$SU(3)_c \otimes SU(4)_L \otimes U(1)_X$ without exotic electric charges

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We present an extension of the standard model to the local gauge group $SU(3)_c \otimes SU(4)_L \otimes U(1)_X$ with a family nonuniversal treatment and anomalies canceled among the three families in a nontrivial fashion. The mass scales, the gauge boson masses, and the masses for the spin 1/2 particles in the model are analyzed. The neutral currents coupled to all neutral vector bosons in the model are studied, and particular values of the parameters are used in order to simplify the mixing between the three neutral currents present in the theory, mixing which is further constrained by experimental results from the CERN LEP, SLAC Linear Collider, and atomic parity violation.

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I. INTRODUCTION

In spite of the overwhelming phenomenological success of the standard model (SM) based on the local gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, with $SU(2)_L \otimes U(1)_Y$ hidden and $SU(3)_c$ confined [1], it fails to explain several issues such as hierarchical fermion masses and mixing angles, charge quantization, strong *CP* violation, replication of families and neutrino oscillations among others. For example, in the weak basis, before symmetry is broken, the three families in the SM are identical to each other; when symmetry breaking takes place, the fermions get masses according to their experimental values and the three families acquire a strong hierarchy. However in the SM there is no mechanism for explaining the origin of families or the fermion mass spectrum.

These drawbacks of the SM have led to a strong belief that the model is still incomplete and that it must be regarded as a low-energy effective field theory originating from a more fundamental one. That belief lies on strong conceptual indications for physics beyond the SM which have produced a variety of theoretically well motivated extensions of the model: left-right symmetry, grand unification, supersymmetry, superstring inspired extensions, etc. [2].

At present the only experimental fact that points toward a beyond the SM structure lies in the neutrino sector, and even there the results are not final yet. So a reasonable approach is to depart from the SM as little as possible, allowing some room for neutrino oscillations [3].

$SU(4)_L \otimes U(1)_X$ as a flavor group has been considered before in the literature [4,5], and, among its best features, provides an alternative to the problem of the number N_f of families, in the sense that anomaly cancellation is achieved when $N_f = N_c = 3$, N_c being the number of colors of $SU(3)_c$ (also known as QCD). In addition, this gauge structure has been used recently in order to implement the so-called little Higgs mechanism [5].

In this paper an analysis of the $SU(3)_c \otimes SU(4)_L \otimes U(1)_X$ local gauge theory (hereafter the 3-4-1 theory) shows that, by restricting the fermion field representations to particles without exotic electric charges and by paying due attention to anomaly cancellation, a few different models are obtained, while by relaxing the condition of the nonexistence of exotic electric charges, an infinite number of models can be generated.

This paper is organized as follows. In the next section we introduce the model based on the local gauge group $SU(3)_c \otimes SU(4)_L \otimes U(1)_X$ which we are going to study. In Sec. III we describe the scalar sector needed to break the symmetry and to produce masses to the fermion fields in the model. In Sec. IV we study the gauge boson sector paying special attention to the neutral currents present in the model and their mixing. In Sec. V we analyze the fermion mass spectrum. In Sec. VI we use experimental results in order to constrain the mixing angle between two of the neutral currents and the mass scale of the new neutral gauge bosons. In the last section we summarize the model and state our conclusions. At the end an Appendix is presented in which we make a systematic analysis of the 3-4-1 symmetry and obtain general conditions to have anomaly free models without exotic electric charges.

II. THE FERMION CONTENT OF THE MODEL

In what follows we assume that the electroweak gauge group is $SU(4)_L \otimes U(1)_X$ which contains $SU(2)_L \otimes U(1)_Y$ as a subgroup, with a nonuniversal hypercharge X in the quark sector, which in turn implies anomaly cancellation among the families in a nontrivial fashion. We also assume that the left-handed quarks (color triplets) and left-handed leptons (color singlets) transform either under the 4 or $\bar{4}$ fundamental representations of $SU(4)_L$, and that as in the SM, $SU(3)_c$ is vectorlike.

With the former assumptions we look for the simplest structure in such a way that, not only it does not contain fields with exotic electric charges, but also that charged exotic leptons are absent from the anomaly-free spectrum. According to the Appendix there is only one model (model A) satisfying all those constraints, for which the electric charge operator is given by $Q=T_{3L}+(1/\sqrt{3})T_{8L}+(1/\sqrt{6})T_{15L}+XI_4$, with the following fermion structure:

$$Q_{aL}=\begin{pmatrix} u_a \\ d_a \\ D_a \\ D'_a \end{pmatrix}_L \quad u^c_{aL} \quad d^c_{aL} \quad D^c_{aL} \quad D'^c_{aL}$$

$$[3,4,-\tfrac{1}{12}] \quad [3,1,-\tfrac{2}{3}] \quad [3,1,\tfrac{1}{3}] \quad [3,1,\tfrac{1}{3}] \quad [3,1,\tfrac{1}{3}]$$

$$Q_{1L}=\begin{pmatrix} d_1 \\ u_1 \\ U_1 \\ U'_1 \end{pmatrix}_L \quad d^c_{1L} \quad u^c_{1L} \quad U^c_{1L} \quad U'^c_{1L}$$

$$[3,\overline{4},\tfrac{5}{12}] \quad [3,1,\tfrac{1}{3}] \quad [3,1,-\tfrac{2}{3}] \quad [3,1,-\tfrac{2}{3}] \quad [3,1,-\tfrac{2}{3}]$$

$$L_{\alpha,L}=\begin{pmatrix} e^-_\alpha \\ \nu_{e\alpha} \\ N^0_\alpha \\ N'^0_\alpha \end{pmatrix}_L \quad e^+_\alpha$$

$$[1,\overline{4},-\tfrac{1}{4}] \quad [1,1,1]$$

where $a=2,3$ and $\alpha=1,2,3$ are two and three family indexes, respectively. The numbers in parentheses refer to the $[SU(3)_C,SU(4)_L,U(1)_X]$ quantum numbers, respectively. Notice that if needed, the lepton structure of the model can be augmented with an undetermined number of neutral Weyl singlet states $N^0_{L,b}\sim[1,1,0]$, $b=1,2,\ldots,$ without violating our assumptions, neither the anomaly constraint relations, because singlets with no X charges are as good as not being present as far as anomaly cancellation is concerned.

III. THE SCALAR SECTOR

Our aim is to break the symmetry following the pattern

$$\begin{aligned} SU(3)_c\otimes SU(4)_L\otimes U(1)_X &\rightarrow SU(3)_c\otimes SU(3)_L\otimes U(1)_X \\ &\rightarrow SU(3)_c\otimes SU(2)_L\otimes U(1)_Y \\ &\rightarrow SU(3)_c\otimes U(1)_Q, \end{aligned}$$

where $SU(3)_c\otimes SU(3)_L\otimes U(1)_X$ refers to the so-called 3-3-1 structure introduced in Ref. [6]. At the same time we want to give masses to the fermion fields in the model. With this in mind we introduce the following three Higgs scalars: $\phi_1[1,4,-3/4]$ with a vacuum expectation value (VEV) aligned in the direction $\langle\phi_1\rangle=(v,0,0,0)^T$; $\phi_2[1,\overline{4},-1/4]$ with a VEV aligned as $\langle\phi_2\rangle=(0,0,V,0)^T$ and $\phi_3[1,\overline{4},-1/4]$ with a VEV aligned as $\langle\phi_3\rangle=(0,0,0,V')^T$, with the hierarchy $V\sim V'\gg v\sim 174$ GeV (the electroweak breaking scale).

