Nonanomalous horizontal $U(1)_H$ gauge model of flavor

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A nonanomalous horizontal $U(1)_H$ gauge symmetry can be responsible for the fermion mass hierarchies of the minimal supersymmetric standard model. Imposing the consistency conditions for the absence of gauge anomalies yields the following results: (i) unification of leptons and down-type quark Yukawa couplings is allowed at most for two generations; (ii) the μ term is necessarily somewhat below the supersymmetry breaking scale; (iii) the determinant of the quark mass matrix vanishes, and there is no strong CP problem; (iv) the superpotential has accidental B and L symmetries. The prediction $m_{up}=0$ allows for an unambiguous test of the model at low energy.

One of the most successful ideas in modern particle physics is that of local gauge symmetries. A huge amount of data is explained in terms of the standard model (SM) gauge group $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$. Identifying this symmetry required a lot of experimental and theoretical effort, since $SU(2)_L \times U(1)_Y$ is hidden and color is confined. Today we understand particle interactions but we do not have any deep understanding of other elementary particle properties, such as fermion masses and mixing angles. The SM can only accommodate but not explain these data. Another puzzle is why CP is preserved by strong interactions to an accuracy $< 10^{-9}$. One solution is to postulate that one quark is massless, but within the SM there are no good justifications for this. Adding supersymmetry does not provide us with any better understanding of these issues. In contrast, it adds new problems. A bilinear coupling for the down-type and up-type Higgs superfields $\mu \phi_d \phi_u$ is allowed both by supersymmetry and by the gauge symmetry. However, phenomenology requires that μ should be close to the scale where these symmetries are broken. With supersymmetry, several operators that violate baryon (B) and lepton (L) numbers can appear. However, none of the effects expected from these operators has ever been observed. Since a few of them can induce fast proton decay, they must be very suppressed or absent.

Relying on the gauge principle, in this paper we attempt to gain insight into these problems. We extend minimally G_{SM} with a nonanomalous horizontal Abelian $U(1)_H$ factor. An unambiguous prediction of the nonanomalous $U(1)_H$ is a massless up quark. This represents the crucial low energy test of our framework. Shall future lattice computations rule out $m_{up}=0$ [1], the whole idea will have to be abandoned.

To explain the fermion mass pattern we follow the approach originally suggested by Froggatt and Nielsen (FN) [2]. $U(1)_H$ forbids most of the fermion Yukawa couplings. The symmetry is spontaneously broken by the vacuum expectation value (VEV) of a SM singlet field S, giving rise to a set of effective operators that couple the SM fermions to the electroweak Higgs field. The hierarchy of fermion masses results from the dimensional hierarchy among the various higher order operators. This idea was recently recon-

sidered by several groups, both in the context of supersymmetry [3] and with a gauged $U(1)_H$ [4–7]. It was argued that consistency with phenomenology implies that $U(1)_H$ must be anomalous, and thus only the anomalous case was studied in detail.

Our theoretical framework is defined by the following assumptions: (1) Supersymmetry and the gauge group $G_{SM} \times U(1)_H$. (2) $U(1)_H$ is broken only by the VEV of a field S with horizontal charge -1.[1] S is a SM singlet and is chiral under $U(1)_H$. (3) The ratio between the VEV $\langle S \rangle$ and the mass scale M of the FN fields is of the order of the Cabibbo angle $\lambda \simeq \langle S \rangle / M \sim 0.2$. (4) The only fields chiral under $U(1)_H$ and charged under G_{SM} are the minimal supersymmetric SM supermultiplets. (5) The lepton and down-type quark mass matrices M^l and M^d satisfy $\det M^l \lesssim \det M^d$ (of course this last assumption is an experimental fact).

