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Compendium of bag-model matrix elements of the weak nonleptonic Hamiltonian

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A comprehensive list of baryon and meson single-particle matrix elements of the nonleptonic weak Hamiltonian is presented.

Quantum-chromodynamics (QCD) radiative corrections have had considerable impact on the modern form of the nonleptonic weak Hamiltonian. The first publications on this subject appeared in 1974.[1] In the following six years, the work of Ref. 1 was extended to include both the effect of mass scales associated with heavy quarks[2] and also the occurrence of weak vertices at which color-gluon emission occurs. The latter effect can arise either from the existence of as-yet undetected right-handed currents,[3] or from purely left-handed currents, appearing in the dimension-5 operators as induced by QCD radiative corrections.[4]

As the subject of QCD radiative corrections progressed, a number of bag-model estimates of single-particle matrix elements associated with various weak Hamiltonians were performed. However, although a partial summary appears in Ref. 5, it remains true that the literature lacks a simple yet up-to-date compilation of these bag-model matrix elements.[6] This addendum serves to fill that gap. Such a collection can serve as a resource in the future as work on nonleptonic transitions continues. That this subject is still in a state of flux can be inferred from the recent computation of a new dimension-5 term in the weak Hamiltonian,[4] and the announcement that two-loop corrections to the QCD enhancement factors have been analyzed.[7]

Our procedure is as follows. We identify the operators which underlie $|\Delta S|=1$ transitions, define the notation employed in our analysis, present the analytical form of our matrix elements, and finally exhibit numerical values for a given set of input parameters.

I. OPERATORS

The $|\Delta S|=1$ nonleptonic weak Hamiltonian is given by

$$H_W=\frac{G_F\sin\theta_C\cos\theta_C}{2\sqrt{2}}\sum_{i=1}^{7}c_i(\mu)O_i(\mu)\,, \tag{1}$$

where μ indicates the energy scale at which the matrix element is to be evaluated. This affects both the coefficient functions c_i and the operators O_i. A reasonable energy scale can be taken as $\mu\simeq(2\to 3)R^{-1}$ where R is a length, such as the bag radius, characteristic of hadron structure. Thus we have (e.g., see Ref. 4)

$$\begin{aligned}
O_1&=H_B-H_A\,,\\
O_2&=H_A+H_B+2H_C+2H_D\,,\\
O_3&=H_A+H_B+2H_C-3H_D\,,\\
O_4&=H_A+H_B-H_C\,,\\
O_5&=\bar{d}\Gamma_L^{\mu}t^A s\bar{Q}\Gamma_{R\mu}t^A Q\,,\\
O_6&=\bar{d}\Gamma_L^{\mu}s\bar{Q}\Gamma_{R\mu}Q\,,\\
O_7&=m_s\bar{d}\lambda^A\sigma^{\mu\nu}(1-\gamma_5)sG^A_{\mu\nu}\\
&\quad+m_d\bar{d}\lambda^A\sigma^{\mu\nu}(1+\gamma_5)sG^A_{\mu\nu}\,,
\end{aligned} \tag{2a}$$

where

$$\begin{aligned}
H_A&=\bar{d}\Gamma_L^{\mu}u\bar{u}\Gamma_{L\mu}s\,,\\
H_B&=\bar{d}\Gamma_L^{\mu}s\bar{u}\Gamma_{L\mu}u\,,\\
H_C&=\bar{d}\Gamma_L^{\mu}s\bar{d}\Gamma_{L\mu}d\,,\\
H_D&=\bar{d}\Gamma_L^{\mu}s\bar{s}\Gamma_{L\mu}s\,.
\end{aligned} \tag{2b}$$

In Eqs. (2a) and (2b), we define $\Gamma_L^{\mu}=\gamma^{\mu}(1+\gamma_5)$, $\Gamma_R^{\mu}=\gamma^{\mu}(1-\gamma_5)$, the t^A are SU(3) matrices normalized as $\mathrm{Tr}(t_A{}^2)=16$, Q runs over quark flavors u, d, s, the quantities m_s, m_d represent current-quark masses, and $G^A_{\mu\nu}$ is the field tensor for color gluons. All color indices are suppressed in Eqs. (2a) and (2b).