IV. THE GAUGE BOSON SECTOR

In the model there are a total of 24 gauge bosons: One gauge field B^μ associated with $U(1)_X$, the 8 gluon fields associated with $SU(3)_c$ which remain massless after breaking the symmetry, and another 15 gauge fields associated with $SU(4)_L$ which we may write as

$$\frac{1}{2}\lambda_\alpha A^\mu_\alpha=\frac{1}{\sqrt{2}}\begin{pmatrix} D^\mu_1 & W^{+\mu} & K^{+\mu} & X^{+\mu} \\ W^{-\mu} & D^\mu_2 & K^{0\mu} & X^{0\mu} \\ K^{-\mu} & \overline{K}^{0\mu} & D^\mu_3 & Y^{0\mu} \\ X^{-\mu} & \overline{X}^{0\mu} & \overline{Y}^{0\mu} & D^\mu_4 \end{pmatrix},$$

where $D^\mu_1=A^\mu_3/\sqrt{2}+A^\mu_8/\sqrt{6}+A^\mu_{15}/\sqrt{12}$, $D^\mu_2=-A^\mu_3/\sqrt{2}+A^\mu_8/\sqrt{6}+A^\mu_{15}/\sqrt{12}$; $D^\mu_3=-2A^\mu_8/\sqrt{6}+A^\mu_{15}/\sqrt{12}$, and $D^\mu_4=-3A^\mu_{15}/\sqrt{12}$.

After breaking the symmetry with $\langle\phi_1\rangle+\langle\phi_2\rangle+\langle\phi_3\rangle$ and using for the covariant derivative for 4-plets $iD^\mu=i\partial^\mu-(g/2)\lambda_\alpha A^\mu_\alpha-g'XB^\mu$, where g and g' are the $SU(4)_L$ and $U(1)_X$ gauge coupling constants, respectively, we get the following mass terms for the charged gauge bosons: $M^2_{W^\pm}=(g^2/2)v^2$ as in the SM, $M^2_{K^\pm}=(g^2/2)(v^2+V^2)$, $M^2_{X^\pm}=(g^2/2)(v^2+V'^2)$, $M^2_{K^0(\overline{K}^0)}=(g^2/2)V^2$, $M^2_{X^0(\overline{X}^0)}=(g^2/2)V'^2$ and $M^2_{Y^0(\overline{Y}^0)}=(g^2/2)(V^2+V'^2)$. Since $W^\pm$ does not mix with $K^\pm$ or with $X^\pm$ we have that $v\approx 174$ GeV as in the SM.

For the four neutral gauge bosons we get mass terms of the form

$$M=\frac{g^2}{2}\left[V^2\left(\frac{g'B^\mu}{2g}-\frac{2A^\mu_8}{\sqrt{3}}+\frac{A^\mu_{15}}{\sqrt{6}}\right)^2+V'^2\left(\frac{g'B^\mu}{2g}-\frac{3A^\mu_{15}}{\sqrt{6}}\right)^2 +v^2\left(A^\mu_3+\frac{A^\mu_8}{\sqrt{3}}+\frac{A^\mu_{15}}{\sqrt{6}}-\frac{3g'B^\mu}{2g}\right)^2\right].$$

M is a 4×4 matrix with a zero eigenvalue corresponding to the photon. Once the photon field has been identified, we remain with a 3×3 mass matrix for three neutral gauge bosons Z^μ, Z'^μ and Z''^μ. Since we are interested now in the low energy phenomenology of our model, we can choose $V=V'$ in order to simplify matters. For this particular case the field $Z''^\mu=A^\mu_8/\sqrt{3}-\sqrt{2/3}A^\mu_{15}$ decouples from the other two and acquires a squared mass $(g^2/2)V^2$. By diagonalizing the remaining 2×2 mass matrix we get the other two physical neutral gauge bosons which are defined through the mixing angle θ between Z_μ, Z'_μ:

$$Z^\mu_1=Z_\mu\cos\theta+Z'_\mu\sin\theta,$$

$$Z^\mu_2=-Z_\mu\sin\theta+Z'_\mu\cos\theta,$$

where

$$\tan(2\theta) = -\frac{2\sqrt{2}C_W}{\sqrt{1+2\delta^2}\left[1+\frac{2V^2}{v^2}C_W^4 - \frac{2}{1+2\delta^2}C_W^2\right]}, \quad (1)$$

with $\delta = g'/(2g)$.

The photon field A^μ and the fields Z_μ and Z'_μ are given by

$$A^\mu = S_W A_3^\mu + C_W\left[\frac{T_W}{\sqrt{3}}\left(A_8^\mu + \frac{A_{15}^\mu}{\sqrt{2}}\right) + (1-T_W^2/2)^{1/2}B^\mu\right],$$

$$Z^\mu = C_W A_3^\mu - S_W\left[\frac{T_W}{\sqrt{3}}\left(A_8^\mu + \frac{A_{15}^\mu}{\sqrt{2}}\right) + (1-T_W^2/2)^{1/2}B^\mu\right],$$

$$Z'^\mu = \sqrt{\frac{2}{3}}(1-T_W^2/2)^{1/2}\left(A_8^\mu + \frac{A_{15}^\mu}{\sqrt{2}}\right) - \frac{T_W}{\sqrt{2}}B^\mu. \quad (2)$$

$S_W = 2\delta/\sqrt{6\delta^2+1}$ and C_W are the sine and cosine of the electroweak mixing angle respectively, and $T_W = S_W/C_W$. We can also identify the Y hypercharge associated with the SM Abelian gauge boson as

$$Y^\mu = \left[\frac{T_W}{\sqrt{3}}\left(A_8^\mu + \frac{A_{15}^\mu}{\sqrt{2}}\right) + (1-T_W^2/2)^{1/2}B^\mu\right]. \quad (3)$$