In the following we will use the same symbol to denote a field and its horizontal charge. Upon $U(1)_H$ breaking, the Yukawa couplings Y^u, Y^d, and Y^l of the up-type and down-type quarks and of the leptons are generated. They satisfy the following relations:

$$Y^u_{ij} = \begin{cases} A^u_{ij} \lambda^{Q_i + u_j + \phi_u} & \text{if } Q_i + u_j + \phi_u \geq 0, \\ 0 & \text{if } Q_i + u_j + \phi_u < 0, \end{cases} \quad (1)$$

and similar ones for Y^d and Y^l. The zero entries arise from holomorphy, while A^u_{ij} are numerical coefficients of order λ^0 that we will often leave understood. Let us introduce the following combinations of charges:

$$n_u = \sum_i (Q_i + u_i), \quad n_d = \sum_i (Q_i + d_i), \quad n_Q = \sum_i Q_i,$$

$$n_l = \sum_i (L_i + l_i), \quad n_\phi = \phi_u + \phi_d, \quad n_L = \sum_i L_i. \quad (2)$$

After electroweak symmetry breaking, the Yukawa couplings (1) give rise to the fermion mass matrices M^u, M^d, and M^l. In the absence of vanishing eigenvalues their determinants read

$$\det M^u = \langle \phi_u \rangle^3 \lambda^{n_u + 3\phi_u} \det A^u, \tag{3}$$

$$\det M^d = \langle \phi_d \rangle^3 \lambda^{n_d + 3\phi_d} \det A^d, \tag{4}$$

$$\det M^l = \langle \phi_d \rangle^3 \lambda^{n_l + 3\phi_d} \det A^l. \tag{5}$$

Since all the entries in $A^{u,d,l}$ are of order λ^0, $\det A^{u,d,l}$ is of order 1. Then the size of the determinants (3)–(5) is fixed by the horizontal charges and by the ratio of the Higgs doublets VEVs $\tan \beta = \langle \phi_u \rangle / \langle \phi_d \rangle$.

The SM Yukawa operators are invariant under a set of global $U(1)$ symmetries: B, L, hypercharge (Y), and a symmetry X with charges $X(d) = X(l) = -X(\phi_d)$ and $X = 0$ for all the other fields. Therefore, shifts of the horizontal charges proportional to L, B, Y, and X do not affect the fermion mass matrices. In the following, we will denote as equivalent two sets of charges that can be transformed one into the other by means of shifts of this kind. Note that the superpotential term $\mu \phi_u \phi_d$ (the μ term) is not invariant under X, and hence it can be different for two equivalent sets. Experimental evidences for nonvanishing neutrino mixings [8] imply that shifts proportional to individual lepton flavor numbers L_a ($a = e, \mu, \tau$) transform between phenomenologically nonequivalent set of charges. In fact, while these shifts do not affect the charged lepton masses, they still produce different patterns of neutrino mixings. In our analysis we will work with the following linear combinations of generators: X, B, B-L, L_τ-L_μ, L_μ-L_e, and Y.

Since $G_{SM} \times U(1)_H$ is a local symmetry, it is mandatory to study the (field theory) consistency conditions for cancellation of the gauge anomalies. The mixed $SU(n)^2 \times U(1)_H$ anomalies, quadratic in $SU(n) = SU(3)_C$, $SU(2)_L$, $U(1)_Y$ and linear in the horizontal charges, can be expressed in terms of the coefficients

$$C_3 = n_u + n_d,$$

$$C_2 = n_\phi + (3n_Q + n_L), \tag{6}$$

$$C_1 = n_\phi + \tfrac{8}{3} n_u + \tfrac{2}{3} n_d + 2n_l - (3n_Q + n_L).$$

The coefficient of the mixed $U(1)_Y \times U(1)_H^2$ anomaly quadratic in the horizontal charges reads

$$C^{(2)} = \phi_u^2 - \phi_d^2 + \sum_i [Q_i^2 - 2u_i^2 + d_i^2 - L_i^2 + l_i^2]. \tag{7}$$

