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Bilinear R-parity violation and small neutrino masses: a self-consistent framework

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Abstract

We study extensions of supersymmetric models without R-parity which include an anomalous $U(1)_H$ horizontal symmetry. Bilinear R-parity violating terms induce a neutrino mass at tree level $m_\nu^{\rm tree} \approx (\theta^2)^\delta$ eV where $\theta \simeq 0.22$ is the $U(1)_H$ breaking parameter and δ is an integer number that depends on the horizontal charges of the leptons. For $\delta = 1$ a unique self-consistent model arises in which (i) all the superpotential trilinear R-parity violating couplings are forbidden by holomorphy; (ii) $m_\nu^{\rm tree}$ falls in the range suggested by the atmospheric neutrino problem; (iii) radiative contributions to neutrino masses are strongly suppressed resulting in $\Delta m^2_{\rm solar} \approx$ few 10^{-8} eV^2 which only allows for the LOW (or quasi-vacuum) solution to the solar neutrino problem; (iv) the neutrino mixing angles are not suppressed by powers of θ and can naturally be large.

1. Introduction

The field content of the standard model (SM) together with the requirement of $G_{\rm SM} = SU(2)_L \times U(1)_Y$ gauge invariance implies that the most general Lagrangian is characterized by additional accidental $U(1)$ symmetries implying baryon (B) and lepton flavor number (L_i, $i = e, \mu, \tau$) conservation at the renormalizable level. When the SM is supersymmetrized, this nice feature is lost. The introduction of the superpartners allows for several new Lorentz invariant couplings. The most general renormalizable superpotential respecting the gauge symmetries reads

$$W = \mu_\alpha H_\alpha \phi_u + \lambda_{\alpha\beta k} H_\alpha H_\beta l_k + \lambda'_{\alpha jk} H_\alpha Q_j d_k + \lambda''_{ijk} u_i d_j d_k + h^u_{jk} \phi_u Q_j u_k, \quad (1)$$

where $i, j, k = 1, 2, 3$ and $\alpha, \beta = 0, 1, 2, 3$, and all the fields appearing in (1) are superfields. In the following we will denote with the same symbol the minimal supersymmetric SM (MSSM) superfields and their SM fermion and scalar components. Since we will soon extend the model to include a horizontal $U(1)_H$ symmetry, we take the fields in (1) in the basis where the horizontal charges are well defined. We have denoted by H_α a vector containing the four hypercharge $Y = -1/2$ $SU(2)_L$ doublets of the MSSM and, without loss of generality, H_0 is the field whose main component is the down-type Higgs field: $H_0 \simeq \phi_d$ (ϕ_d is defined as the direction in H_α field space that acquires a vacuum expectation). It follows

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that H_1, H_2 and H_3 have as main components the lepton doublets L_e, L_μ and L_τ, with $\langle L_i \rangle = 0$ by definition. ϕ_u denotes the $Y = +1/2$ Higgs doublet, u_i, d_j and l_k ($i, j, k = 1, 2, 3$) are the $SU(2)_L$ singlets up-type quarks, down-type quarks and leptons of the three generations, and Q_j denotes the $SU(2)_L$ quark doublet. The Yukawa couplings responsible of the up-type quark masses are denoted by h^u_{jk} and, given our definition of the down-type Higgs field, in first approximation the leptons and down-type quarks Yukawa couplings are given by $h^l_{jk} \simeq \lambda_{0jk}$ and $h^d_{jk} \simeq \lambda'_{0jk}$. As it stands, (1) has potentially dangerous phenomenological consequences:

(i) The dimensionful parameters μ_α are gauge and supersymmetric invariant, and thus their natural value is expected to be much larger than the electroweak and supersymmetry breaking scales. A large value of μ_0 would result in too large Higgsino mixing term (this is the supersymmetric μ problem) while $\mu_i \sim \mu_0$ would give a large mass to one neutrino [1–3].

(ii) The dimensionless Yukawa couplings h^l_{jk} ($\simeq \lambda_{0jk}$), $h^d_{jk} (\simeq \lambda'_{0jk})$ and h^u_{jk} are expected to be of order unity, suggesting that all the fermion masses should be close to the electroweak breaking scale.

(iii) The trilinear couplings $\lambda_{ijk}, \lambda'_{ijk}, \lambda''_{ijk}$ are also expected to be of order unity, implying unsuppressed B and L violating processes.

The approach originally suggested by Froggatt and Nielsen (FN) [4] to solve (ii) and account for the fermion mass hierarchy turns out to be quite powerful in the context of the MSSM to solve also the μ problem. FN postulated an horizontal $U(1)_H$ symmetry that forbids most of the fermion Yukawa couplings. The symmetry is spontaneously broken by the vacuum expectation value (vev) of a SM singlet field χ and a small parameter of the order of the Cabibbo angle $\theta = \langle \chi \rangle / M \simeq 0.22$ (where M is some large mass scale) is introduced. The breaking of the symmetry induces a set of effective operators coupling the SM fermions to the electroweak Higgs fields, which involve enough powers of θ to ensure an overall vanishing horizontal charge. Then the observed hierarchy of fermion masses results from the dimensional hierarchy among the various higher order operators. When the FN idea is implemented within the MSSM, it is often assumed that the breaking of the horizontal symmetry is triggered by a single vev, for example the vev of the scalar component of a chiral supermultiplet χ with horizontal charge $H(\chi) = -1$. Then, because the superpotential is holomorphic all the operators carrying a negative charge are forbidden in the supersymmetric limit. If under $U(1)_H$ the bilinear term $H_0\phi_u$ has a charge $n_0 < 0$, a μ_0 term can only arise from the (non-holomorphic) Kähler potential, suppressed with respect the supersymmetry breaking scale $m_{3/2}$ as [5]

$$\mu_0 \simeq m_{3/2}\,\theta^{|n_0|}. \tag{2}$$

A too large suppression ($|n_0| > 1$) would result in unacceptably light Higgsinos, so that in practice on phenomenological grounds $n_0 = -1$ is by far the preferred value.

