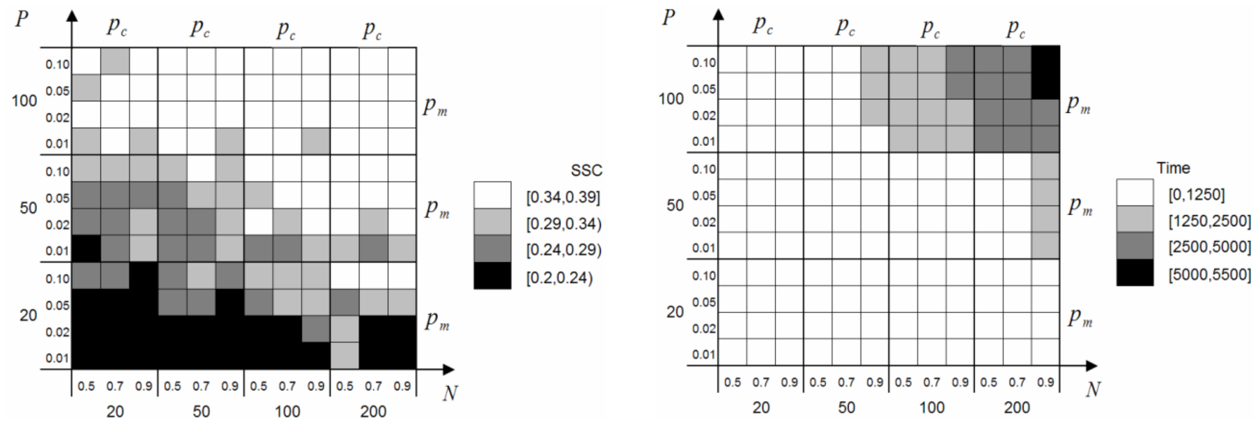


Figure 2: Implementation of the GA-GAH and GA-MIP on MO-JGA



a) SSC **a) Time (ms)**

Figure 3: Tuning the common parameters for the GA-GAH and GA-MIP

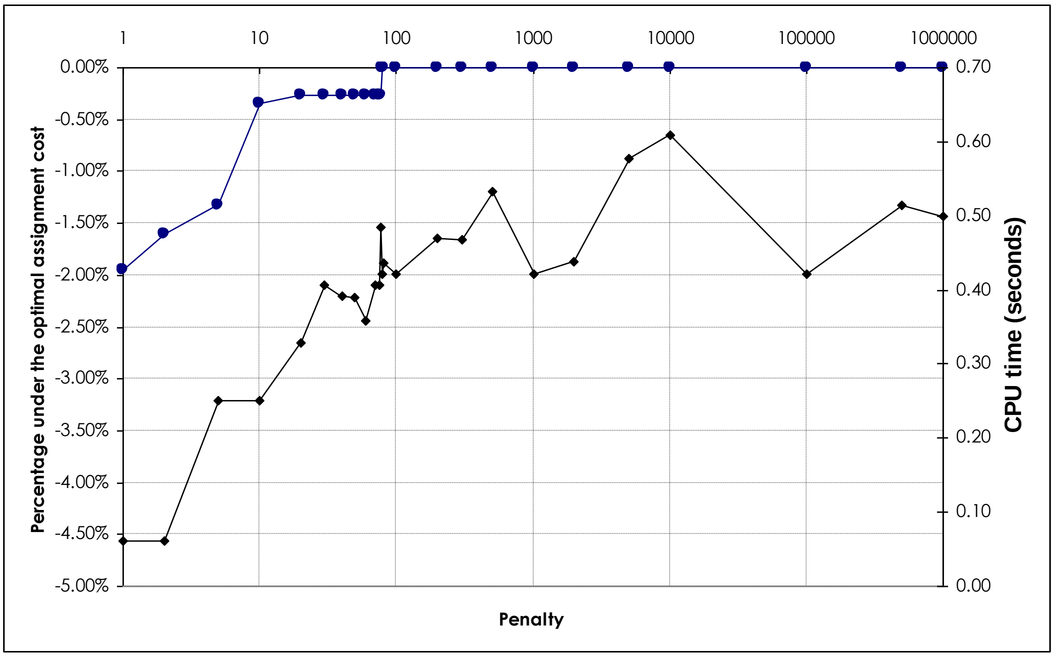
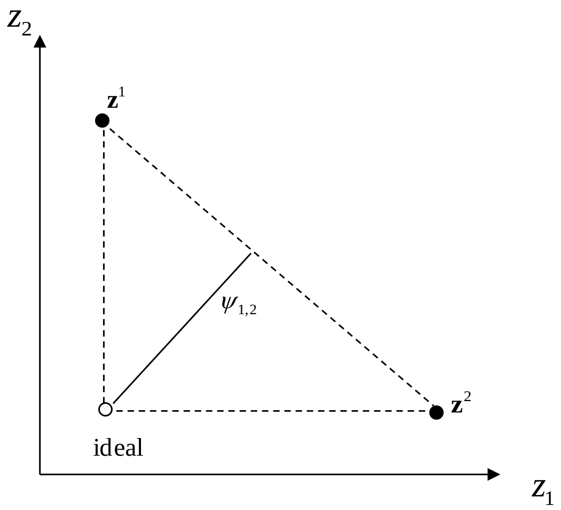
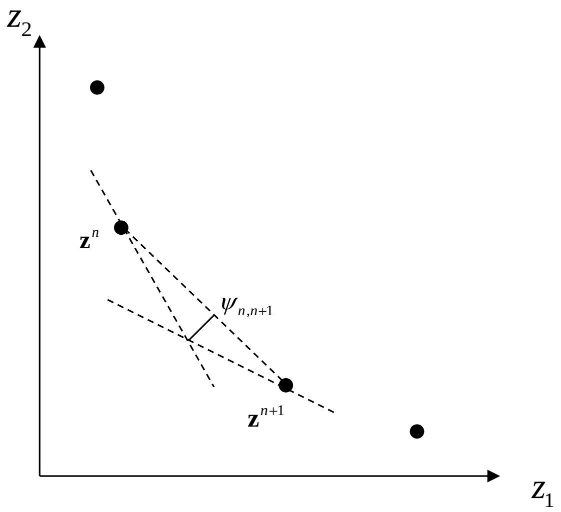


