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Talbot effect for the periodical object limited by a finite aperture

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ABSTRACT

The communicate presents a new interpretation of the Talbot effect for periodical objects limited by finite apertures. According to the proposed approach, a self-image of a real, finite object is a superposition of deformed images of an elementary cell. The singular elementary cell image is equivalent to that formed in a proper optical system. The theoretical description makes possible to define a structure of self-images. Particularly, the approach enables a determination of apertures' dimensions which lead to self-images of a reasonable quality in a desired region of an image plane. The theory is illustrated and verified by numerical simulations.

Keywords: Talbot effect, imaging, diffraction

1. INTRODUCTION

When a periodical object is illuminated by a plane wave, then in the Fresnel region of diffraction exact object images appear. The images are formed in a free space and are localized in planes periodically situated along the illumination direction [1]. This phenomenon is known as the self-imaging effect or the Talbot effect. According to the theory of the Talbot effect, exact replicas are formed in the diffractive process only when the periodical structure is infinite. This crucial condition cannot be fulfilled in reality. Although many scientific works presented different aspects of the Talbot effect, only few of them were devoted to the analysis of the influence of finite dimensions of real objects. The qualitative explanation of the Talbot effect in a case of the finite periodical object was based on the walk-off effect [2, 3]. The quantitative, however approximate approach was given by Smirnov [4].

This article presents a new interpretation of the Talbot effect in a case of periodical objects limited by apertures of finite dimensions. The approach is based on the lens-like theory of the sampling filter [5, 6]. The description is precise within the Fresnel paraxial approximation. According to our approach, a self-image is equivalent to the superposition of differently spoiled images of the elementary cell. The singular elementary cell image is equivalent to that formed in a proper optical system. The filtering process depending on the aperture function of the object deforms the image.

The theory enables determination of the self-image structure in a case of a given elementary cell transmittance and an aperture function limiting a periodical input object. The presented description is verified by numerical simulations.

For simplicity all calculations correspond to the one-dimensional case of diffractive gratings. A transition to the two-dimensional case of periodical objects arranged in square arrays is straightforward and requires only an addition of the second variable. All conclusions can be extended onto arbitrary arrays fulfilling the self-imaging condition [7, 8].

2. THEORY

The transmittance of the real, periodical object can be expressed as follows:

$$T(x) = A(x)\left[\sum_{n=-\infty}^{\infty} t(x-nd)\right] = \frac{1}{d}A(x)\left[t(x)\otimes comb\left(\frac{x}{d}\right)\right], \tag{1}$$

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where d is the object period, $t(x)$ defines transmittance of the object elementary cell, $A(x)$ is the aperture function limiting the object dimension, and the symbol $\otimes$ denotes the convolution operator. According to the theory of the sampling filter, the comb function in Eq.(1) can be rewritten with accuracy to a constant in the following equivalent form [5, 6]:

$$comb\left(\frac{x}{d}\right)=\sum_{n=-\infty}^{\infty}\exp\left[-\frac{ik}{2Z_N}(x-nd-D)^2\right], \tag{2}$$

where $Z_N=\frac{Nd^2}{\lambda}$ (N=1, 2, 3…), $k=\frac{2\pi}{\lambda}$, D=0 for N even and $D=\frac{d}{2}$ for N odd, λ is a wavelength of monochromatic light used in an optical system. According to Eqs. (1), (2), when the infinite object ($A(x)$=1) with the transmittance T(x) is illuminated by a plane wave of a wavelength λ, then in planes at distances Z_N behind the object its images are formed [6]. Self-images being the exact object images correspond to N even. For N odd the self-image is additionally shifted by $\frac{d}{2}$ in respect to the object. Eqs. (1), (2) lead to the following form of the diffractive field in the plane Z_N behind the real periodic object:

$$U(x)=\sum_{n=-\infty}^{\infty}\left\{A(x)\left[t(x-nd-D)\otimes\exp\left(-\frac{ikx^2}{2Z_N}\right)\right]\right\}\otimes\exp\left(\frac{ikx^2}{2Z_N}\right). \tag{3}$$

The non-important constant was omitted in the above formula.