More recently it has been realized that the FN mechanism can play a crucial role also in keeping under control the trilinear B and L violating terms in (1) without the need of introducing an ad hoc R-parity quantum number [6–11]. For example in [6] it was argued that under a set of mild phenomenological assumptions about the size of neutrino mixings a non-anomalous $U(1)_H$ symmetry together with the holomorphy conditions implies the vanishing of all the superpotential B and L violating couplings. A systematic analysis on the restrictions on trilinear R-parity violating couplings in the framework of $U(1)_H$ horizontal symmetries was also recently presented in [11].

In this paper we argue that if the μ_0 problem is solved by the horizontal symmetry in the way outlined above, and if the additional bilinear terms μ_i are also generated from the Kähler potential and satisfy the requirement of inducing a neutrino mass below the eV scale, as indicated by data on atmospheric neutrinos [12,13], then in the basis where the horizontal charges are well defined, all the trilinear R-parity violating couplings are automatically absent. This hints at a self-consistent theoretical framework in which R-parity is violated only by bilinear terms that induce a tree level neutrino mass in the range suggested by the atmospheric neutrino anomaly, L and B violating processes are strongly suppressed, and the radiative contributions to neutrino masses are safely small so that $m_\nu^{\text{loop}} \approx 10^{-4}$ eV, which barely allows for the LOW or quasi-vacuum solutions to the solar neutrino problem [14,15].

2. Tree level neutrino mass

Our theoretical framework is defined by the following assumptions: (i) Supersymmetry and the gauge group $G_{\rm SM} \times U(1)_H$. (ii) $U(1)_H$ is broken only by the vev of a field χ with horizontal charge -1; the field χ is a SM singlet, chiral under $U(1)_H$. (iii) The ratio between the vev $\langle\chi\rangle$ and the mass scale M of the FN fields is of the order of the Cabibbo angle $\theta \simeq \langle\chi\rangle/M \simeq 0.22$. In the following we will denote a field and its horizontal charge with the same symbol, e.g., $H(l_i) = l_i$ for the lepton singlets, $H(Q_i) = Q_i$ for the quark doublets, etc. It is also useful to introduce the notation $f_{ij} = f_i - f_j$ to denote the difference between the charges of two fields. For example H_{i0} denotes the difference between the charges of the $H_i \simeq L_i$ 'lepton doublet' and the $H_0 \simeq \phi_d$ 'Higgs field'. On phenomenological grounds we will assume that the charge of the μ_0 term is $n_0 = -1$ and we will also assume negative charges $n_i = H_i + \phi_u < n_0$ for the other three bilinear terms $H_i\phi_u$. It is worth stressing that the theoretical constraints from the cancellation of the mixed $G_{\rm SM} \times U(1)_H$ anomalies hint at the same value $n_0 = -1$ both in the anomalous [16] and in the non-anomalous [6] $U(1)_H$ models (see Section 6). With the previous assumptions the four components of the vector μ_α in (1) read

$$\mu_\alpha \simeq m_{3/2}\big(\theta^{|n_0|}, \theta^{|n_1|}, \theta^{|n_2|}, \theta^{|n_3|}\big), \tag{3}$$

where coefficients of order unity multiplying each entry have been left understood. It is well known that if μ_α and the vector of the hypercharge $Y = -1/2$ vevs $v_\alpha \equiv \langle H_\alpha\rangle$ are not aligned [1,3]:

$$\sin\xi \equiv \frac{\mu \wedge v}{\sqrt{v_\alpha v^\alpha\, \mu_\beta \mu^\beta}} \neq 0 \tag{4}$$

the neutrinos mix with the neutralinos [17], and one neutrino mass is induced at the tree level [3]:

$$m_\nu^{\rm tree} \simeq \frac{\mu \cos^2\beta}{\sin 2\beta \cos\xi - \frac{\mu M_1 M_2}{M_Z^2 M_\gamma}} \sin^2\xi, \tag{5}$$

where $M_\gamma = M_1 \cos^2\theta_W + M_2 \sin^2\theta_W$, M_1 and M_2 are the $U(1)_Y$ and $SU(2)_L$ gaugino masses, and $\tan\beta = \langle\phi_u\rangle/\langle\phi_d\rangle$. Since $m_b/\langle\phi_u\rangle \tan\beta \approx \theta^{2.7} \tan\beta$ (with $m_b(m_t) \sim 2.9$ GeV [18]) in the following we will use the parameterization $\tan\beta = \theta^{x-3}$ that ranges between 90 and 1 for x between 0 and 3. Keeping in mind that we are always neglecting coefficients of order unity, we can approximate $\cos^2\beta = (1 + \tan^2\beta)^{-1} \approx \theta^{2(3-x)}$. Taking also $M_1 \simeq M_\gamma$, $\mu M_2/M_Z^2 \gg \sin 2\beta \cos\xi$ and 100 GeV $\lesssim M_2 \lesssim$ 500 GeV we obtain from (5)

$$m_\nu^{\rm tree} \approx \big[\theta^{-(5+x)} \sin\xi\big]^2 \text{ eV}. \tag{6}$$