Figure 4: Tuning the penalty parameter α of the GA-MIP



a) Computing the maximum error (upper bound)



b) Computing the upper bound between two adjacent points in the frontier

Figure 5: NISE

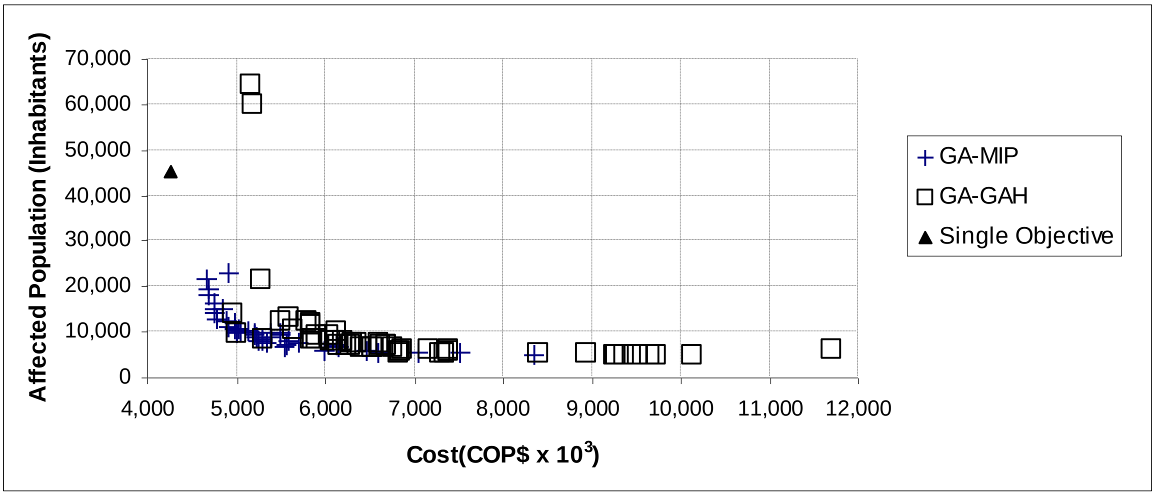


Figure 6: Projected approximate efficient frontier for Boyacá's hospital waste management network

