Finite object Talbot effect as a lens produced image

Article *in* Proceedings of SPIE - The International Society for Optical Engineering · November 2003

DOI: 10.1117/12.524039

3 authors, including:

John Fredy Barrera Ramírez
University of Antioquia

Andrzej Kolodziejczyk
Warsaw University of Technology

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John F. Barrera * [a], Rodrigo Henao[a], Andrzej Kolodziejczyk** [b]
[a]Universidad de Antioquia; [b]Warsaw University of Technology

ABSTRACT
In this work we theoretically calculate the self-image field amplitude of a finite periodic object. It is compared with the field of the image of an unitary cell of the same object formed by a lens. The results are verified by simulations of the two processes.

1. INTRODUCTION

When we illuminate an infinite periodic object of transmittance t(x,y) with a monochromatic plane wave, at a distance Z_N in front of the object, we find a plane where the field amplitude is identical to the transmitance function of the object [1]. These planes are called " Talbot" planes or "self-images". There also exist other planes Z_{NT} between them where the field amplitude reproduces t(x,y) but shifted half period (See Figure 1).

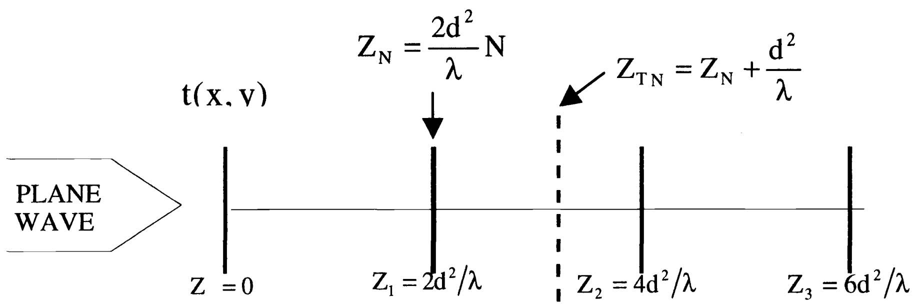

Figure 1. Talbot Effect.

Where d is the period, N is a natural number and λ is the wave length of the incident light.

2. THEORETICAL DESCRIPTION

The transmittance function of an ideal sampling filter, i.e. an infinite periodic object formed by points that are separated a distance d, can be represented as [2,3]:

$$T_1(x,y) = \left[\exp\left(-i\frac{k}{2f}\left(x^2+y^2\right)\right) \otimes t_u(x,y)\right] \cdot A\left(x+nd, y+md\right) \tag{1}$$

Where (n,m) represents the point coordinates (raw, column), f is the self-image distance in Talbot effect, $\otimes$ is the convolution operation and $\kappa = 2\pi/\lambda$. Now we consider the transmittance of a "finite" periodic object with unitary cells $t_u(x,y)$ with pupil function $A(x,y)$. For the cell (n,m), in particular, we obtain the transmittance function:

$$T_1(x,y) = \left[\exp\left(-i\frac{k}{2f}\left(x^2+y^2\right)\right) \otimes t_u(x,y)\right] \cdot A\left(x+nd, y+md\right) \tag{2}$$

Using the Fresnel approximation, the properties of the Fourier transform and of the convolution, we obtain, for the object whose transmittance is given by equation (2), a field amplitude in the self-image plane:

* Grupo de Óptica, Instituto de Física, Universidad de Antioquia, . FAX: +57 - 4 – 712771, Medellín, Colombia; **Faculty of Physics, Warsaw University of Technology, Koszykowa 75,00-662 Warsaw, Poland

19th Congress of the International Commission for Optics: Optics for the Quality of Life, Giancarlo C. Righini, Anna Consortini, Editors, SPIE Vol. 4829 (2003) © 2003 SPIE · 0277-786X/03/$15.00

$$U(x, y) = A \cdot \exp\left(\frac{i\kappa}{2f}\left(x^2 + y^2\right)\right) \cdot \left\{\left(t_u(x, y) \cdot \exp\left(\frac{-i\kappa x^2}{2f}\right)\right) \otimes \left(T_A\left(\frac{x}{\lambda f}\right) \cdot \exp\left(\frac{i2\pi d}{\lambda f}\right)(nx + my)\right)\right\} \tag{3}$$

Where A is a constant and $T_A(x/\lambda f)$ is the Fourier transform of the periodic object pupil function.