In referring to matrix elements of the operator O_7, it is convenient to employ the decomposition $\sigma^{\mu\nu}G^A_{\mu\nu}=2\vec{\sigma}\cdot\vec{\mathrm{B}}^A-2i\vec{\alpha}\cdot\vec{\mathrm{E}}^A$. That is, we consider gluon magnetic and electric field effects separately.

II. NOTATION

We employ the following notation in writing the matrix elements of the O_i in analytical form. For a quark of mass m existing within a hadron bag of radius R in mode ω, we define $p=(\omega^2-m^2R^2)^{1/2}$. The spatial dependence of the upper and lower components of the bag wave functions is given in terms of spherical Bessel functions j_0, $\epsilon^{1/2}j_1$, respectively, where $\epsilon=(\omega-mR)/(\omega+mR)$. The bag normalization[8] for the positive-parity modes considered in this paper is $N=p^2|[2\omega(\omega-1)+mR]^{1/2}\sin p|^{-1}$. Throughout, unprimed and primed quantities are to be evaluated in terms of nonstrange- and strange-quark kinematics, respectively.

The bag matrix elements are proportional to wave-function overlap integrals. For operators O_i $(i=1,\ldots,6)$, the relevant integrals are

$$A=\frac{1}{4\pi}\int_0^1 u^2du[j_0^2(pu)-\epsilon j_1^2(pu)][j_0(pu)j_0(p'u)-(\epsilon\epsilon')^{1/2}j_1(pu)j_1(p'u)] \tag{3a}$$

and

$$B=\frac{1}{4\pi}\int_0^1 u^2du\, 2\sqrt{\epsilon}j_0(pu)j_1(pu)[\sqrt{\epsilon}j_0(p'u)j_1(pu)+\sqrt{\epsilon'}j_0(pu)j_1(p'u)]\,. \tag{3b}$$

The electric and magnetic matrix elements of the operator O_7 are proportional, respectively, to integrals I_E, I_M which are defined as

$$I_E=\frac{1}{4\pi}\int_0^1 du[\sqrt{\epsilon'}j_0(pu)j_1(p'u)+\sqrt{\epsilon}j_0(p'u)j_1(pu)]\left[\frac{1}{\bar{p}^2}\frac{2\bar{\omega}-mR}{\bar{\omega}+\bar{m}R}\left(u-\frac{\sin2\bar{p}u}{2\bar{p}}\right)-u^3\bar{\epsilon}^2[j_0^2(\bar{p}u)+j_1^2(\bar{p}u)]\right] \tag{4a}$$

and

$$I_M=\frac{1}{4\pi}\int_0^1 du\left\{-\tfrac{4}{3}(\epsilon\epsilon')^{1/2}j_1(pu)j_1(p'u)\bar{X}\right.$$
$$\left.+u^2[j_0(pu)j_0(p'u)+\tfrac{1}{3}(\epsilon\epsilon')^{1/2}j_1(pu)j_1(p'u)]\left[\bar{Y}+\frac{2}{3}\frac{\sqrt{\bar{\epsilon}}}{\bar{p}}[j_0^2(\bar{p}u)-j_0^2(\bar{p})]\right]\right\}, \tag{4b}$$

where

$$\bar{X}=\frac{\sqrt{\bar{\epsilon}}}{3\bar{p}^3}\left(1+\frac{1}{2}\cos2\bar{p}u-\frac{3}{4}\frac{\sin2\bar{p}u}{\bar{p}u}\right), \tag{5a}$$

$$\bar{Y}=\frac{\sqrt{\bar{\epsilon}}}{3\bar{p}^3}\left(1+\frac{1}{2}\cos2\bar{p}-\frac{3}{4}\frac{\sin2\bar{p}}{\bar{p}}\right). \tag{5b}$$

The barred quantities in Eqs. (4a)–(5b) take on strange- or nonstrange-quark kinematics, respectively, according to whether or not the integrals I_E, I_M appear with or without a prime in the following section.