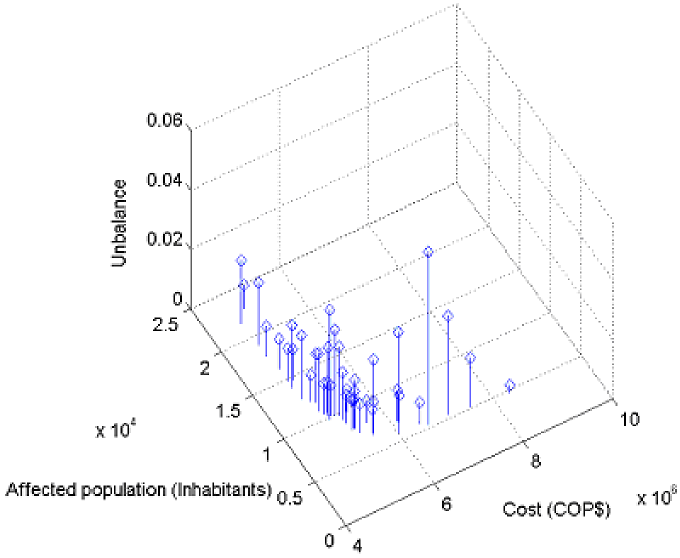
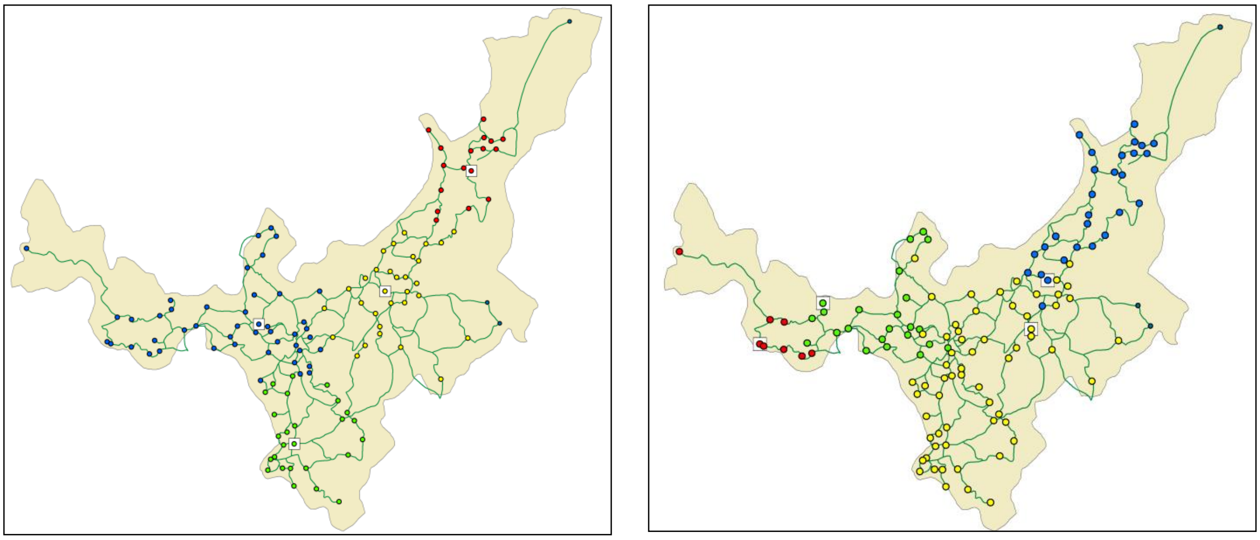


Figure 7: Three-dimensional approximate efficient frontier obtained using a GA-MIP with α=2



a) Lowest cost solution (Rodríguez, 2005) **b) GA-MIP efficient solution # 1**

Figure 8: Different design alternatives for Boyacá's hospital waste management network