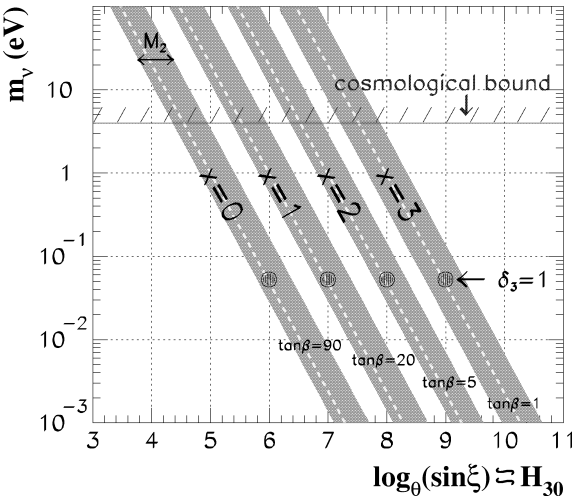

Fig. 1. Tree-level neutrino mass dependence on $\log_\theta \sin\xi \approx H_{30}$ for different assignments of the charge difference $H_{30} = H_3 - H_0$ and for different values of $\tan\beta$. Details in the text.

The magnitude of the tree-level neutrino mass as a function of $\log_\theta \sin\xi \approx H_{30}$ for different values of x (which in our notations parameterizes $\tan\beta$) is illustrated in Fig. 1. The grey bands correspond to Eq. (5) with M_2 ranging between 100 GeV and 500 GeV, while the dashed lines correspond to the approximate expression (6). In general, two conditions have to be satisfied to ensure exact μ_α–v_α alignment and $m_\nu^{\rm tree} = 0$ [3]: (1) $\mu_\alpha \propto B_\alpha$ and (2) $\tilde{m}^2_{\alpha\beta}\mu_\beta = \tilde{m}^2\mu_\alpha$, where B_α is the bilinear soft-breaking term coupling the H_α and ϕ_u scalar components, and $\tilde{m}^2_{\alpha\beta}$ is the matrix of the soft scalar masses for the H_α fields.

In our case the goodness of the alignment between μ_α and v_α is controlled by the horizontal symmetry, and in particular there is no need of assuming universality of the soft breaking terms to suppress $m_\nu^{\rm tree}$ to an acceptable level. This is because the previous two conditions are automatically satisfied in an approximate way up to corrections of the order $\theta^{|H_{i0}|}$, where

the minimum charge difference between H_0 and the H_i 'lepton' fields is responsible for the leading effects. Thus we can estimate

$$\sin\xi \approx \theta^{|H_{i0}|} = \theta^{|n_i - n_0|} \simeq \frac{\mu_i}{\mu_0}. \tag{7}$$

Confronting (7) with (6) it follows that in order to ensure that $m_\nu^{\rm tree}$ is *parametrically* suppressed below the eV scale we need

$$|n_i - n_0| > 5 + x \quad (i = 1, 2, 3). \tag{8}$$

3. Vanishing of the λ and λ' couplings

As we have shown in the previous section, requiring a sufficient suppression of tree level neutrino mass with respect to the Higgsino mass implies that the charges H_i should be much larger in absolute value than H_0. Then it follows that in the basis where the charges are well defined, the relations $H_0 \simeq \phi_d$ and $h_{ij}^{l(d)} \simeq \lambda_{0ij}^{(\prime)}$ are satisfied to a very good approximation. Let us introduce the parameterization

$$|n_i - n_0| - (5 + x) = \delta_i. \tag{9}$$

Without loss of generality, we can also assume $n_1 \leqslant n_2 \leqslant n_3$ which implies

$$m_\nu^{\rm tree} \approx \theta^{2\delta_3}\ \text{eV}. \tag{10}$$

It is worth stressing that the parameter that controls the scaling of $m_\nu^{\rm tree}$ with respect to changes in the values of the horizontal charges is $\theta^2 \simeq 0.05$, and thus neutrino masses are much more sensitive to the horizontal symmetry than the other fermion masses that scale with θ. For example $\delta_3 = -1$ yields $m_\nu^{\rm tree} \sim 20$ eV in conflict with cosmological structure formation [19]; $\delta_3 = 0$ yields $m_\nu^{\rm tree} \sim 1$ eV which implies a sizeable amount of hot dark matter; however, as we will see, it also allows for non-vanishing λ and λ'' couplings; for $\delta_3 = 1$ all the trilinear R-parity violating couplings are forbidden, and at the same time $m_\nu^{\rm tree} \sim 5 \times 10^{-2}$ eV (see Fig. 1) is in the correct range for a solution to the atmospheric neutrino problem [12,13]; finally, $\delta_3 = 2$ would suppress $m_\nu^{\rm tree}$ too much to allow for such a solution.