The pure $U(1)_H^3$ and the mixed gravitational anomalies can always be canceled by adding SM singlet fields with suitable charges, and we assume they vanish. If the C_n's in Eq. (6) do not vanish, the Green-Schwarz (GS) mechanism [9] can be invoked to remove the anomalies by means of a $U(1)_H$ gauge shift of an axion field $\eta(x) \to \eta(x) - \xi(x) \delta_{GS}$. The consistency conditions for this cancellation read [10]

$$C_1 / k_1 = C_2 = C_3 = \delta_{GS}, \tag{8}$$

where the Kac-Moody levels of the $SU(2)_L$ and $SU(3)_C$ gauge groups have been assumed to be unity and, since we are not postulating any grand unified theory (GUT) symmetry, the $U(1)_Y$ normalization factor k_1 is arbitrary. Then the weak mixing angle (at some large scale Λ) is given by $\tan^2 \theta_W = g'^2/g^2 = 1/k_1$. Using Eq. (6), conditions (8) translate into

$$2(n_\phi - n_d + n_l) = (k_1 - \tfrac{5}{3}) \delta_{GS}. \tag{9}$$

Now, one can assume that the gauge couplings unify for the canonical value $\tan^2 \theta_W = 3/5$ [7]. Then $n_\phi = n_d - n_l$ is obtained. Alternatively, one can assume that for some reasons the left-hand side in Eq. (9) vanishes, and thus predict canonical gauge couplings unification [5]. However, in the absence of a GUT symmetry the value $k_1 = 5/3$ is not compelling. Other values of k_1 can be in reasonable agreement with unification at scales $\Lambda \neq \Lambda_{GUT}$ [10], so that n_ϕ and $n_d - n_l$ are not necessarily related in any simple way. For a nonanomalous $U(1)_H$, Eqs. (8) and (9) still hold with $\delta_{GS} = 0$, so that the interplay with gauge couplings unification is lost. However, $n_\phi = n_d - n_l$ now follows as an unavoidable consistency condition, giving a first constraint on the permitted horizontal charges.

Let us now study the symmetry properties of the coefficients (6). Since for each $SU(2)_L$ multiplet $\text{Tr}[T_3 Y H] = YH \text{Tr}[T_3] = 0$, the mixed electromagnetic-$U(1)_H$ anomaly can be expressed in terms of C_1 and C_2 as $C_Q = \tfrac{1}{2}(C_1 + C_2)$. Being $SU(3)_C \times U(1)_Q$ vectorlike, it is free of B and L anomalies, and then C_3 and C_Q must be invariant under shifts of the horizontal charges proportional to B and L. Clearly, the same is not true for C_1 and C_2 separately. However, the SM is free of B-L anomalies, and thus C_1 and C_2 are invariant under the corresponding shift. Also L_τ-L_μ and L_μ-L_e have vanishing anomalies with G_{SM}, so they identify two more possible shifts that leave invariant the C_n's. In the following we state the consistency conditions for cancellation of the $G_{SM} \times U(1)_H$ gauge anomalies.

A set of horizontal charges $\{H\}$ is equivalent to a second set $\{H''\}$ for which the coefficients C_n'' of the mixed linear anomalies vanish, if and only if the mixed $U(1)_Q^2$-$U(1)_H$ and $SU(3)_C^2$-$U(1)_H$ anomaly coefficients are equal:

$$C_Q - C_3 = 0 \Leftrightarrow C_1'' = C_2'' = C_3'' = 0. \tag{10}$$

Moreover, if for $\{H''\}$ the charge of the μ term n_ϕ'' is different from zero, the coefficient of the quadratic anomaly $\tilde{C}^{(2)}$ can always be set to zero:

$$n_\phi'' \neq 0 \Rightarrow \tilde{C}^{(2)} = 0. \tag{11}$$

[As it stands, this condition is sufficient but not necessary. However, if all the neutrinos are mixed at a measurable level (11) turns out to be necessary [11]. In the following we take $n_\phi \neq 0$ in the strong sense.]