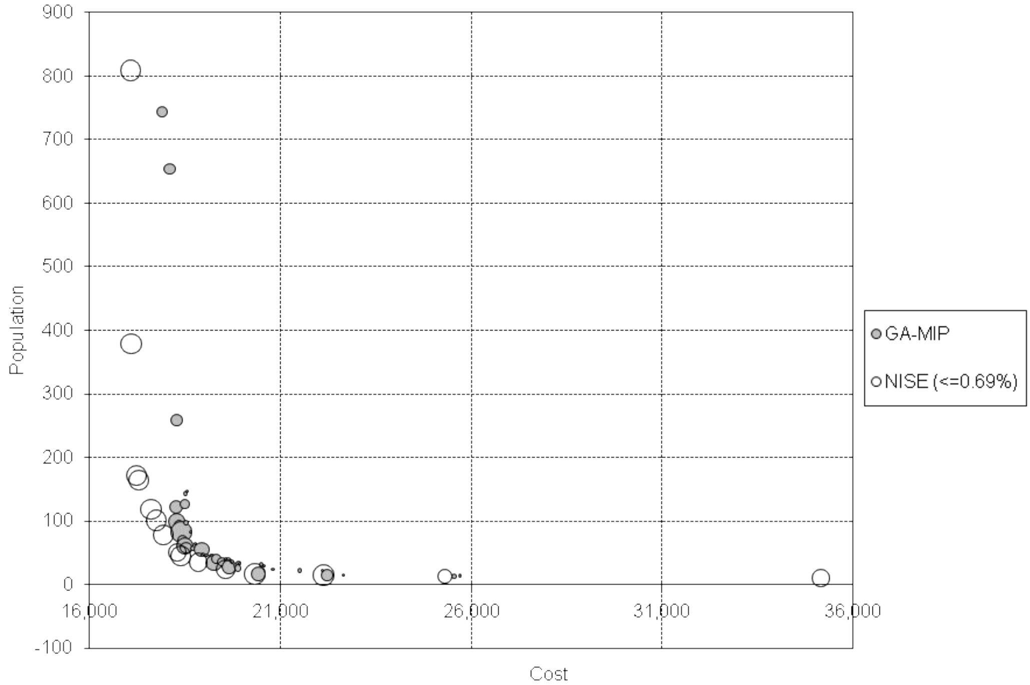


Figure 9 Approximate efficient frontier found by GA-MIP and NISE in problem SJC-100-10

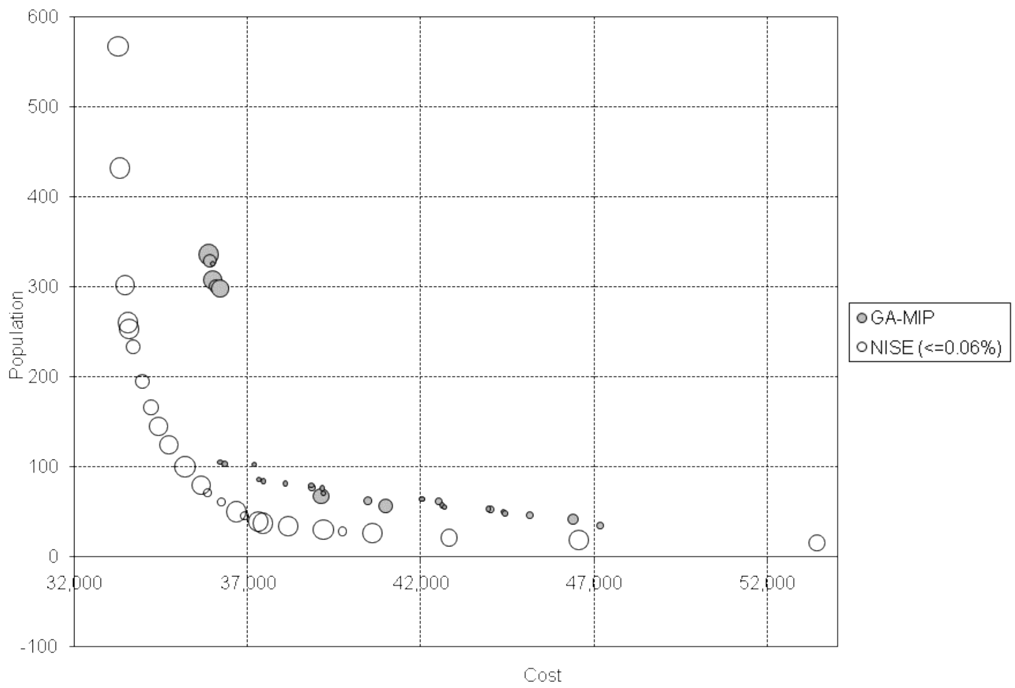


Figure 10 Approximate efficient frontier found by GA-MIP and NISE in problem SJC-200-15