A. Charged currents

After some algebra, the Hamiltonian for the charged currents can be written as

$$\begin{aligned}
H^{CC} = \frac{g}{\sqrt{2}}\Bigg[& W_\mu^+\left(\left(\sum_{a=2}^{3}\overline{u}_{aL}\gamma^\mu d_{aL}\right) - \overline{u}_{1L}\gamma^\mu d_{1L} - \left(\sum_{\alpha=1}^{3}\overline{\nu}_{e\alpha L}\gamma^\mu e_{\alpha L}^-\right)\right) \\
& + K_\mu^+\left(\left(\sum_{a=2}^{3}\overline{u}_{aL}\gamma^\mu D_{aL}\right) - \overline{U}_{1L}\gamma^\mu d_{1L} - \left(\sum_{\alpha=1}^{3}\overline{N}^0_{\alpha L}\gamma^\mu e_{\alpha L}^-\right)\right) \\
& + X_\mu^+\left(\left(\sum_{a=2}^{3}\overline{u}_{aL}\gamma^\mu D'_{aL}\right) - \overline{U'}_{1L}\gamma^\mu d_{1L} - \left(\sum_{\alpha=1}^{3}\overline{N'}^0_{\alpha L}\gamma^\mu e_{\alpha L}^-\right)\right) \\
& + K_\mu^0\left(\left(\sum_{a=2}^{3}\overline{d}_{aL}\gamma^\mu D_{aL}\right) - \overline{U}_{1L}\gamma^\mu u_{1L} - \left(\sum_{\alpha=1}^{3}\overline{N}^0_{\alpha L}\gamma^\mu \nu_{e\alpha L}\right)\right) \\
& + X_\mu^0\left(\left(\sum_{a=2}^{3}\overline{d}_{aL}\gamma^\mu D'_{aL}\right) - \overline{U'}_{1L}\gamma^\mu u_{1L} - \left(\sum_{\alpha=1}^{3}\overline{N'}^0_{\alpha L}\gamma^\mu \nu_{e\alpha L}\right)\right) \\
& + Y_\mu^0\left(\left(\sum_{a=2}^{3}\overline{D}_{aL}\gamma^\mu D'_{aL}\right) - \overline{U'}_{1L}\gamma^\mu U'_{1L} - \left(\sum_{\alpha=1}^{3}\overline{N'}^0_{\alpha L}\gamma^\mu N^0_{\alpha L}\right)\right)\Bigg] + \text{H.c.} \quad (4)
\end{aligned}$$

B. Neutral currents

The neutral currents $J_\mu(EM)$, $J_\mu(Z)$, $J_\mu(Z')$, and $J_\mu(Z'')$ associated with the Hamiltonian

$$H^0 = eA^\mu J_\mu(EM) + (g/C_W)Z^\mu J_\mu(Z) + (g'/\sqrt{2})Z'^\mu J_\mu(Z') + (g/2)Z''^\mu J_\mu(Z'')$$

are

$$\begin{aligned}
J_\mu(EM) = & \frac{2}{3}\left(\left(\sum_{a=2}^{3}\overline{u}_a\gamma_\mu u_a\right) + \overline{u}_1\gamma_\mu u_1 + \overline{U}_1\gamma_\mu U_1 + \overline{U'}_1\gamma_\mu U'_1\right) \\
& - \frac{1}{3}\left(\left(\sum_{a=2}^{3}\overline{d}_a\gamma_\mu d_a + \overline{D}_a\gamma_\mu D_a + \overline{D'}_a\gamma_\mu D'_a\right) + \overline{d}_1\gamma_\mu d_1\right) - \sum_{\alpha=1}^{3}\overline{e_\alpha^-}\gamma_\mu e_\alpha^- \\
= & \sum_f q_f\overline{f}\gamma_\mu f,
\end{aligned}$$

$$J_\mu(Z)=J_{\mu,L}(Z)-S_W^2 J_\mu(EM),$$

$$J_\mu(Z')=T_W J_\mu(EM)-J_{\mu,L}(Z'),$$

$$\begin{aligned}J_\mu(Z'')=&\sum_{a=2}^{3}(\overline{D'}_{aL}\gamma_\mu D'_{aL}-\overline{D}_{aL}\gamma_\mu D_{aL})\\&+(-\overline{U'}_{1L}\gamma_\mu U'_{1L}+\overline{U}_{1L}\gamma_\mu U_{1L})\\&+\sum_{\alpha=1}^{3}(-\overline{N'}^0_{\alpha L}\gamma_\mu N'_{\alpha L}0+\overline{N}^0_{\alpha L}\gamma_\mu N^0_{\alpha L}),\end{aligned}\tag{5}$$

where $e=gS_W=g'C_W\sqrt{1-T_W^2/2}>0$ is the electric charge, q_f is the electric charge of the fermion f in units of e and $J_\mu(EM)$ is the electromagnetic current. Notice that the Z''_μ current couples only to exotic fields. The left-handed currents are

$$\begin{aligned}J_{\mu,L}(Z)=&\frac{1}{2}\Bigg[\sum_{a=2}^{3}(\overline{u}_{aL}\gamma_\mu u_{aL}-\overline{d}_{aL}\gamma_\mu d_{aL})\\&-(\overline{d}_{1L}\gamma_\mu d_{1L}-\overline{u}_{1L}\gamma_\mu u_{1L})\\&-\sum_{\alpha=1}^{3}(\overline{e^-_{\alpha L}}\gamma_\mu e^-_{\alpha L}-\overline{\nu}_{e\alpha L}\gamma_\mu \nu_{e\alpha L})\Bigg]\\=&\sum_f T_{4f}\overline{f}_L\gamma_\mu f_L,\end{aligned}$$

$$\begin{aligned}&J_{\mu,L}(Z')\\&=T_W^{-1}\Bigg[\sum_{a=2}^{3}\left(\overline{u}_{aL}\gamma_\mu u_{aL}-\frac{1}{2}\overline{D}_{aL}\gamma_\mu D_{aL}-\frac{1}{2}\overline{D'}_{aL}\gamma_\mu D'_{aL}\right)\\&\quad-\overline{d}_{1L}\gamma_\mu d_{1L}+\frac{1}{2}\overline{U}_{1L}\gamma_\mu U_{1L}+\frac{1}{2}\overline{U'}_{1L}\gamma_\mu U'_{1L}\\&\quad+\sum_{\alpha=1}^{3}\left(-\overline{e^-_{\alpha L}}\gamma_\mu e^-_{\alpha L}+\frac{1}{2}\overline{N}^0_{\alpha L}\gamma_\mu N^0_{\alpha L}+\frac{1}{2}\overline{N'}^0_{\alpha L}\gamma_\mu N'_{\alpha L}0\right)\Bigg]\\&=\sum_f T'_{4f}\overline{f}_L\gamma_\mu f_L,\end{aligned}\tag{6}$$

where $T_{4f}=Dg(1/2,-1/2,0,0)$ is the third component of the weak isospin and $T'_{4f}=(1/T_W)Dg(1,0,-1/2,-1/2)=(1/T_W)(\lambda_3/2+\lambda_8/\sqrt{3}+\lambda_{15}/\sqrt{6})$ is a convenient 4×4 diagonal matrix, acting both of them on the representation 4 of $SU(4)_L$. Notice that $J_\mu(Z)$ is just the generalization of the neutral current present in the SM. This allows us to identify Z_μ as the neutral gauge boson of the SM, which is consistent with Eqs. (2) and (3).