3. IMAGE FORMED BY A LENS

If we illuminate an unitary cell with a spherical wave and we use a lens to form its image (See figure 2), the field amplitude in the image plane is identically to the expression of the equation (3).

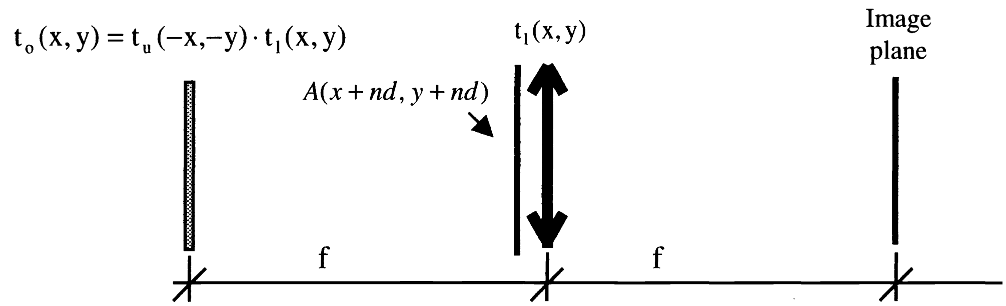

Figure 2. Unitary cell image by a lens

Where $t_l(x, y)$ is the lens transmittance whose focal length is f/2 and A(x+nd,y+md) is the lens pupil function equivalent to the finite size object in Talbot effect.

4. SIMULATIONS

Making the simulation of the processes that are shown in figures 1 and 2 we can observe the coincidence among the field amplitude for an unitary cell in Talbot effect (Figure 3) with the image of an unitary cell formed by a lens (Figure 4).

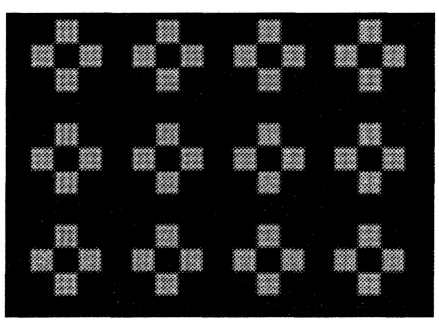

Figure 3

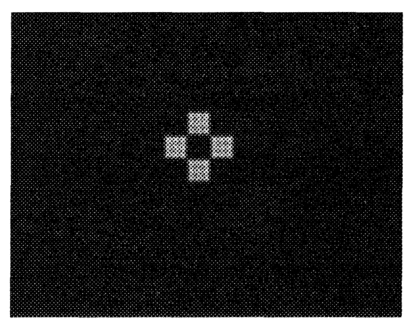

Figure 4

5. CONCLUSIONS

Equation (3) shows that the field amplitude in the self-image plane is affected by the finite size of the object, since the quality of the image falls toward the borders when having larger values for n and m. We tested, by using simulations, (Figures 3 and 4) that the unitary cell field amplitudes in both cases are the same.

6. REFERENCIAS

1. K. Patorsky, *The Self-imaging Phenomenon and its Applications,essevier science pubklishers B.V, Poland,1989.*
2. A. Kolodziejczyk, *''Self-imaging effect a new approach''*, Optics Communications. **65,** pp. 84-86, 1988.
3. A. Kolodziejczyk, *''Lensless multiple imege formation by using sampling filter''*, Optics Communications. **59,** pp. 867-875, 1986.