III. THE MATRIX ELEMENTS

Meson and baryon matrix elements are addressed separately below. In all cases we consider only quark wave functions in the valence model, leaving out quark-sea contributions of the type studied in Ref. 9.

A. Mesons

The only meson-to-meson matrix element relevant to $|\Delta S|=1$ transitions is $\langle\pi^+|H_W|K^+\rangle$. Others such as $\langle\pi^0|H_W|K^0\rangle$ can be obtained from isospin relations. Thus we list

$$\begin{aligned}
&\langle\pi^+|O_1|K^+\rangle=-4N^3N'R^{-3}(2m_K^2)^{1/2}(A-B)\,,\\
&\langle\pi^+|O_i|K^+\rangle=8N^3N'R^{-3}(2m_K^2)^{1/2}(A-B),\quad i=2,3,4\,,\\
&\langle\pi^+|O_5|K^+\rangle=\tfrac{16}{3}\langle\pi^+|O_6|K^+\rangle\,,\\
&\langle\pi^+|O_6|K^+\rangle=-4N^3N'R^{-3}(2m_K^2)^{1/2}(A+B)\,,\\
&\langle\pi^+|O_7^{(E)}|K^+\rangle\\
&\qquad=-\tfrac{16}{3}g(m_s+m_d)NN'R^{-2}(2m_K^2)^{1/2}(N^2I_E-N'^2I_E')\,,\\
&\langle\pi^+|O_7^{(M)}|K^+\rangle\\
&\qquad=16g(m_s+m_d)NN'R^{-2}(2m_K^2)^{1/2}(3N^2I_M+N'^2I_M')\,,
\end{aligned} \tag{6}$$

where the quantity g appearing in the O_7 matrix elements is the quark-gluon coupling constant evaluated at the energy μ discussed earlier, and the factor $(2m_K^2)^{1/2}$ is our estimate for the meson normalization factor [which equals $(4E_\pi E_K)^{1/2}$ for plane-wave states] for bag states.[10]

In order to obtain the corresponding K^0-to-π^0 matrix elements, we observe that all the O_i are $\Delta I=\frac{1}{2}$ operators excepting O_4, which is pure $\Delta I=\frac{3}{2}$. From this, we infer

$$\langle \pi^0 | O_i | K^0 \rangle = -\frac{1}{\sqrt{2}} \langle \pi^+ | O_i | K^+ \rangle, \quad i \neq 4$$
$$= \sqrt{2} \langle \pi^+ | O_i | K^+ \rangle, \quad i = 4. \qquad (7)$$

B. Baryons

It is convenient to use the SU(3) d, f parametrization for the baryon matrix elements. With one exception, this is valid in our approach because the quark content of the wave functions and weak Hamiltonian respects the SU(3) algebra. Symmetry breaking associated with quark masses is taken into account in the overlap integrals. In our phase convention, the matrix elements for individual transitions are given by

$$\begin{aligned} &\Lambda \to n: \ d + 3f, \\ &\Xi^0 \to \Lambda: \ d - 3f, \\ &\Sigma^+ \to p: \ \sqrt{6}(f - d), \\ &\Sigma^0 \to n: \ \sqrt{3}(d - f), \\ &\Xi^0 \to \Sigma^0: \ \sqrt{3}(d + f), \\ &\Xi^- \to \Sigma^-: \ -\sqrt{6}(d + f) \end{aligned} \qquad (8)$$