Let us now write the down-quarks and lepton Yukawa matrices as

$$\begin{aligned} h_{jk}^d &\simeq \theta^{H_0+Q_j+d_k} = \theta^{Q_{j3}+d_{k3}+x}, \\ h_{jk}^l &\simeq \theta^{H_0+H_j+l_k} = \theta^{H_{j3}+l_{k3}+x}, \end{aligned} \tag{11}$$

where $x = H_0 + Q_3 + d_3 = H_0 + H_3 + l_3$ consistently with our parameterization of $\tan\beta$ and with the approximate equality between the bottom and tau masses at sufficiently high energies (which in particular allows for b–τ Yukawa unification). The order of magnitude of the trilinear R-parity violating couplings is then:

$$\begin{aligned} \lambda'_{ijk} &\simeq \theta^{n_i - n_0}\, h_{jk}^d \simeq \theta^{Q_{j3}+d_{k3}-(5+\delta_i)}, \\ \lambda_{ijk} &\simeq \theta^{n_i - n_0}\, h_{jk}^l \simeq \theta^{H_{j3}+l_{k3}-(5+\delta_i)}. \end{aligned} \tag{12}$$

One can show that the phenomenological information on the charged fermion mass ratios and quark mixing angles

$$\begin{aligned} &m_u : m_c : m_t \simeq \theta^8 : \theta^4 : 1, \\ &m_d : m_s : m_b \simeq \theta^4 : \theta^2 : 1, \\ &m_e : m_\mu : m_\tau \simeq \theta^5 : \theta^2 : 1, \\ &V_{us} \simeq \theta, \quad V_{cb} \simeq \theta^2, \end{aligned} \tag{13}$$

which gives rise to eight conditions on the fermion charges [1] can be re-expressed in terms of the following sets of eight charge differences [7,10,20–22]

Model	Q_{13}	Q_{23}	d_{13}	d_{23}	u_{13}	u_{23}
MQ1 :	3	2	1	0	5	2
MQ2 :	−3	2	7	0	11	2
Model	$H_{13}+l_{13}$	$H_{23}+l_{23}$				
ML1 :	5	2				
ML2 :	9	−2				

(14)

We will not repeat here the phenomenological analysis leading to these sets of charge differences, since this has been extensively discussed in the literature [7, 10,20–22]; however, let us comment briefly on the different models listed in (14). The first set of charge differences labeled as MQ1 and ML1 corresponds the simplest solution where all the charges are fixed before supersymmetry breaking by the phenomenological

[1] Note that $V_{ub} \simeq V_{us}V_{cb} \simeq \theta^3$ is a prediction of the model (in agreement with the experimental measurements) and does not give additional constraints.

conditions listed in (13). Note however, that the charge differences in the second row labeled as MQ2 and ML2 are also compatible with (13). This is due to the fact that in MQ2 and ML2 some entries in the mass matrices have negative values of the charges, and initially correspond to holomorphic zeroes. After canonical diagonalization of the field kinetic terms these zeroes are lifted to non-vanishing values which are the correct ones to reproduce the same pattern (13) of mass ratios and quark mixing angles [7,10,22]. For example, with the restriction $x \neq 3$, the overall charge of the $(1,2)$ entry in the down quark mass matrix $Q_{13} + d_{23} + x = -3 + x$ is negative and implies $h^d_{12} = 0$. However, after Q_i and d_j field redefinition this entry is lifted to to $h^d_{12} \simeq \theta^{x+3}$ [7] which yields the correct value of the Cabibbo mixing angle $V_{us} \simeq \theta$. Similarly, with the restriction $x \neq 2, 3$ ML2 reproduces correctly the lepton mass ratios in (13).

Confronting now (12) with (14) we can conclude the following

- In MQ1, $\delta_i \geqslant 0$ is a sufficient condition to ensure that the overall charges of the λ' couplings are negative, implying that in the charge basis all these couplings are forbidden by holomorphy.
- In ML1, $\delta_i \geqslant 1$ is only a necessary condition to achieve $\lambda_{ijk} = 0$. Since in the leptonic sector the single values of the charge differences that control the mixing angles are not known, we need more assumptions to make a definite statement about these couplings. Let us note that the values $H_{12} = -1, -2$ are always excluded since they would result in incorrect values of the m_e/m_μ mass ratio, while $H_{12} = -3$ is allowed only for $x = 0$. Therefore in the leptonic sector the condition $n_i < 0$ forces the mixing between the first two generation neutrinos $V_{e\mu} \simeq \theta^{|H_{12}|}$ to be either very strongly suppressed ($\lesssim \theta^3$) or of order unity. The first case excludes the possibility of explaining the solar neutrino data through ν_e–ν_μ oscillations. The other possibility $H_{12} = 0$ corresponds to ν_e–ν_μ mixing not suppressed by powers of θ, and hence gives the possibility of implementing a large mixing angle solution for the solar neutrino problem. On the other hand, since a maximal ν_μ–ν_τ mixing is strongly supported by the atmospheric neutrino data, we will take $H_{23} = 0$ as a phenomenological assumption. Then from Eq. (12) it is easy to see that $H_{23} = H_{12} = 0$ is enough to guarantee the vanishing of all the λ_{ijk} couplings.
- In MQ2, $Q_{23} + d_{13} = 9$ so that to eliminate the λ' couplings we would need $\delta_i \geqslant 5$. This results in a very large suppression of the tree level neutrino mass $m_\nu^{\text{tree}} \lesssim 10^{-7}$ eV so that this case is not very interesting from the point of view of neutrino phenomenology. Insisting on $\delta_i = 1$ results in $\lambda'_{i21} \simeq \theta^3$ and $\lambda'_{i31} \simeq \theta$ while all the others λ' couplings vanish. Apparently, this is not in conflict with the existing experimental limits. However, after Q_i and d_i field redefinition a tiny coupling $\lambda'_{i12} \simeq \lambda'_{i31}\theta^{|Q_{13}|+|d_{12}|} \simeq \theta^{11}$ is generated. This is enough to conflict with the strong limit $\lambda'_{i21}\lambda'_{i12} \lesssim \theta^{15}$ from K–$\overline{K}$ mixing [23]. We conclude that in MQ2 either the neutrino masses are uninterestingly small, or the λ' conflicts with existing experimental limits. [2]
- In ML2, once we set $H_{23} = 0$ to allow for maximal ν_μ–ν_τ mixing, the lepton mass ratios (13) can be correctly reproduced only if $H_{12} \geqslant 4$, which would again exclude the possibility of explaining the solar neutrinos deficit through ν_e–ν_μ oscillations.