2.1 The imaging system with a lens

For the further calculations it is convenient to introduce new coordinate systems x_1 which centres coincide with those of the elementary cells. In the case of the fixed index n we have the relation $x_1=x-nd-D$ and the term under the sum in Eq. (3) can be rewritten in the following way:

$$U_n(x_1)=\left\{A(x_1+nd+D)\left[t(x_1)\otimes\exp\left(-\frac{ikx_1^2}{2Z_N}\right)\right]\right\}\otimes\exp\left(\frac{ikx_1^2}{2Z_N}\right), \tag{4}$$

After the elementary calculations we obtain the identity:

$$t(x_1)\otimes\exp\left(-\frac{ikx_1^2}{2Z_N}\right)=\left\{\left[t(-x_1)\exp\left(-\frac{ikx_1^2}{Z_N}\right)\right]\otimes\exp\left(\frac{ikx_1^2}{2Z_N}\right)\right\}\exp\left(-\frac{ikx_1^2}{Z_N}\right). \tag{5}$$

Then Eq. (4) can be expressed as follows:

$$U_n(x_1)=\left\langle\left\{\left[t(-x_1)\exp\left(-\frac{ikx_1^2}{Z_N}\right)\right]\otimes\exp\left(\frac{ikx_1^2}{2Z_N}\right)\right\}\exp\left(-\frac{ikx_1^2}{Z_N}\right)A(x_1+nd+D)\right\rangle\otimes\exp\left(\frac{ikx_1^2}{2Z_N}\right) \quad .(6)$$

According to the theory of the thin lens and the Fresnel paraxial approximation [9], the elementary cell image described by the function $U_n(x_1)$ is exactly the same as the real image created by a thin lens in the optical arrangement shown in Fig. 1.

The lens has a focal length $Z_N/2$ and is limited by the aperture function $A(x_1+nd+D)$. The object and image planes are distant by Z_N from the lens so the magnification is equal to one. The input object is an inverted elementary cell illuminated by a convergent wave of the curvature radius $Z_N/2$. The self image is a superposition of the spoiled elementary cell images. Using the previous coordinate system *x* we obtain the following formula for the output field in the self image plane:

$$U(x)=\sum_{n=-\infty}^{\infty}U_n(x-nd-D). \qquad (7)$$

The singular image of the elementary cell is deformed because of the pupile function of the lens $A(x_1+nd+D)$. A limited aperture of the lens filters the diffractive Fresnel field behind the object.

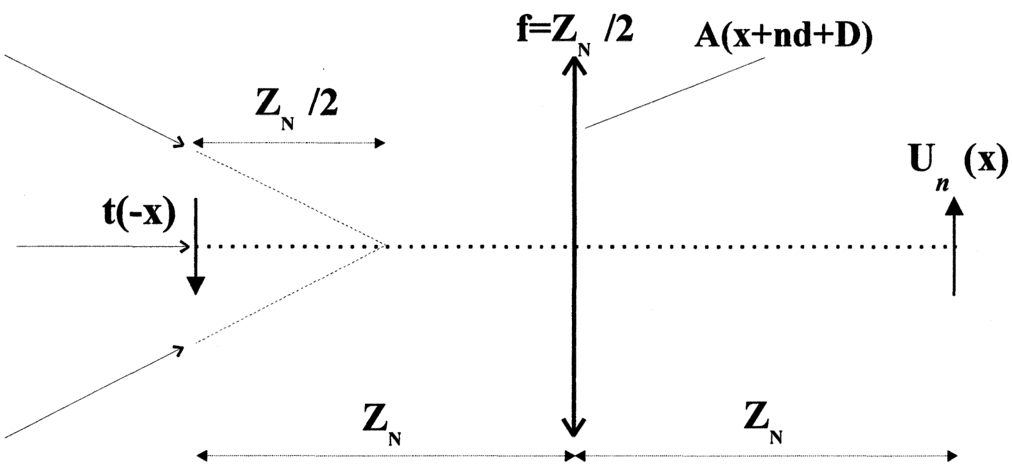
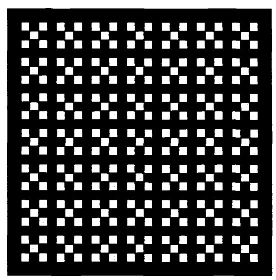