and the f, d parameters are

$$\begin{aligned} &\langle O_1 \rangle: \ f = -d = -\sqrt{6} N^3 N' R^{-3}(A + B), \\ &\langle O_i \rangle: \ f = d = 0 \quad (i = 2, 3, 4), \\ &\langle O_5 \rangle: \ f = \tfrac{4}{27}\sqrt{6} N^3 N' R^{-3}(3A + 7B), \\ &\qquad d = \tfrac{4}{9}\sqrt{6} N^3 N' R^{-3}(3A - B), \\ &\langle O_6 \rangle: \ \langle O_6 \rangle = -\tfrac{3}{8}\langle O_5 \rangle, \\ &\langle O_7^{(E)} \rangle: \ f = \tfrac{1}{3} d \\ &\qquad = \tfrac{4}{9}\sqrt{6}\, g N N' R^{-2}(m_s + m_d)(N^2 I_E - N'^2 I'_E). \end{aligned} \qquad (9)$$

TABLE I. Numerical values of bag normalization factors and overlap integrals. The overlap integrals should each be multiplied by 10^{-3}.

	$R = 5.0$ GeV^{-1}	$R = 3.3$ GeV^{-1}
N	2.27	2.27
N'	2.94	2.74
A	3.38	3.43
B	4.22	4.69
I_E	1.51	1.69
I'_E	1.11	1.35
I_M	1.43	1.48
I'_M	0.670	0.868

The matrix element $\langle O_7^{(M)} \rangle$ turns out *not* to have an exact f, d parametrization for unequal quark masses. In this case, it is simplest to employ equal-mass kinematics and we find

$$\langle O_7^{(M)} \rangle: \ f = -\tfrac{7}{3} d = -\tfrac{56}{9}\sqrt{6}\, g N^4 R^{-2}(m_s + m_d) I_M.$$

In our numerical evaluation of the integral I_M, we take $m_s = m_u = 0$. The effect of unequal quark masses on I_M is roughly 20%. The reader interested in a more accurate prescription than that given above can consult Ref. 11.

Observe that we are able to express all the above baryon matrix elements in terms of $\Delta I = \frac{1}{2}$ *octet* parameters because matrix elements of O_4, the only $\Delta I = \frac{3}{2}$ 27-plet operator, vanish identically.[5]

IV. NUMERICS

Our choice of input parameters is $m_u = m_d = 0$, $m_s = 0.280$ GeV, $R = 5.0$ GeV^{-1} for the baryons, and $R = 3.3$ GeV^{-1} for the mesons. The corresponding bag normalization factors N, N' and overlap inte-

TABLE II. Selected matrix elements of $\Delta S = 1$ nonleptonic operators. Each entry gives the matrix element for operator $\bar{O}_i$ defined as $G_F \sin\theta_C \cos\theta_C O_i / 2\sqrt{2}$. Units are (a) 10^{-9} GeV for the baryon matrix elements and (b) 10^{-8} GeV^2 for the meson matrix elements. The quark-gluon coupling constant g is left unspecified.

		(a) Baryons			
Mode	$\bar{O}_1$	$\bar{O}_{2,3,4}$	$\bar{O}_5$	$\bar{O}_7^{(E)}$	$\bar{O}_7^{(M)}$
$\Lambda \to n$	−9.18	0.0	12.3	$-0.806g$	$14.9g$
$\Xi^0 \to \Lambda$	18.4	0.0	−9.11	0.0	$-19.9g$
$\Sigma^+ \to p$	−22.5	0.0	4.83	$0.658g$	$20.3g$
$\Xi^0 \to \Sigma^0$	0.0	0.0	8.94	$0.931g$	$5.75g$

			(b) Mesons			
Mode	$\bar{O}_1$	$\bar{O}_{2,3}$	$\bar{O}_4$	$\bar{O}_5$	$\bar{O}_7^{(E)}$	$\bar{O}_7^{(M)}$
$K^0 \to \pi^0$	−0.203	0.405	−0.811	6.92	$-0.0537g$	$-3.37g$

TABLE III. Comparison of experimental and theoretical parity-violating $|\Delta S|=1$ amplitudes. The hyperon and meson amplitudes are given in units of $G_F m_\pi^2$ and $10^{-7} m_{K^0}$, respectively. The quantities in parentheses are the "left-right factorization" amplitudes described in the text. All entries are to be multiplied by the appropriate coefficient c_i before comparison with experiment.