To prove this, let us assume that for the initial set $\{H\}$ $C_n \neq 0$. Then we start by shifting the charges proportionally to the X quantum numbers. $H \rightarrow H + (a/3)X$ yields

$$C_n \rightarrow C'_n = C_n + \alpha_n a, \tag{12}$$

with $\alpha_3 = 1$, $\alpha_2 = -1/3$, and $\alpha_1 = +7/3$. We fix $a = -C_3/\alpha_3$ so that $C'_3 = 0$. Note that the combination $(C_1 + C_2)/(\alpha_1 + \alpha_2) - C_3/\alpha_3 = C_Q - C_3$ besides being B and L invariant, is also X invariant by construction. Now a shift proportional to B can be used to set $C''_2 = 0$. Since C_3 is B invariant, $C''_3 = C'_3 = 0$. The sum $C'_1 + C'_2$ is also B invariant and thus $C''_1 = C'_1 + C'_2 = 2C'_Q$. However, by assumption $C'_Q = C'_3 (=0)$ and then the set $\{H''\}$ has vanishing mixed linear anomalies. Now, in order to cancel the quadratic anomaly while keeping vanishing C''_n, we can use any of the SM anomaly free symmetries B-L, L_τ-L_μ, L_μ-L_e [that in general will have nonvanishing mixed anomalies with U(1)$_H$]. Since L_τ-L_μ and L_μ-L_e transform between nonequivalent set of charges, we keep this freedom to account for two neutrino mixings (the third one results as a prediction) and we use B-L. Under the charge redefinition $H \rightarrow H + \beta(B-L)$

$$C^{(2)''} \rightarrow \tilde{C}^{(2)} = C^{(2)''} + \beta[\tfrac{4}{3}n''_u - \tfrac{2}{3}n''_d + 2n''_l] = C^{(2)''} - 2\beta n''_\phi, \tag{13}$$

where in the last step we have used the identity $\tfrac{4}{3}n_u - \tfrac{2}{3}n_d + 2n_l = C_1 + C_2 - \tfrac{4}{3}C_3 - 2n_\phi$ and the vanishing of the C''_n. If, as we have assumed, $n''_\phi \neq 0$, we can always set $\tilde{C}^{(2)} = 0$ by choosing $\beta = C^{(2)''}/(2n''_\phi)$. The constraint derived here is again stronger than in the anomalous case: $C^{(2)}$ cannot be canceled with the GS mechanism, and one has to redefine the charges so that it vanishes identically. Assuming $k_1 = \tfrac{5}{3}$, the GS consistency conditions (8) yield $C_1 + C_2 - \tfrac{4}{3}C_3 = \tfrac{4}{3}C_3 \neq 0$ and then $C^{(2)} = 0$ does not constrain the charges in any useful way.

A set of horizontal charges $\{H\}$ for which $C_n = C^{(2)} = 0$ identifies a one parameter family of anomaly free charges generated by shifts proportional to hypercharge: $H \rightarrow H + yY$. For the C_n's this is trivial due to the vanishing of the SM anomalies $\text{Tr}[\text{SU}(n)^2 Y] = 0$. For $C^{(2)}$ we have $\text{Tr}[YH^2] \rightarrow \text{Tr}[Y(H+Y)^2] = 2C_1 = 0$.

This property can be useful for model building: if the charges of the ith family satisfy $L_i - d_i = u_i - Q_i = Q_i - l_i$, then it is possible to arrange the corresponding fermions into $\bar{\mathbf{5}} + \mathbf{10}$ representations of SU(5). Alternatively we can choose to fix, e.g., $\phi_d = \phi_u = n_\phi/2$.

In summary, imposing cancellation of the $G_{SM} \times U(1)_H$ gauge anomalies results in the following constraints on the fermions charges

$$n_\phi \neq 0, \quad n_\phi = n_d - n_l \simeq \ln_\lambda \frac{\det M^d}{\det M^l}, \tag{14}$$

where the last relation follows from Eqs. (4) and (5). Since $n_d \neq n_l$ we conclude that (i) Yukawa coupling unification is permitted at most for two families. Together with assumption 5, we also obtain $n_\phi < 0$ so that (ii) the superpotential μ term