Fig. 1: A scheme of the imaging system with a lens. The convergent wave of a curvature radius $Z_N/2$ illuminates the inverted elementary cell with a transmittance $t(-x)$. The lens with a focal length $Z_N/2$ and the aperture function $A(x+nd+D)$ forms the cell's image corresponding to the complex amplitude $U_n(x)$.

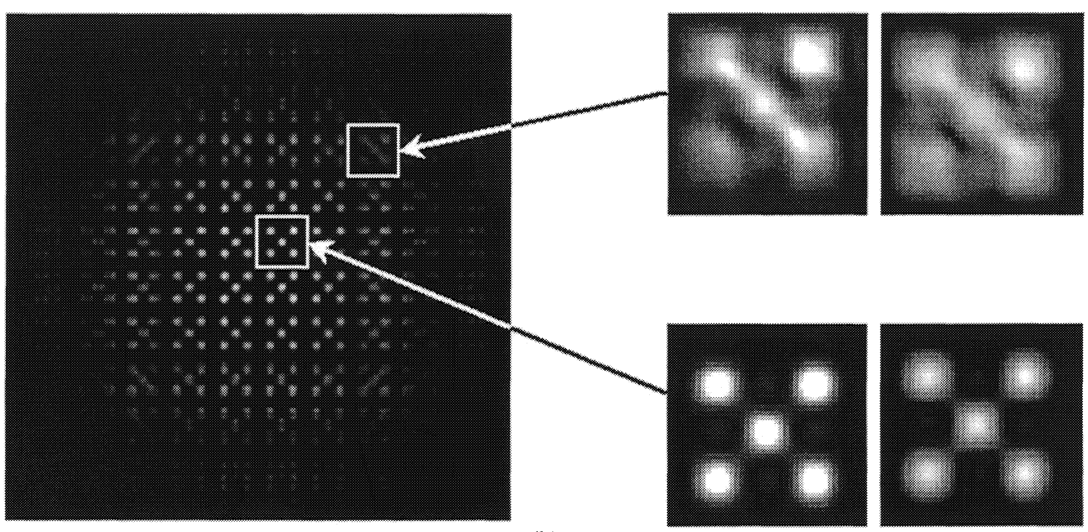
The dimension of the elementary cell is limited by the period *d.* Hence $\frac{k}{Z_N}\left(\frac{d}{2}\right)^2=\frac{\pi}{2N}$ defines the maximal phase module of the illuminating cylindrical wave within the input object region corresponding to the range $x_1\in\left(-\frac{d}{2},\frac{d}{2}\right)$. This module never overcomes π/2. Therefore in many cases seems to be reasonable to neglect the cylindrical factor in the product $t(-x_1)\exp\left(-\frac{ikx_1^2}{Z_N}\right)$. According to this approximation the discussed imaging system has a coherent transfer function described by $A(\lambda Z_N \nu + nd + D)$, where ν denotes a spatial frequency corresponding to the elementary cell transmittance $t(x_1)$ [9]. This conclusion coincides with that given in ref. 4.

3. NUMERICAL SIMULATIONS

Numerical simulations have been performed in order to verify the presented description. The simulations were conducted for the wavelength of He-Ne laser (λ=632.8 nm) using a diffractive modeling package working according to the modified convolution approach [10] on a matrix 2048×2048 points with a sampling interval of 1 μm covering an area in a form of a square of the width about 2.05 mm The numerical results are shown in Fig.2. The performed simulations compare adequate elementary cell images formed in the self-imaging process and created in the optical arrangements described in the subsection 2.1.The compared images are very similar. The slight difference is caused by the additional interference between neighbouring elementary cell images what occurs in the self-imaging process. Singular images of elementary cells created in the optical systems are not modified by the above interference. Nevertheless the compared images have congruent structures. The obtained results confirm the theoretical approach. By means of numerical simulations one can evaluate a quality of self-images corresponding to limited periodical objects.