In conclusion, we have shown that in the framework of models of Abelian horizontal symmetries, the phenomenological information on the charged fermion mass ratios and quark mixing angles listed in (13) and re-expressed in terms of the eight horizontal charge differences in (14), when complemented with the requirement that m_ν^{tree} is adequately suppressed below the eV scale ($\delta_i \geqslant 1$) hints at one self-consistent model (MQ1 + ML1) where all the λ and λ' couplings vanish. It is interesting to note that $\delta_3 = 1$ which yields $m_\nu^{\text{tree}} \approx \theta^2$ eV in the correct range required by the atmospheric neutrino problem is also the minimum value that ensures $\lambda = 0$, $\lambda' = 0$ and, as we will see in the next section, $\lambda'' = 0$.

4. Vanishing of the λ'' couplings

Even if the trilinear lepton number violating couplings are absent in the basis where the horizontal charges are well defined, field rotation to the physical basis (ϕ_d, L_i) will still induce tiny $\delta\lambda$ and $\delta\lambda'$

[2] As we will see in the next section, MQ2 with $\delta_i = 1$ is also excluded by the requirement that the λ'' couplings vanish.

terms. In general the couplings induced in this way remain safely small to satisfy most of the experimental constraints, however some combination of the $\delta\lambda'$ with the B violating λ'' couplings can endanger proton stability. In this section we will show that the additional theoretical constraints from cancellation of the mixed $G_{\rm SM} \times U(1)_H$ anomalies, which are mandatory if $U(1)_H$ is a local symmetry, ensure that all the λ'' charges are negative and that the couplings are forbidden by holomorphy.[3] Since for the λ'' a change of basis or a field redefinition cannot lift any of the holomorphic zeroes, proton stability is not in jeopardy.

Let us introduce the notation $n_Q = \sum_i Q_i$ for the sum of the charges of the quark doublets and let us write the charge of a generic λ''_{ijk} coupling as

$$d_i + d_j + u_k = d_{i1} + d_{j2} + u_{k3} + (Q_1 + d_1 + H_0) + (Q_2 + d_2 + H_0) + \phi_u - n_Q - 2n_0, \tag{15}$$

where we have used $Q_3 + u_3 + \phi_u = 0$ as implied by $m_t \sim \langle\phi_u\rangle$. The consistency conditions for cancellation of the anomalies via the Green–Schwarz mechanism [24] imply that the coefficients of the mixed $SU(2)_L^2 \times U(1)_H$ and $SU(3)_C^2 \times U(1)_H$ anomalies $C_2 = \sum_\alpha H_\alpha + \phi_u + 3n_Q$ and $C_3 = \sum_i (2Q_i + d_i + u_i)$ must be equal [26]. This equality can be written as

$$\sum_{\alpha=0}^{3} n_\alpha + 3(n_Q - \phi_u) = 3(6 + x - n_0), \tag{16}$$

where for C_2 on the left-hand side of (16) we have used $\sum_\alpha H_\alpha = \sum_\alpha n_\alpha - 4\phi_u$, and the expression for C_3 on the right-hand side can be easily derived from the charge differences given in (14) and holds for both MQ1 and MQ2.

Inserting in (15) the value of $\phi_u - n_Q$ derived from the anomaly cancellation condition (16) and writing the explicit values of the m_d and m_s charges appearing inside the parentheses in (15) (respectively, $4 + x$ and $2 + x$) we obtain

$$\begin{aligned} d_i + d_j + u_k &= d_{i1} + d_{j2} + u_{k3} + (x - n_0) + \frac{1}{3}\sum_{\alpha=0}^{3} n_\alpha \\ &\leqslant d_{i1} + d_{j2} + u_{k3} - 5 - \frac{1}{3}, \end{aligned} \tag{17}$$

where in the last step we have used $n_0 = -1$ and $n_1 \leqslant n_2 \leqslant n_3 \leqslant -(6 + x)$ as suggested by the analysis in the previous sections. Now it is straightforward to verify that the charge differences in (14) imply $d_{i1} + d_{j2} \leqslant 0$ both in MQ1 and MQ2 (remember that $i \neq j$ because of the antisymmetry of the λ'') and $u_{k3} \leqslant 5$ (MQ1), $u_{k3} \leqslant 11$ (MQ2). The values that saturate these relations are the most conservative ones. Therefore in MQ1 $d_i + d_j + u_k < 0$ for all values of the indices and independently of $\tan\beta$, thus ensuring the vanishing of all the λ'' couplings, while in MQ2 some of the λ'' do not vanish.