The couplings of the mass eigenstates Z_1^μ and Z_2^μ are given by

$$\begin{aligned}H^{NC}=&\frac{g}{2C_W}\sum_{i=1}^{2}Z_i^\mu\sum_f\{\overline{f}\gamma_\mu[a_{iL}(f)(1-\gamma_5)\\&+a_{iR}(f)(1+\gamma_5)]f\}\\=&\frac{g}{2C_W}\sum_{i=1}^{2}Z_i^\mu\sum_f\{\overline{f}\gamma_\mu[g(f)_{iV}-g(f)_{iA}\gamma_5]f\},\end{aligned}$$

where

$$\begin{aligned}a_{1L}(f)&=\cos\theta(T_{4f}-q_fS_W^2)+\frac{g'\sin\theta C_W}{g\sqrt{2}}(T'_{4f}-q_fT_W),\\a_{1R}(f)&=-q_fS_W\left(\cos\theta S_W+\frac{g'\sin\theta}{g\sqrt{2}}\right),\\a_{2L}(f)&=-\sin\theta(T_{4f}-q_fS_W^2)+\frac{g'\cos\theta C_W}{g\sqrt{2}}(T'_{4f}-q_fT_W),\\a_{2R}(f)&=q_fS_W\left(\sin\theta S_W-\frac{g'\cos\theta}{g\sqrt{2}}\right),\end{aligned}\tag{7}$$

and

$$\begin{aligned}g(f)_{1V}&=\cos\theta(T_{4f}-2S_W^2q_f)+\frac{g'\sin\theta}{g\sqrt{2}}(T'_{4f}C_W-2q_fS_W),\\g(f)_{2V}&=-\sin\theta(T_{4f}-2S_W^2q_f)\\&\quad+\frac{g'\cos\theta}{g\sqrt{2}}(T'_{4f}C_W-2q_fS_W),\\g(f)_{1A}&=\cos\theta T_{4f}+\frac{g'\sin\theta}{g\sqrt{2}}T'_{4f}C_W,\\g(f)_{2A}&=-\sin\theta T_{4f}+\frac{g'\cos\theta}{g\sqrt{2}}T'_{4f}C_W.\end{aligned}\tag{8}$$

The values of g_{iV}, g_{iA} with $i=1{,}2$ are listed in Tables I and II.

As we can see, in the limit $\theta=0$ the couplings of Z_1^μ to the ordinary leptons and quarks are the same as in the SM; due to this we can test the new physics beyond the SM predicted by this particular model.

V. FERMION MASSES

The Higgs scalars introduced in Sec. III not only break the symmetry in an appropriate way, but produce the following mass terms for the fermions of the model.

A. Quark masses

For the quark sector we can write the following Yukawa terms:

TABLE I. The $Z_1^\mu \to \overline{f} f$ couplings.

f	$g(f)_{1V}$	$g(f)_{1A}$
$u_{2,3}$	$\cos\theta\left(\frac{1}{2}-\frac{4S_W^2}{3}\right)+\frac{\sin\theta}{(3C_W^2-1)^{1/2}}\left(1-\frac{7S_W^2}{3}\right)$	$\frac{1}{2}\cos\theta+\sin\theta C_W^2/(3C_W^2-1)^{1/2}$
$d_{2,3}$	$\left(-\frac{1}{2}+\frac{2S_W^2}{3}\right)\cos\theta+\frac{2}{3}\sin\theta S_W^2/(3C_W^2-1)^{1/2}$	$-\frac{1}{2}\cos\theta$
$D_{2,3}$	$\frac{2S_W^2}{3}\cos\theta-\frac{1}{2}\sin\theta\left(1-\frac{7S_W^2}{3}\right)/(3C_W^2-1)^{1/2}$	$-\frac{1}{2}\sin\theta C_W^2/(3C_W^2-1)^{1/2}$
$D'_{2,3}$	$\frac{2S_W^2}{3}\cos\theta-\frac{1}{2}\sin\theta\left(1-\frac{7S_W^2}{3}\right)/(3C_W^2-1)^{1/2}$	$-\frac{1}{2}\sin\theta C_W^2/(3C_W^2-1)^{1/2}$
d_1	$\left(-\frac{1}{2}+\frac{2S_W^2}{3}\right)\cos\theta-\sin\theta\left(1-\frac{5S_W^2}{3}\right)/(3C_W^2-1)^{1/2}$	$-\frac{1}{2}\cos\theta-\sin\theta C_W^2/(3C_W^2-1)^{1/2}$
u_1	$\cos\theta\left(\frac{1}{2}-\frac{4S_W^2}{3}\right)-\frac{4\sin\theta}{3(3C_W^2-1)^{1/2}}S_W^2$	$\frac{1}{2}\cos\theta$
U_1	$-\frac{4S_W^2\cos\theta}{3}+\sin\theta(1-\frac{11}{3}S_W^2)/[2(3C_W^2-1)^{1/2}]$	$C_W^2\sin\theta/[2(3C_W^2-1)^{1/2}]$
U'_1	$-\frac{4S_W^2\cos\theta}{3}+\sin\theta(1-\frac{11}{3}S_W^2)/[2(3C_W^2-1)^{1/2}]$	$C_W^2\sin\theta/[2(3C_W^2-1)^{1/2}]$
$e^-_{1,2,3}$	$\cos\theta(-\frac{1}{2}+2S_W^2)-\frac{\sin\theta}{(3C_W^2-1)^{1/2}}(1-3S_W^2)$	$-\frac{\cos\theta}{2}-\frac{\sin\theta}{(3C_W^2-1)^{1/2}}C_W^2$
$\nu_{1,2,3}$	$\frac{1}{2}\cos\theta$	$\frac{1}{2}\cos\theta$
$N^0_{1,2,3}$	$C_W^2\sin\theta/[2(3C_W^2-1)^{1/2}]$	$C_W^2\sin\theta/[2(3C_W^2-1)^{1/2}]$
$N'^0_{1,2,3}$	$C_W^2\sin\theta/[2(3C_W^2-1)^{1/2}]$	$C_W^2\sin\theta/[2(3C_W^2-1)^{1/2}]$

$$
\begin{aligned}
\mathcal{L}_Y^Q = \sum_{a=2}^{3} Q_{aL}^T C\Bigg\{ & \phi_1^*\left(\sum_{\alpha=1}^{3} h_{u\alpha}^a u_{\alpha L}^c + h_U^a U_L^c + h_U^{a'} U_L'^c\right) \\
& +(\phi_2+\phi_3)\left[\sum_{\alpha=1}^{3} h_{\alpha d}^a d_{\alpha L}^c + \sum_{b=2}^{3}\left(h_{bD}^a D_{bL}^c + h_{bD}^{a'} D_{bL}'^c\right)\right]\Bigg\} \\
& + Q_{1L}^T C\Bigg\{\phi_1\left[\sum_{\alpha=1}^{3} h_{d\alpha}^1 d_{\alpha L}^c + \sum_{a=2}^{3}\left(h_{aD}^1 D_{aL}^c + h_{aD}^{1'} D_{aL}'^c\right)\right] \\
& +(\phi_2^*+\phi_3^*)\left(\sum_{\alpha=1}^{3} h_{u\alpha}^1 u_{\alpha L}^c + h_U^1 U_L^c + h_U^{1'} U_L'^c\right)\Bigg\} + \text{H.c.}
\end{aligned}
$$

where the h's are Yukawa couplings and C is the charge conjugate operator. This Lagrangian produces the following tree level quark masses:

U_1', D_2' and D_3' acquire heavy masses of the order of $V' \gg v$.

U_1, D_2 and D_3 acquire heavy masses of the order of $V \gg v$.

u_3, u_2 and d_1 acquire masses of the order of $v \approx 174$ GeV.

u_1, d_2 and d_3 remain massless at the tree level.