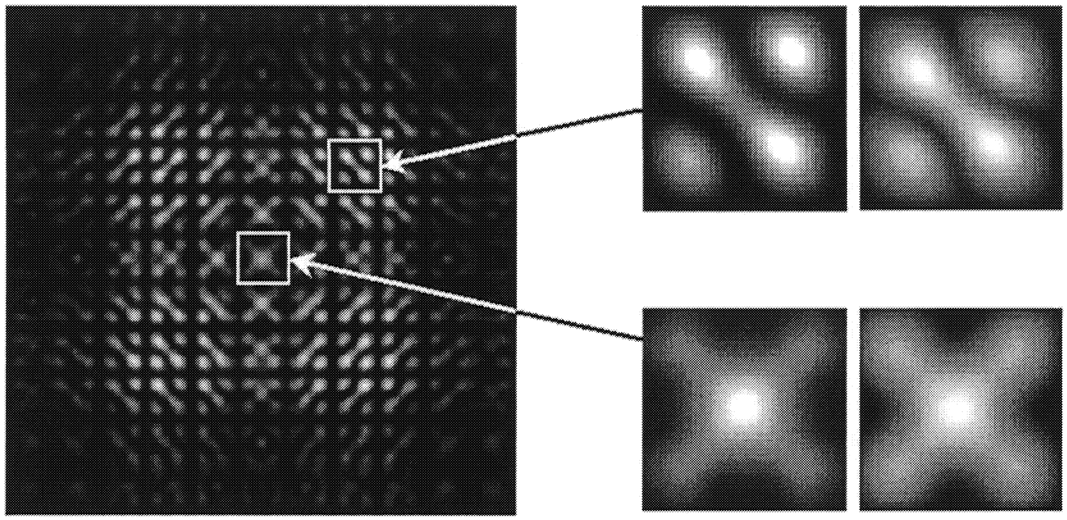


(a)



(b)

Fig. 2: (a) The periodical object with a period 40 μm, limited by a square aperture of a width 280 μm. (b) The intensity distribution of the diffractive field behind the object corresponding to the distance Z_1 = 2.528 mm.



(c)

Fig. 2: (c) The intensity distribution of the diffractive field behind the object corresponding to the distance $Z_2 = 5.057$ mm. The arrows indicate magnified fragments of the intensity distribution corresponding to images of the central elementary cell. On the right side of the images there are shown their counterparts obtained in the described optical arrangement with a lens.

4. CONCLUSIONS

The communicate describes the Talbot effect in a case of periodical objects limited by finite apertures. According to the proposed approach based on the lens theory of the sampling filter [5, 6], a self-image is a superposition of differently spoiled images of elementary cells of a periodical object. An image of a singular elementary cell is equivalent to that formed in a proper optical arrangement with a lens. The proper aperture of the lens filters the diffractive Fresnel field behind the elementary cell. The filtering process causes an image deformation and depends on elementary cell localization within the object's aperture. The presented approach makes possible to evaluate aperture dimensions, which lead to a high self-image quality within a desired region of a self-image plane. In this case an influence of the Fresnel field filtration should be negligible for proper elementary cells. The evaluation of aperture dimensions requires a detailed analysis of a structure of the Fresnel field corresponding to the elementary cell.

A self-image is a sum of complex amplitudes corresponding to elementary cells' images created in the described optical systems. Therefore one can determine a quality of self-images of a given limited periodical object. Appropriate numerical simulations confirming the presented theory were performed. The structures of elementary cell images formed in the optical system coincide with structures of their counterparts in self-image planes.

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