is forbidden by holomorphy and vanishes in the supersymmetric limit. Let us confront these results with phenomenology. To a good approximation the mass ratios $m_e/m_\mu \sim \lambda^3$ or λ^4, $m_\mu/m_\tau \sim \lambda^2$, $m_d/m_s \sim \lambda^2$ and $m_s/m_b \sim \lambda^2$ are renormalization group invariant. Then, since Yukawa coupling unification works remarkably well for the third family, $\det M^l/\det M^d \sim \lambda$ or λ^2, and the allowed values of n_ϕ are -1 or -2. Then a μ term arising from the (nonholomorphic) Kähler potential [12] will have a value somewhat below the supersymmetry breaking scale $m_{3/2}$:

$$\mu \sim \lambda^{|n_\phi|} m_{3/2} \quad \text{with } n_\phi = -1 \quad \text{or } -2. \tag{15}$$

As we have explicitly shown, the anomaly cancellation condition $C_Q - C_3 = 0$ (10) is Y, B, L, and X invariant (as it should be), and hence it shares the same invariance of the Yukawa couplings. Therefore, any product of the determinants (3)–(5) for which the overall horizontal charge can be recast just in terms of the C_n's must depend precisely on this combination. Such a relation was first found in [7]. Given that $C_Q - C_3 = n_l - \tfrac{2}{3}n_d + \tfrac{1}{3}n_u + n_\phi$ we can write it down at once:

$$\left(\frac{\det M^l}{\langle \phi_d \rangle^3} \right) \left(\frac{\det M^d}{\langle \phi_d \rangle^3} \right)^{-2/3} \left(\frac{\det M^u}{\langle \phi_u \rangle^3} \right)^{1/3} \simeq \lambda^{C_Q - C_3}. \tag{16}$$

Let us confront this relation with phenomenology. Anomaly cancellation implies that the right-hand side is unity, while the left-hand side is bounded by an upper limit of order $[(\det M^d/\langle \phi_d \rangle^3)(\det M^u/\langle \phi_u \rangle^3)]^{1/3} \ll 1$ [7]. This inconsistency (or similar ones) led several authors to conclude that U(1)$_H$ must be anomalous [4–7]. However, Eq. (16) is meaningful only under the assumption that none of the determinants vanishes, and since low energy phenomenology is still compatible with a massless up quark [1,13] (see, however, [14]), this might not be the case. In the following we prove that insisting on the vanishing of the gauge anomalies yields $m_{\text{up}} = 0$ as a prediction.

We start by noticing that if the determinant of the matrix $U_{ij} \sim \lambda^{Q_i + u_j + \phi_u}$ has an overall negative charge $\eta^U \equiv n_u + 3\phi_u \sim \log_\lambda \det U < 0$, then M^u has vanishing eigenvalues. This is because $\det U$ consists of the sum of six terms of the form $\lambda^{n_1} \cdot \lambda^{n_2} \cdot \lambda^{n_3}$ where $n_1 + n_2 + n_3 = \eta^U < 0$. Then at least one of the n_i must be negative, corresponding to a holomorphic zero in the mass matrix. Hence each one of the six terms vanishes.

Now, if U(1)$_H$ is anomaly free and assumption 5 holds, it is easy to see that (iii) the determinant of the six quark mass matrix $\mathcal{M}_q$ vanishes:

$$\left. \begin{array}{r} n_l \geqslant n_d \\ C_n = C^{(2)} = 0 \end{array} \right\} \Rightarrow \det \mathcal{M}_q = 0. \tag{17}$$

In fact adding and subtracting $3n_\phi$ to $C_3 = 0$ yields

$$\eta^U + \eta^D = 3n_\phi < 0. \tag{18}$$

Then at least one of the two η must be negative, and the corresponding determinant vanishes. Of course, on phenom-

enological grounds, a massless up quark is the only viable possibility [1,13]. Using the d-quark mass ratios given above, assuming $m_b/m_t \sim \lambda^3$ (as is preferred at large scales), and choosing $n_\phi = -1$, we obtain

$$\eta^U \simeq -9 - 3\log_\lambda\left(\frac{m_b}{m_t}\tan\beta\right), \qquad (19)$$