The former mass spectrum is far from being realistic, but it can be improved by implementing the following program:

To introduce a discrete symmetry in order to avoid a tree-level mass for d_1 (and maybe for u_2 too).

To introduce a new Higgs field $\phi_4[1,\overline{4},-1/4]$ which does not acquire VEV but that introduces a quartic coupling $\phi_1^*\phi_2\phi_3\phi_4$ in the Higgs potential in order to generate radiative masses for the ordinary quarks.

To tune the Yukawa couplings in order to obtain the correct mixing between flavors (ordinary and exotic) with the same electric charge.

B. Lepton masses

For the charged leptons we have the following Yukawa terms:

$$
\mathcal{L}_Y^l = \sum_{\alpha=1}^{3}\sum_{\beta=1}^{3} h_{\alpha\beta}^e L_{\alpha L}^T C \phi_1 e_{\beta L}^+ + \text{H.c.} \tag{9}
$$

Notice that for $h_{\alpha\beta}^e = h\delta_{\alpha\beta}$ we get a mass only for the heavi-

TABLE II. The $Z_2^\mu \to \overline{f}f$ couplings.

f	$g(f)_{2V}$	$g(f)_{2A}$
$u_{2,3}$	$-\sin\theta\left(\frac{1}{2}-\frac{4S_W^2}{3}\right)+\frac{\cos\theta}{(3C_W^2-1)^{1/2}}\left(1-\frac{7S_W^2}{3}\right)$	$-\frac{1}{2}\sin\theta+\cos\theta C_W^2/(3C_W^2-1)^{1/2}$
$d_{2,3}$	$\left(\frac{1}{2}-\frac{2S_W^2}{3}\right)\sin\theta+\frac{2}{3}\cos\theta S_W^2/(3C_W^2-1)^{1/2}$	$\frac{1}{2}\sin\theta$
$D_{2,3}$	$-\frac{2S_W^2}{3}\sin\theta-\frac{1}{2}\cos\theta\left(1-\frac{7S_W^2}{3}\right)/(3C_W^2-1)^{1/2}$	$-\frac{1}{2}\cos\theta C_W^2/(3C_W^2-1)^{1/2}$
$D'_{2,3}$	$-\frac{2S_W^2}{3}\sin\theta-\frac{1}{2}\cos\theta\left(1-\frac{7S_W^2}{3}\right)/(3C_W^2-1)^{1/2}$	$-\frac{1}{2}\cos\theta C_W^2/(3C_W^2-1)^{1/2}$
d_1	$\left(\frac{1}{2}-\frac{2S_W^2}{3}\right)\sin\theta-\cos\theta\left(1-\frac{5S_W^2}{3}\right)/(3C_W^2-1)^{1/2}$	$\frac{1}{2}\sin\theta-\cos\theta C_W^2/(3C_W^2-1)^{1/2}$
u_1	$-\sin\theta\left(\frac{1}{2}-\frac{4S_W^2}{3}\right)-\frac{4\cos\theta}{3(3C_W^2-1)^{1/2}}S_W^2$	$-\frac{1}{2}\sin\theta$
U_1	$\frac{4S_W^2\sin\theta}{3}+\cos\theta(1-\frac{11}{3}S_W^2)/[2(3C_W^2-1)^{1/2}]$	$C_W^2\cos\theta/[2(3C_W^2-1)^{1/2}]$
U'_1	$\frac{4S_W^2\sin\theta}{3}+\cos\theta(1-\frac{11}{3}S_W^2)/[2(3C_W^2-1)^{1/2}]$	$C_W^2\cos\theta/[2(3C_W^2-1)^{1/2}]$
$e^-_{1,2,3}$	$\sin\theta(\frac{1}{2}-2S_W^2)-\frac{\cos\theta}{(3C_W^2-1)^{1/2}}(1-3S_W^2)$	$\frac{\sin\theta}{2}-\frac{\cos\theta}{(3C_W^2-1)^{1/2}}C_W^2$
$\nu_{1,2,3}$	$-\frac{1}{2}\sin\theta$	$-\frac{1}{2}\sin\theta$
$N^0_{1,2,3}$	$C_W^2\cos\theta/[2(3C_W^2-1)^{1/2}]$	$C_W^2\cos\theta/[2(3C_W^2-1)^{1/2}]$
$N'^0_{1,2,3}$	$C_W^2\cos\theta/[2(3C_W^2-1)^{1/2}]$	$C_W^2\cos\theta/[2(3C_W^2-1)^{1/2}]$

est lepton (the τ). So, in the context of this model the masses for the charged leptons can be generated in a consistent way, with the masses for e^- and μ^- suppressed by differences of Yukawa couplings.

The neutral leptons remain massless as far as we use only the original fields introduced in Sec. II. But as mentioned earlier, we may introduce one or more Weyl singlet states $N^0_{L,b}$, $b=1,2,\ldots$ which may implement the appropriate neutrino oscillations [3].

VI. CONSTRAINS ON THE $(Z^\mu - Z'^\mu)$ MIXING ANGLE AND THE Z_2^μ MASS

To bound $\sin\theta$ and M_{Z_2} we use parameters measured at the Z pole from CERN e^+e^- collider (LEP), SLAC Linear Collider (SLC), and atomic parity violation constraints which are given in Table III.

The expression for the partial decay width for $Z_1^\mu \to f\overline{f}$ is

$$\Gamma(Z_1^\mu \to f\overline{f}) = \frac{N_C G_F M_{Z_1}^3}{6\pi\sqrt{2}}\rho\left\{\frac{3\beta-\beta^3}{2}[g(f)_{1V}]^2 + \beta^3[g(f)_{1A}]^2\right\}(1+\delta_f)R_{EW}R_{QCD}, \qquad (10)$$

where f is an ordinary SM fermion, Z_1^μ is the physical gauge boson observed at LEP, $N_C=1$ for leptons while for quarks $N_C=3(1+\alpha_s/\pi+1.405\alpha_s^2/\pi^2-12.77\alpha_s^3/\pi^3)$, where the 3 is due to color and the factor in parentheses represents the universal part of the QCD corrections for massless quarks (for fermion mass effects and further QCD corrections which are different for vector and axial-vector partial widths see Ref. [7]); R_{EW} are the electroweak corrections which include the leading order QED corrections given by $R_{QED}=1+3\alpha/(4\pi)$. R_{QCD} are further QCD corrections (for a comprehensive review see Ref. [8] and references therein), and $\beta=\sqrt{1-4m_f^2/M_{Z_1}^2}$ is a kinematic factor which can be taken equal to 1 for all the SM fermions except for the bottom quark. The factor δ_f contains the one loop vertex contribution which is negligible for all fermion fields except for the bottom quark for which the contribution coming from the top quark at the one loop vertex radiative correction is parametrized as $\delta_b \approx 10^{-2}[-m_t^2/(2M_{Z_1}^2)+1/5]$ [9]. The ρ parameter can be expanded as $\rho=1+\delta\rho_0+\delta\rho_V$ where the oblique correction $\delta\rho_0$ is given by $\delta\rho_0\approx 3G_F m_t^2/(8\pi^2\sqrt{2})$, and $\delta\rho_V$ is the tree level contribution due to the $(Z_\mu - Z'_\mu)$ mixing which can be parametrized as $\delta\rho_V \approx (M_{Z_2}^2/M_{Z_1}^2-1)\sin^2\theta$. Fi-

TABLE III. Experimental data and SM values for the parameters.