5. One loop neutrino masses

It has long been realized that loop effects may lead to radiative neutrino masses [27]. In order to estimate the size of these effects in the present framework, first we need to evaluate the $\delta\lambda$ and $\delta\lambda'$ terms induced by the rotation from the basis (H_0, H_i) in which the charges are well defined to the basis (ϕ_d, L_i) in which the Yukawa couplings are well defined. Given that $H_0 \simeq \phi_d + \sum_i \theta^{|H_{i0}|} L_i$ we obtain

$$(\delta\lambda')_{ijk} \simeq \theta^{|H_{i0}|}\, h^d_{jk} \simeq \theta^{5+\delta_i+x}\, \theta^{\,Q_{j3}+d_{k3}+x}, \tag{18}$$

$$(\delta\lambda)_{ijk} \simeq \theta^{|H_{i0}|}\, h^l_{jk} \simeq \theta^{5+\delta_i+x}\, \theta^{\,H_{j3}+l_{k3}+x}. \tag{19}$$

Once non-vanishing λ and λ' couplings are generated, quark–squark and lepton–slepton loop diagrams will induce a mass for the two neutrinos that are massless at the tree level [28–31]. An approximate expression for the leading one-loop contributions to the neutrino mass matrix reads [32]

$$\left(m_\nu^{\rm loop}\right)_{ij} \simeq \frac{3(\delta\lambda')_{ikl}(\delta\lambda')_{jmn}}{8\pi^2}\,\frac{(m^d)_{kn}(\widetilde{M}^{d^2}_{LR})_{lm}}{\tilde{m}^2} + \frac{(\delta\lambda)_{ikl}(\delta\lambda)_{jmn}}{8\pi^2}\,\frac{(m^l)_{kn}(\widetilde{M}^{l^2}_{LR})_{lm}}{\tilde{m}^2}. \tag{20}$$

Here m^d (m^l) is the d-quark (lepton) mass matrix, $\widetilde{M}^{d(l)^2}_{LR}$ is the left–right sector in the mass-squared

[3] Here we assume that the $U(1)_H$ is anomalous, so that the anomaly cancellation is achieved via the Green–Schwarz mechanism [24]. This is the only possibility consistent with the implicit assumption $m_u \neq 0$ made in (13) [6]. A study of the non-anomalous case is presented in [25].

matrix for the $\tilde{d}$ ($\tilde{l}$) scalars, $\tilde{m}$ represents a slepton or squark mass, and the expression holds at leading order in $\widetilde{M}^2_{LR}/\tilde{m}^2$. As was discussed in [29] the largest loop contribution comes from quark–squark loops involving $(m^d)_{32} \sim (m^d)_{33} \sim m_b$ and $(\widetilde{M}^{d^2}_{LR})_{32} \sim (\widetilde{M}^{d^2}_{LR})_{33} \sim \tilde{m} m_b$, and gives a mass of the order

$$\begin{aligned}\left(m_\nu^{\text{loop}}\right)_{ij} &\approx \frac{3}{8\pi^2}\,\frac{m_b^2}{\tilde{m}}\,(\delta\lambda')_{i33}\,(\delta\lambda')_{j33}\\ &\approx \theta^{\delta_i+\delta_j+4x}\ \text{eV},\end{aligned} \tag{21}$$

where we have used $3/(8\pi^2)(m_b/\tilde{m})(m_b/1\ \text{eV}) \approx \theta^{-10}$ corresponding to $\tilde{m} \approx 100$ GeV. We see that for $\delta_2 = \delta_3 = 1$ (that allows for a ν_μ–ν_τ mixing angle without parametric suppression) we have two main possibilities: (i) $x = 0$ ($\tan\beta \sim m_t/m_b$) and $m_\nu^{\text{loop}} \approx m_\nu^{\text{tree}} \approx \theta^2 \sim$ few 10^{-2} eV. While this allows for a $m^2_{\nu_\tau} - m^2_{\nu_\mu}$ difference in the correct range for the atmospheric neutrino problem, ν_e–ν_μ oscillations do not solve the solar neutrino problem. Only for $\tilde{m} \gtrsim 1$ TeV we obtain enough suppression and $m_\nu^{\text{loop}} \sim$ few 10^{-3} eV can fall in the correct range for the large mixing angle solutions to the solar neutrino problem. Of course, $x = 0$ implies that the value of $\tan\beta$ is very large ($\gtrsim 60$) and therefore this case is phenomenologically disfavored [33,34]. (ii) $x = 1$ ($\tan\beta \approx 10$–40) yields $m_\nu^{\text{loop}} \approx \theta^6 \sim 10^{-4}$ eV which allows for the LOW or the quasi-vacuum solution to the solar neutrino problem. Finally $x = 2$ ($\tan\beta \sim 5$) would yield a too large suppression $m_\nu^{\text{loop}} \approx \theta^{10} \sim 10^{-7}$ eV to be interesting for the solar neutrinos.