that ranges between -9 and -18 for $\tan\beta$ between m_t/m_b and 1. Because of the constraints from holomorphy, $\eta^U < 0$ results in an accidental U(1)$_u$ symmetry acting on the SU(2)$_L$ singlet up quark: $u_1 \to e^{i\alpha}u_1$. Then the QCD CP violating parameter $\bar\theta \equiv \theta + \arg\det \mathcal{M}_q$ is no more physical, and can be rotated away by means of a chiral transformation of the massless quark field. However, holomorphy is a crucial ingredient to achieve this result, and one has to check that after supersymmetry is broken this result is not badly spoiled. Supergravity effects induce mixings in the kinetic terms. Canonical form is recovered by means of the field redefinitions $Q = V^Q Q'$ and $u = V^u u'$. Then the matrix of the Yukawa couplings Y^u transforms into $Y^{u\prime} = V^{QT}Y^u V^u$. Since $\det Y^u = 0$, $\det Y^{u\prime} = 0$ follows, so that kinetic terms mixing, while it can lift mass matrix holomorphic zeroes, it does not lift vanishing eigenvalues. In general, soft supersymmetry breaking terms will not respect the U(1)$_u$ symmetry, so that a mass for the up quark can be induced radiatively. A conservative estimate of these effect gives $m_{\rm up} \lesssim (\alpha_s/\pi)\lambda^{|\eta^U - 4|}\langle\phi_u\rangle \lesssim 10^{-6}$ (10) eV [for $\tan\beta \sim 1, (m_t/m_b)$] where $\eta^U - 4$ is the charge of the up-quark mass operator when $m_c/m_t \sim \lambda^4$ is used. Following [15] we have estimated the possible contribution to the neutron electric dipole moment: $d_n/e \lesssim 10^{-28}\bar\theta$, $(10^{-22}\bar\theta)$ cm. Therefore, for moderate values of $\tan\beta$, the neutron dipole moment remains safely below the experimental limit $d_n/e < 6.3 \times 10^{-26}$ cm [16] even for $\bar\theta \sim 1$.

Gauge symmetry and supersymmetry, together with constraints from fermion charges relations, imply that (iv) the superpotential has accidental B and L symmetries. This result is deeply related to the solutions of the μ and strong CP problems ($n_\phi < 0$, $\eta^U < 0$). The proof of (iv) requires phenomenological inputs, like fermion mass ratios and Cabibbo-Kobayashi-Maskawa (CKM) mixings plus the assumption that neutrinos mixings are sizeable. Since it is somewhat lengthy, we will present it elsewhere [11]. An intuitive (but not rigorous) argument goes as follows: given a set of minimal charges that fit well the fermion masses and mixings, the (shifts invariant) value of η^U (19) implies that $C^{(2)}$ in Eq. (7) is negative. To cancel $C^{(2)}$ the shift $H \to H + \beta \cdot (B-L)$ is required, where $\beta = C^{(2)}/2n_\phi$ is positive. All the R-parity violating operators $\mu_L L\phi_u$, λLLl, $\lambda' LQd$, and $\lambda'' udd$ have B-$L = -1$, so that under this shift their charges are driven to negative values implying that they are not allowed in the superpotential. Of course, dimension five seesaw operators for neutrino masses are also forbidden. However, the same mechanism that generates μ will generate (with larger suppressions) also $\mu_L L\phi_u$ terms, which induce s-neutrinos VEVs. Canonical diagonalization of L-ϕ_d mixed kinetic terms will produce tiny λ and λ' from the Yukawa couplings. Both these effects can result in small neutrino masses [11]. However, since none of the λ'' can be generated in this way, proton stability is not in jeopardy.

Finally, let us stress that except for $\eta^U < 0$ the condition $C_Q - C_3 = 0$ does not imply other serious constraints on charge assignments, so that a suitable choice of horizontal charges can account for the observed pattern of fermion masses and mixings. The mass matrices of popular models [3,6] can be easily reproduced and, apart from $m_{\rm up} = 0$, also the same phenomenology [11].

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