	Experimental results	SM
Γ_Z(GeV)	2.4952 ± 0.0023	2.4966 ± 0.0016
Γ(had) (GeV)	1.7444 ± 0.0020	1.7429 ± 0.0015
$\Gamma(l^+l^-)$ (MeV)	83.984 ± 0.086	84.019 ± 0.027
R_e	20.804 ± 0.050	20.744 ± 0.018
R_μ	20.785 ± 0.033	20.744 ± 0.018
R_τ	20.764 ± 0.045	20.790 ± 0.018
R_b	0.21664 ± 0.00068	0.21569 ± 0.00016
R_c	0.1729 ± 0.0032	0.17230 ± 0.00007
Q_W^{Cs}	$-72.65\pm0.28\pm0.34$	-73.10 ± 0.03
M_{Z_1} (GeV)	91.1872 ± 0.0021	91.1870 ± 0.0021

nally, $g(f)_{1V}$ and $g(f)_{1A}$ are the coupling constants of the physical Z_1^μ field with ordinary fermions which are listed in Table I.

In what follows we are going to use the experimental values [10]: $M_{Z_1}=91.188$ GeV, $m_t=174.3$ GeV, $\alpha_s(m_Z)=0.1192$, $\alpha(m_Z)^{-1}=127.938$, and $\sin^2\theta_W$=0.2333. The experimental values are introduced using the definitions $R_\eta \equiv \Gamma(\eta\eta)/\Gamma(\text{hadrons})$ for $\eta=e,\mu,\tau,b,c$.

As a first result notice from Table I that our model predicts $R_e=R_\mu=R_\tau$, in agreement with the experimental results in Table III.

The effective weak charge in atomic parity violation, Q_W, can be expressed as a function of the number of protons (Z) and the number of neutrons (N) in the atomic nucleus in the form

$$Q_W=-2[(2Z+N)c_{1u}+(Z+2N)c_{1d}], \tag{11}$$

where $c_{1q}=2g(e)_{1A}g(q)_{1V}$. The theoretical value for Q_W for the cesium atom is given by [11] $Q_W({}^{133}_{55}\text{Cs})=-73.09 \pm 0.04+\Delta Q_W$, where the contribution of new physics is included in ΔQ_W which can be written as [12]

$$\Delta Q_W=\left[\left(1+4\frac{S_W^4}{1-2S_W^2}\right)Z-N\right]\delta\rho_V+\Delta Q_W'. \tag{12}$$

The term $\Delta Q_W'$ is model dependent and it can be obtained for our model by using $g(e)_{1A}$ and $g(q)_{1V}$ from Table I. The value we obtain is

$$\Delta Q_W'=(8.29Z+16.14N)\sin\theta+(11.64Z+14.47N)\frac{M_{Z_1}^2}{M_{Z_2}^2}. \tag{13}$$

The discrepancy between the SM and the experimental data for ΔQ_W is given by [13]

$$\Delta Q_W=Q_W^{exp}-Q_W^{SM}=1.03\pm0.44, \tag{14}$$

which is 2.3 σ away from the SM predictions.

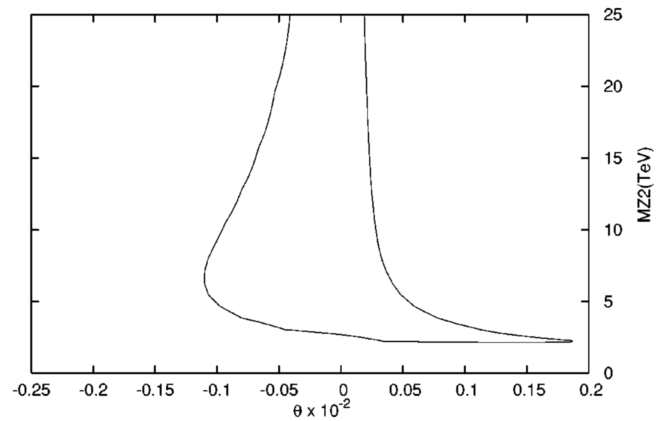

FIG. 1. Contour plot displaying the allowed region for θ vs M_{Z_2} at 95% C.L.

Introducing the expressions for Z pole observables in Eq. (10), with ΔQ_W in terms of new physics in Eq. (12) and using experimental data from LEP, SLC and atomic parity violation (see Table III), we do a χ^2 fit and we find the best allowed region in the $(\theta - M_{Z_2})$ plane at 95% confidence level (C.L.). In Fig. 1 we display this region which gives us the constraints

$$-0.0011\leqslant\theta\leqslant0.0019, \quad 2\ \text{TeV}\leqslant M_{Z_2}. \tag{15}$$

As we can see the mass of the new neutral gauge boson is compatible with the bound obtained in $p\bar{p}$ collisions at the Fermilab Tevatron [14]. From our analysis we can also see that for $|\theta|\to0$, M_{Z_2} peaks at a finite value larger than 100 TeV which still copes with the experimental constraints on the ρ parameter.

VII. CONCLUSIONS

We have presented an anomaly-free model based on the local gauge group $SU(3)_c\otimes SU(4)_L\otimes U(1)_X$. We break the gauge symmetry down to $SU(3)_c\otimes U(1)_Q$ and at the same time give masses to the fermion fields in the model in a consistent way by using three different Higgs scalars ϕ_i, $i=1,2,3$ which set two different mass scales: $V\sim V'\gg v=174$ GeV. By using experimental results we bound the mixing angle θ between the SM neutral current and a new one to be $-0.0011<\theta<0.0019$ and the lowest bound for M_{Z_2} is 2 TeV$\leqslant M_{Z_2}$.

Our model includes four exotic down type quarks D_a, D_a', $a=2,3$ of electric charge $-1/3$ and two exotic up quarks U_1, U_1' of electric charge 2/3. The six exotic quarks acquire large masses of the order of $V\simeq V'\gg v=174$ GeV and are useful in two ways: first they mix with the ordinary up and down quarks in the three families with a mixing that can be used in order to produce a consistent mass spectrum (masses and mixings) for ordinary quarks; second, they can be used in order to implement the so-called little Higgs mechanism [5].

Notice also the consistence of our model in the charged lepton sector. Not only it predicts the correct ratios R_η, η

$=e,\mu,\tau$ in the Z decays, but the model also allows for a consistent mass pattern of the particles, which do not include leptons with exotic electric charges.

In the main body of this paper we have analyzed an specific model based on the 3-4-1 gauge structure. This model is just one of a large variety of models based on the same gauge structure. A systematic analysis of models without exotic electric charges with the same gauge structure is presented in the Appendix at the end of the paper. A phenomenological analysis for all those model can be done, but we think it is not profitable since all of them must produce similar results at low energies.