In conclusion, our analysis results in the following set of fields charge differences and of $n_\alpha = H_\alpha + \phi_u$ charge sums:

Q_{13}	Q_{23}	d_{13}	d_{23}	u_{13}	u_{23}	H_{13}	H_{23}	l_{13}	l_{23}	n_i	n_0
3	2	1	0	5	2	0	0	5	2	−8	−1

(22)

where we have used the value $x = 1$ (corresponding to $\tan\beta \approx 10$–40) as suggested by the analysis of the loop effects. The corresponding structure of the charged fermion mass matrices is:

$$\frac{M^u}{\langle\phi_u\rangle} \simeq \begin{bmatrix}\theta^8 & \theta^5 & \theta^3\\ \theta^7 & \theta^4 & \theta^2\\ \theta^5 & \theta^2 & 1\end{bmatrix}, \quad \frac{M^d}{\langle\phi_d\rangle} \simeq \theta\begin{bmatrix}\theta^4 & \theta^3 & \theta^3\\ \theta^3 & \theta^2 & \theta^2\\ \theta & 1 & 1\end{bmatrix},$$

$$\frac{M^l}{\langle\phi_d\rangle} \simeq \theta\begin{bmatrix}\theta^5 & \theta^2 & 1\\ \theta^5 & \theta^2 & 1\\ \theta^5 & \theta^2 & 1\end{bmatrix}. \tag{23}$$

In the appendix we will derive the individual charges of an anomaly free model that reproduces these results.

6. Inputs versus predictions

Models based on a single $U(1)_H$ Abelian factor are completely specified in terms of the horizontal charges of the SM fields. There are five charges for each fermion family plus two charges for the Higgs doublets, for a total of 17 charges that a priori can be considered as free parameters (the charge of the $U(1)_H$ breaking parameter θ is just a normalization factor). The individual value of these charges is determined by a set of phenomenological and theoretical conditions. To some extent it is a matter of taste what is taken as an input condition, and what is derived as a model prediction. However it is important to understand to what extent the model has a predictive power, and to what extent it just has enough freedom to fit the experimental data. The purpose of this section is to clarify this issue.

The six mass ratios plus two CKM mixing angles listed in (13) provide the first eight constraints on the fermion charges. There are two additional constraints from the absolute values of the masses of the third generation fermions, corresponding to a top mass unsuppressed with respect to the electroweak scale and to the approximate equality between the b and τ masses at high energy

$$\begin{aligned} m_t \sim \langle\phi_u\rangle \quad &\Longrightarrow \quad Q_3 + u_3 + \phi_u = 0,\\ m_b \sim m_\tau \quad &\Longrightarrow \quad x \equiv Q_3 + d_3 + H_0\\ &\qquad\quad = H_3 + l_3 + H_0. \end{aligned} \tag{24}$$

In this paper we have also assumed that the supersymmetric μ problem is solved by the horizontal symmetry and we have taken the phenomenologically preferred value of the charge of the μ term

$$n_0 = H_0 + \phi_u = -1 \tag{25}$$

as an additional input. If we assume that $U(1)_H$ is a gauge symmetry, then additional constraints arise from the requirement of cancellation of the mixed $G_{\text{SM}} \times U(1)_H$ anomalies. The vanishing of the coefficient

of the $U(1)_Y \times U(1)_H^2$ anomaly quadratic in the horizontal charges

$$C^{(2)} = \phi_u^2 - \sum_\alpha H_\alpha^2 + \sum_i \left[Q_i^2 - 2u_i^2 + d_i^2 + \ell_i^2 \right] \tag{26}$$

gives a first condition. If, as we are assuming here, the non-vanishing mixed anomalies linear in the horizontal charges are canceled through the Green–Schwarz mechanism by a $U(1)_H$ gauge shift of an axion field $\eta(x) \to \eta(x) - \xi(x)\,\delta_{\rm GS}$ [24] the following consistency condition must be also satisfied [26]

$$C_3 = C_2 = \frac{C_1}{k_1} = \delta_{\rm GS}, \tag{27}$$

where $C_1 = \phi_u + \sum_\alpha H_\alpha + \frac{1}{3}\sum_i [Q_i + 8u_i + 2d_i - 3l_i]$ is the coefficient of the mixed $U(1)_Y^2 \times U(1)_H$ anomaly, and C_2 and C_3 have been defined before Eq. (16). While the first equality in (27) represents an additional constrain on the horizontal charges, the second condition depends on the hypercharge normalization factor k_1 that, since we are not postulating any GUT symmetry, must be considered as a new arbitrary parameter. When written explicitly in terms of horizontal charges, Eq. (27) yields the following interesting relation [6]:

$$n_0 + \eta_l - \eta_d = \left(k_1 - \frac{5}{3} \right) \delta_{\rm GS}/2, \tag{28}$$

where we have introduced the notation $\eta_d \equiv \sum_i (Q_i + d_i + H_0) \simeq \log_\theta(\det M^d / \langle \phi_d \rangle)$ and η_l is defined in a similar way. From the fermion mass ratios in (13) we obtain $\eta_l - \eta_d = 1$ that, together with the assumption (25) implies $k_1 = 5/3$. Therefore, while the second equality in (28) does not provide additional constraints on the horizontal charges, it predicts gauge coupling unification for the canonical value $\sin^2\theta_W = 3/8$. Of course, we could have equivalently taken the running of the gauge couplings in the MSSM as a good reason to assume canonical gauge couplings unification [16], then $n_0 = -1$ would have resulted as a theoretical prediction. In summary, the 17 horizontal charges are constrained by eleven phenomenological conditions (including $n_0 = -1$) and by two theoretical conditions from anomaly cancellation. This leaves us with four free parameters, and we can chose them to be the charges n_i $(i = 1, 2, 3)$ of the bilinear terms μ_i, and $x = Q_3 + d_3 + H_0$ that fixes the value of $\tan\beta$. The expressions of the horizontal charges for all the SM fields as a function of these four parameters is given in the appendix.