(a) Hyperons

Mode	Expt.	$\bar{O}_1$	$\bar{O}_{2,3,4}$	$\bar{O}_5$ ($\bar{O}_5^{(c)}$)	$\bar{O}_6$ ($\bar{O}_6^{(c)}$)	$\bar{O}_7^{(E)}$	$\bar{O}_7^{(M)}$
$\Lambda n\pi^0$	1.07	−0.222	0.0	0.295 (−0.710)	−0.111 (0.266)	−0.019g	0.3591
$\Lambda p\pi^-$	−1.47	0.313	0.0	−0.418 (1.00)	0.157 (−0.377)	0.027g	−0.508g
$\Xi^0\Lambda\pi^0$	−1.54	0.443	0.0	−0.219 (0.710)	0.082 (−0.266)	0.0	−0.479g
$\Xi^-\Lambda\pi^-$	2.04	−0.627	0.0	0.310 (−1.00)	−0.116 (0.377)	0.0	0.677g
$\Sigma^+p\pi^0$	1.48	−0.543	0.0	0.116 (−0.580)	−0.044 (0.217)	0.016g	0.489g
$\Sigma^-n\pi^-$	−1.93	0.768	0.0	−0.164 (0.820)	0.062 (−0.308)	−0.022g	−0.691g
$\Sigma^+n\pi^+$	0.06	0.0	0.0	0.0 (0.0)	0.0 (0.0)	0.0	0.0

(b) Mesons

Mode	Expt.	$\bar{O}_1$	$\bar{O}_{2,3}$	$\bar{O}_4$	$\bar{O}_5$ ($\bar{O}_5^{(c)}$)	$\bar{O}_6$ ($\bar{O}_6^{(c)}$)	$\bar{O}_7^{(E)}$	$O_7^{(M)}$
$K^0\pi^0\pi^0$	−5.28	−0.741	1.48	−2.97	25.3 (4.18)	4.75 (0.784)	−0.196g	−12.3g

grals of Eqs. (3a)–(5b) take on the numerical values given in Table I. Note that each entry for the overlap integrals in Table I is to be multiplied by 10^{-3}.

For the convenience of the reader, we summarize certain meson and baryon matrix elements of H_W in Table II. Incidentally, in view of the strong cancellation between integrals A, B in $\langle\pi|O_i|K\rangle$ ($i=1,2,3,4$), we caution the reader that this effect (a consequence of helicity suppression[5]) renders the exact magnitude and even sign of the particular combination $A-B$ uncertain. In Table III, we go beyond the bag model and display the result of using PCAC (partial conservation of axial-vector current) to connect the calculated matrix elements with experimental $K\to\pi\pi$ and S-wave hyperon-decay transitions. Although the latter interpolation is generally conceded to be relatively momentum independent, the former is not. We employ here the plausible but by no means unique form of Ref. 5; other approaches could conceivably lead to values differing by as much as a factor of 2. Note that each entry in Table III is to be multiplied by the corresponding coefficient c_i ($i=1,\ldots,7$) before comparing theory to experiment. Finally, we point out the existence of additional contributions, the so-called "factorization diagrams," to the physical decay amplitudes.[5] In the case of O_5, O_6 these factorization contributions are comparable in size to the "four-quark" integrals already computed and included in Table III under the column headings $\bar{O}_5^{(c)}$, $\bar{O}_6^{(c)}$.

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