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APPENDIX

In what follows we present a systematic analysis of models without exotic electric charges, based on the local gauge structure $SU(3)_c\otimes SU(4)_L\otimes U(1)_X$.

We assume that the electroweak group is $SU(4)_L\otimes U(1)_X\supset SU(3)_L\otimes U(1)_Z\supset SU(2)_L\otimes U(1)_Y$, where the gauge structure $SU(3)_L\otimes U(1)_Z$ refers to the one presented in Ref. [6]. We also assume that the left-handed quarks (color triplets), left-handed leptons (color singlets) and scalars, transform either under the 4 or the $\overline{4}$ fundamental representations of $SU(4)_L$. Two classes of models will be discussed: one family models where the anomalies cancel in each family as in the SM, and family models where the anomalies cancel by an interplay between the several families. As in the SM, $SU(3)_c$ is vectorlike.

The most general expression for the electric charge generator in $SU(4)_L\otimes U(1)_X$ is a linear combination of the four diagonal generators of the gauge group

$$Q=aT_{3L}+\frac{1}{\sqrt{3}}bT_{8L}+\frac{1}{\sqrt{6}}cT_{15L}+XI_4, \tag{A1}$$

where $T_{iL}=\lambda_{iL}/2$, being λ_{iL} the Gell-Mann matrices for $SU(4)_L$ normalized as $\mathrm{Tr}(\lambda_i\lambda_j)=2\delta_{ij}$, $I_4=Dg(1,1,1,1)$ is the diagonal 4×4 unit matrix, and a, b and c are free parameters to be fixed next. Notice that we can absorb an eventual coefficient for X in its definition.

If we assume that the usual isospin $SU(2)_L$ of the SM is such that $SU(2)_L\subset SU(4)_L$, then $a=1$ and we have just a two-parameter set of models, all of them characterized by the values of b and c. So, Eq. (A1) allows for an infinite number of models in the context of the 3-4-1 theory, each one associated to particular values of the parameters b and c, with characteristic signatures that make them different from each other.

There are a total of 24 gauge bosons in the gauge group under consideration, 15 of them associated with $SU(4)_L$ which can be written as

$$\frac{1}{2}\lambda_\alpha A^\alpha_\mu=\frac{1}{\sqrt{2}}\begin{pmatrix} D^0_{1\mu} & W^+_\mu & K^{(b+1)/2}_\mu & X^{(3+b+2c)/6}_\mu \\ W^-_\mu & D^0_{2\mu} & K^{(b-1)/2}_\mu & X^{(-3+b+2c)/6}_\mu \\ K^{-(b+1)/2}_\mu & \overline{K}^{-(b-1)/2}_\mu & D^0_{3\mu} & Y^{-(b-c)/3}_\mu \\ X^{-(3+b+2c)/6}_\mu & \overline{X}^{(3-b-2c)/6}_\mu & \overline{Y}^{(b-c)/3}_\mu & D^0_{4\mu} \end{pmatrix}, \tag{A2}$$

where $D^\mu_1=A^\mu_3/\sqrt{2}+A^\mu_8/\sqrt{6}+A^\mu_{15}/\sqrt{12}$, $D^\mu_2=-A^\mu_3/\sqrt{2}+A^\mu_8/\sqrt{6}+A^\mu_{15}/\sqrt{12}$; $D^\mu_3=-2A^\mu_8/\sqrt{6}+A^\mu_{15}/\sqrt{12}$, and $D^\mu_4=-3A^\mu_{15}/\sqrt{12}$. The upper indices in the gauge bosons in the former expression stand for the electric charge of the corresponding particle, some of them functions of the b and c parameters as they should be. Notice that if we demand for gauge bosons with electric charges $0,\pm1$ only, there are not more than four different possibilities for the simultaneous values of b and c; they are: $b=c=1$; $b=c=-1$; $b=1,c=-2$, and $b=-1,c=2$.

Now, contrary to the SM where only the Abelian $U(1)_Y$ factor is anomalous, in the 3-4-1 theory both, $SU(4)_L$ and $U(1)_X$ are anomalous [$SU(3)_c$ is vectorlike]. So, special combinations of multiplets must be used in each particular model in order to cancel the possible anomalies, and obtain renormalizable models. The triangle anomalies we must take care of are: $[SU(4)_L]^3$, $[SU(3)_c]^2U(1)_X$, $[SU(4)_L]^2U(1)_X$, $[grav]^2U(1)_X$ and $[U(1)_X]^3$.

Now let us see how the charge operator in Eq. (A1) acts on the representations 4 and $\overline{4}$ of $SU(4)_L$:

$$Q[4]=Dg\left(\frac{1}{2}+\frac{b}{6}+\frac{c}{12}+X,-\frac{1}{2}+\frac{b}{6}+\frac{c}{12}+X,\right.$$
$$\left.-\frac{2b}{6}+\frac{c}{12}+X,-\frac{3c}{12}+X\right),$$

$$Q[\overline{4}]=Dg\left(-\frac{1}{2}-\frac{b}{6}-\frac{c}{12}+X,\frac{1}{2}-\frac{b}{6}-\frac{c}{12}+X,\right.$$
$$\left.\frac{2b}{6}-\frac{c}{12}+X,\frac{3c}{12}+X\right).$$

TABLE IV. Anomalies for sets with values $b=c=1$.

Anomaly	S_1^q	S_2^q	S_3^l	S_4^l	S_5^l	S_6^l
$[U(1)_X]^3$	$-9/16$	$-27/16$	$21/16$	$-15/16$	$15/16$	$-21/16$
$[SU(4)_L]^2U(1)_X$	$-1/4$	$5/4$	$-3/4$	$1/4$	$-1/4$	$3/4$
$[SU(4)_L]^3$	3	-3	1	1	-1	-1

Notice that, if we accommodate the known left-handed quark and lepton isodoublets in the two upper components of 4 and $\overline{4}$ (or $\overline{4}$ and 4), do not allow for electrically charged antiparticles in the two lower components of the multiplets [antiquarks violate $SU(3)_c$ and e^+,μ^+ and τ^+ violate lepton number at the tree level] and forbid the presence of exotic electric charges in the possible models, then the electric charge of the third and fourth components in 4 and $\overline{4}$ must be equal either to the charge of the first and/or second component, which in turn implies that b and c can take only the four sets of values stated above. So, these four sets of values for b and c are necessary and sufficient conditions in order to exclude exotic electric charges in the fermion sector too.

A further analysis also shows that models with $b=c=-1$ are equivalent, via charge conjugation, to models with $b=c=1$. Similarly, models with $b=-1$, $c=2$ are equivalent to models with $b=1$, $c=-2$. So, with the constraints imposed, we have only two different sets of models; those for $b=c=1$ and those for $b=1$, $c=-2$.