The main predictions of the model is the vanishing of all the trilinear R-parity violating couplings in the charge basis, as well as $x = 1$ which corresponds to $\tan\beta$ in the range $\approx$ 10–40. In what concerns the pattern of neutrino mixings, our model is most naturally realized with no parametric suppression of the mixing angles, in agreement with the solar and atmospheric neutrino observations, and in sharp contrast with the pattern of mixings in the quark sector. The exact values of the mixings depend on the unknown coefficients of order unity, which are not determined by the Abelian symmetry. Finally, the absence of parametric suppression also applies to the mixing angle which is restricted by reactor neutrino experiments [35], whose small value in the present framework can only arise from a conspiracy between the unknown coefficients of order unity.

7. Conclusions

We have studied extensions of supersymmetric models without R-parity which include an anomalous horizontal symmetry. We have assumed that all the bilinear superpotential terms coupling the up-type Higgs doublet with the four hypercharge $-1/2$ doublets carry negative horizontal charges, and hence are forbidden by holomorphy. We have constrained the value of these charges by several theoretical and phenomenological requirements, such as having an acceptable higgsino mass (μ problem) and neutrino masses suppressed below the electron-volt scale, as suggested by present neutrino data. We have found that under these conditions all the trilinear R-parity violating superpotential couplings vanish, yielding a consistent model which is defined by the charge differences in (22), where lepton number is mildly violated only by small bilinear terms. This implies that the magnitude of the tree level neutrino mass is automatically suppressed by the goodness of the alignment between μ_α and v_α implied by the horizontal symmetry. From this it follows that neutrino masses can be in the correct ranges suggested by the atmospheric neutrino problem and by the LOW and quasi-vacuum solutions to the solar neutrino problem. Moreover, having only bilinear R-parity violating terms as the origin of the neutrino masses im-

Table 1
The anomaly free set of charges of model MQ1 + ML1 for $x = 1$ and $n_1 = n_2 = n_3 = -8$

Q_1	Q_2	Q_3	u_1	u_2	u_3	d_1	d_2	d_3	H_1	H_2	H_3	l_1	l_2	l_3	ϕ_u	H_0
$\frac{161}{30}$	$\frac{131}{30}$	$\frac{71}{30}$	$\frac{103}{15}$	$\frac{58}{15}$	$\frac{28}{15}$	$-\frac{18}{5}$	$-\frac{23}{5}$	$-\frac{23}{5}$	$-\frac{113}{30}$	$-\frac{113}{30}$	$-\frac{113}{30}$	$\frac{98}{15}$	$\frac{53}{15}$	$\frac{23}{15}$	$-\frac{127}{30}$	$\frac{97}{30}$

plies that also the three neutrino mixing angles (assuming CP conservation in the leptonic sector) are determined as functions of the three bilinear $\not{R}_p$ terms, leading to a predictive scenario, independently of the particular structures of the charged lepton mass matrix. However, no precise theoretical information can be obtained about the neutrino mixing angles except for the fact that, unlike the quark mixings, there is no parametric suppression of their values and thus they can be naturally large.

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Appendix A

In this appendix we derive the general expressions for the individual field charges satisfying the set of 13 phenomenological and theoretical constraints corresponding to the six mass ratios for the quarks and the charged leptons plus the two quark mixing angles listed in (13); the two relations provided by the absolute value of the masses of the third generation fermions given in (24); one phenomenological assumption about the charge of the μ term (25); one theoretical constraint corresponding to the consistency conditions (27) for the coefficients of the mixed linear anomalies (the second constraint fixes $k_1 = 5/3$) and one additional constraint from the vanishing of the mixed anomaly quadratic in the horizontal charges (26). As discussed in Section 6, this leaves us with four free parameters that we choose to be n_i ($i = 1, 2, 3$) and x. We obtain

$$\begin{aligned} Q_3 = {} & \frac{1}{15(7+x)} \\ & \times \big[-180 - 45x - 3x^2 + Q_{13}(41+5x) \\ & - 7L_{23} + L_{23}^2 + n_1(2 + x + L_{23}) \\ & + n_2(9 + x - L_{23}) + n_3(9+x) \big], \end{aligned} \tag{A.1}$$

$$\begin{aligned} H_3 = {} & \frac{1}{15(7+x)} \\ & \times \big[20 + 50x + 6x^2 + 18Q_{13} - 21L_{23} + 3L_{23}^2 \\ & - n_1(29 + 2x - 3L_{23}) - n_2(8 + 2x + 3L_{23}) \\ & + n_3(97 + 13x) \big], \end{aligned} \tag{A.2}$$

where $L_{23} = H_{23} + l_{23}$ and Q_{13} parametrize the two different possibilities for the quark and lepton charge differences given in (14). In terms of Q_3 and H_3 and of our four free parameters we have

$$\begin{aligned} \phi_u &= n_3 - H_3, & u_3 &= -Q_3 - \phi_u, \\ H_0 &= -1 + \phi_u, & d_3 &= -Q_3 + x - H_0, \\ & & l_3 &= -H_3 + x - H_0 \end{aligned} \tag{A.3}$$

and from these all the other individual charges can be straightforwardly determined from the charge differences in Eq. (14). The solution for the charges in model MQ1 + ML1 for the preferred values $n_1 = n_2 = n_3 = -8$ and $x = 1$ is given in Table 1.

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