1. Models for $b=c=1$

First let us define the following complete sets of spin 1/2 Weyl spinors (complete in the sense that each set contains its own charged antiparticles):

$$S_1^q=\{(u,d,D,D')_L\sim[3,4,-\tfrac{1}{12}],\ u_L^c\sim[\overline{3},1,-\tfrac{2}{3}],$$
$$d_L^c\sim[\overline{3},1,\tfrac{1}{3}],\ D_L^c\sim[\overline{3},1,\tfrac{1}{3}],\ D_L'^c\sim[\overline{3},1,\tfrac{1}{3}]\}.$$

$$S_2^q=\{(d,u,U,U')_L\sim[3,\overline{4},\tfrac{5}{12}],\ u_L^c\sim[\overline{3},1,-\tfrac{2}{3}],$$
$$d_L^c\sim[\overline{3},1,\tfrac{1}{3}],\ U_L^c\sim[\overline{3},1,-\tfrac{2}{3}],\ U_L'^c\sim[\overline{3},1,-\tfrac{2}{3}]\}.$$

$$S_3^l=\{(\nu_e^0,e^-,E^-,E'^-)_L\sim[1,4,-\tfrac{3}{4}],\ e_L^+\sim[1,1,1],$$
$$E_L^+\sim[1,1,1],\ E_L'^+\sim[1,1,1]\}.$$

$$S_4^l=\{(E^+,N_1^0,N_2^0,N_3^0)_L\sim[1,4,\tfrac{1}{4}],\ E_L^-\sim[1,1,-1]\}.$$

$$S_5^l=\{(e^-,\nu_e^0,N^0,N'^0)_L\sim[1,\overline{4},-\tfrac{1}{4}],\ e_L^+\sim[1,1,1]\}.$$

$$S_6^l=\{(N,E_1^+,E_2^+,E_3^+)_L\sim[1,\overline{4},\tfrac{3}{4}],\ E_{1L}^-\sim[1,1,-1],$$
$$E_{2L}^-\sim[1,1,-1],\ E_{3L}^-\sim[1,1,-1]\}.$$

Due to the fact that each set includes charged particles together with their corresponding antiparticles, and since $SU(3)_c$ is vectorlike, the anomalies $[\text{grav}]^2U(1)_X$, $[SU(3)_c]^3$ and $[SU(3)_c]^2U(1)_X$ automatically vanish. So, we only have to take care of the remaining three anomalies whose values are shown in Table IV.

Several anomaly free models can be constructed from this table. Let us see.

a. Three family models

We found two three family structures which are:

Model $A=2S_1^q\oplus S_2^q\oplus 3S_5^l$. (The model analyzed in the main text.)

Model $B=S_1^q\oplus 2S_2^q\oplus 3S_3^l$.

b. Two family models

We find only one two family structure given by: Model $C=S_1^q\oplus S_2^q\oplus S_3^l\oplus S_5^l$.

c. One family models

A one family model can not be directly extracted from S_i, $i=1,2,\ldots,6$, but we can check that the following particular arrangement is an anomaly free one family structure: Model $D=S_1^q\oplus(e^-,\nu_e^0,N^0,N'^0)_L\oplus(E_1^-,N_1^0,N_2^0,N_3^0)_L\oplus(N_4^0,E_1^+,e^+,E_2^+)_L\oplus E_2^-$. As can be checked, this model reduces to the model in Ref. [15] for the breaking chain $SU(4)_L\otimes U(1)_X\rightarrow SU(3)_L\otimes U(1)_\alpha\otimes U(1)_X\rightarrow SU(3)_L\otimes U(1)_Z$, for the value $\alpha=1/12$. In an analogous way, other one family models with more exotic charged leptons can also be constructed.

2. Models for $b=1, c=-2$

As in the previous case, let us define the following complete sets of spin 1/2 Weyl spinors:

$$S_1'^q=\{(u,d,D,U)_L\sim[3,4,\tfrac{1}{6}],\ u_L^c\sim[\overline{3},1,-\tfrac{2}{3}],$$
$$d_L^c\sim[\overline{3},1,\tfrac{1}{3}],\ D_L^c\sim[\overline{3},1,\tfrac{1}{3}],\ U_L^c\sim[\overline{3},1,-\tfrac{2}{3}]\}.$$

$$S_2'^q=\{(d,u,U,D)_L\sim[3,\overline{4},\tfrac{1}{6}],\ u_L^c\sim[\overline{3},1,-\tfrac{2}{3}],$$
$$d_L^c\sim[\overline{3},1,\tfrac{1}{3}],\ U_L^c\sim[\overline{3},1,-\tfrac{2}{3}],\ D_L^c\sim[\overline{3},1,\tfrac{1}{3}]\}.$$

$$S_3'^l=\{(\nu_e^0,e^-,E^-,N^0)_L\sim[1,4,-\tfrac{1}{2}],\ e_L^+\sim[1,1,1],$$
$$E_L^+\sim[1,1,1]\}.$$

$$S_4'^l=\{(e^-,\nu_e^0,N^0,E^-)_L\sim[1,\overline{4},-\tfrac{1}{2}],\ e_L^+\sim[1,1,1],$$
$$E_L^+\sim[1,1,1]\}.$$

TABLE V. Anomalies for sets with values $b=1$, $c=-2$.

Anomaly	S_1^q	S_2^q	S_3^l	S_4^l	S_5^l	S_6^l
$[U(1)_X]^3$	$-3/2$	$-3/2$	$3/2$	$3/2$	$-3/2$	$-3/2$
$[SU(4)_L]^2U(1)_X$	$1/2$	$1/2$	$-1/2$	$-1/2$	$1/2$	$1/2$
$[SU(4)_L]^3$	3	-3	1	-1	1	-1

$$S_5'^{\,l}=\{(E^+,N_1^0,N_2^0,e^+)_L\sim[1,4,\tfrac{1}{2}],\ E_L^-\sim[1,1,-1],\ e_L^-\sim[1,1,-1]\}.$$

$$S_6'^{\,l}=\{(N_3^0,E^+,e^+,N_4^0)_L\sim[1,\overline{4},\tfrac{1}{2}],\ E_L^-\sim[1,1,-1],\ e_L^-\sim[1,1,-1]\}.$$

For the former sets the anomalies $[\mathrm{grav}]^2U(1)_X$, $[SU(3)_c]^3$ and $[SU(3)_c]^2U(1)_X$ vanish. The other anomalies are shown in Table V. Again, several anomaly free models can be constructed from this table. Let us see.

a. Three family models

We found two three family structures which are:
Model E$=2S_1'^{\,q}\oplus S_2'^{\,q}\oplus 3S_4'^{\,l}$.
Model F$=S_1'^{\,q}\oplus 2S_2'^{\,q}\oplus 3S_3'^{\,l}$.

b. Two family models

We find again only one two family structure given by: Model G$=S_1'^{\,q}\oplus S_2'^{\,q}\oplus S_3'^{\,l}\oplus S_4'^{\,l}$.

c. One family models

Two one family models can be constructed using S_i', $i=1,\ldots,6$. They are:

Model H$=S_2'^{\,q}\oplus 2S_3'^{\,l}\oplus S_5'^{\,l}$.

Model I$=S_1'^{\,q}\oplus 2S_4'^{\,l}\oplus S_6'^{\,l}$.

To conclude this appendix let us mention that for the values of the parameters b and c used in our analysis, many more anomaly-free models can be constructed, all of them featuring the SM phenomenology at energies below 100 GeV. Model A, discussed in the main text, is just one example.

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