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# One-step non-SUSY unification

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**Abstract.** – We show that it is possible to achieve one-step gauge coupling unification in a general class of non-supersymmetric models which at low energies have only the standard particle content and extra Higgs fields doublets. The constraints are the experimental values of $\alpha_{\rm em}$, $\alpha_s$ and $\sin^2\theta_{\rm W}$ at $10^2$ GeV s, and the lower bounds for FCNC and proton decay rates. Specific examples are pointed out.

Although the Standard Model (SM) is a successful theory which is in good agreement with experiments [1], it is a common belief that there must exist a more fundamental theory, not far away from the present experimental energies, capable to provide information on the several aspects unanswered in the SM, especially on the so-called flavor problem which is related to the fermion mass spectrum and mixing angles, and on the number of families in nature. The two most popular trends in this direction in today's literature are Supersymmetry (SUSY) [2], and Grand Unified Theories (GUTs) [3] with and without SUSY. The hope is that the extra symmetry provides the lacking information.

A well-known fact nowadays is that the measured values of the SM coupling constants at the $m_Z$ scale and the bounds on proton lifetime rule out models like minimal $SU(5)$ [4], and other models that contain $SU(5)$ as an intermediate stage in the symmetry-breaking chain. Another well-known result (somehow related to the analysis we are going to present next) is that SUSY is a sufficient ingredient in order to achieve one-step unification in GUT models [5].

In what follows we are going to show that one-step unification is also possible in a class of non-SUSY GUT models. We restrict our analysis to models in which the low-energy matter consists only of the standard particle content and more SM Higgs doublet fields. Our analysis excludes at the same time some of the most popular GUT models.

In the SM the coupling constants are defined as effective parameters which include loop corrections in the gauge boson propagators according to the renormalization group equations (rge). They are, therefore, energy scale dependent, and to one loop they read

$$\mu \frac{\mathrm{d}\alpha_i}{\mathrm{d}\mu} \simeq -b_i \alpha_i^2, \tag{1}$$

where $\mu$ is the energy at which the coupling constants $\alpha_i = g_i^2/4\pi$ are evaluated, with $g_1$, $g_2$, and $g_3$ the coupling constants of the SM factor groups $U(1)_\mathrm{Y}$, $SU(2)_\mathrm{L}$ and $SU(3)_\mathrm{c}$, respectively. The constants $b_i$ are completely determined by the particle content in the model by

$$4\pi b_i = \frac{11}{3} C_i(\text{vectors}) - \frac{2}{3} C_i(\text{fermions}) - \frac{1}{3} C_i(\text{scalars}), \tag{2}$$

where $C_i(\cdots)$ is the index of the representation to which the $(\cdots)$ particles are assigned, and where we are considering Weyl fermion and complex scalar fields. The boundary conditions at the $m_Z \simeq 10^2$ GeV scale for these equations are determined by the relationships

$$\alpha_{\mathrm{em}}^{-1} = \alpha_1^{-1} + \alpha_2^{-1}, \quad \text{and} \quad \tan^2 \theta_\mathrm{W} = \frac{\alpha_1}{\alpha_2}, \tag{3}$$

valid at all energy scales, and by the experimental values

$$\begin{aligned} \alpha_{\mathrm{em}}^{-1} &= 127.90 \pm 0.09 \text{ [1], [6]}, \\ \sin^2 \theta_\mathrm{W} &= 0.2315 \pm 0.0002 \text{ [1]} \quad \text{and} \\ \alpha_3 = \alpha_s &= 0.1123 \pm 0.006 \text{ [1], [7]}. \end{aligned} \tag{4}$$

The unification of the SM gauge coupling constants is achieved if they merge together into a common value $\alpha = g^2/4\pi$ at a certain energy scale $M$, where $g$ is the gauge coupling constant of the unifying group $G$. However, since $G \supset G_s$, the normalization of the generators corresponding to the subgroups $U(1)_\mathrm{Y}$, $SU(2)_\mathrm{L}$ and $SU(3)_\mathrm{c}$ is in general different for each particular group $G$, and therefore the SM coupling constants $\alpha_i$ differ at the unification scale from $\alpha$ by numerical factors $c_i$ ($\alpha_i = c_i \alpha$) which are pure rational numbers satisfying $c_i \leq 1$ (due to the normalization of the generators in $G$). For example, in $SU(5)$, $c_1 = \frac{3}{5}$ and $c_2 = c_3 = 1$. These values are the same in $SO(10)$ [8] and $E_6$ [9], but they are different for other cases which do not contain $G_s$ embedded into an $SU(5)$ subgroup [3] as is the case for $E_7$ [10], $SU(5) \otimes SU(5)$ [11], $[SU(6)]^3 \times Z_3$ [12], $[SU(6)]^4 \times Z_4$ [13], $SU(8) \otimes SU(8)$ [14] or the Pati-Salam models [15].

The constants $c_i$ are fixed once we fix the unifying gauge structure. Then, from eq. (3) it follows that at the unification scale the value of $\sin^2 \theta_\mathrm{W}$ is given by

$$\sin^2 \theta_\mathrm{W} = \frac{\alpha_{\mathrm{em}}}{\alpha_2} = \frac{c_1}{c_1 + c_2}. \tag{5}$$

In this paper we shall consider for $c_3$ only two values, $c_3 = 1$ for those models which contain $SU(3)_\mathrm{c}$ embedded into a simple group, or $c_3 = \frac{1}{2}$ for those which contain $SU(3)_\mathrm{c}$ embedded into the chiral color extension $SU(3)_{\mathrm{cL}} \otimes SU(3)_{\mathrm{cR}}$ [16].

To compute the $b_i$ coefficients in the rge we will assume that only the standard particles are light so that, according to the decoupling theorem [17], only they contribute. We obtain

$$2\pi \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{22}{3} \\ 11 \end{pmatrix} - \begin{pmatrix} \frac{20}{9} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix} F - \begin{pmatrix} \frac{1}{6} \\ \frac{1}{6} \\ 0 \end{pmatrix} H, \tag{6}$$

where $F$ is the number of families and $H$ is the number of low-energy complex Higgs doublets (whose contribution was neglected in the early analysis, see for example the first references in [4], [8], [9], [11]). Notice that we are not including in the former equation the normalization factor $\frac{3}{5}$ into $b_1$ coming from the $SU(5)$ theory and wrongly included in some general discussions. In the minimal SM, $F = 3$ and $H = 1$. Nevertheless, a more general model could have more than one low energy Higgs field doublet, then $H$ may be taken as a free parameter. Notice also that we are including in our analysis only doublet Higgs fields, due to the facts that singlets do not contribute to the rge, and the presence of higher multiplets may spoil the $\Delta I = 1/2$ weak isospin rule.

The solutions to (1) are

$$\alpha_i^{-1}(m_Z) = \frac{1}{c_i}\alpha^{-1} - b_i(F,H)\,\ln\left(\frac{M}{m_Z}\right), \tag{7}$$

which for $i = 1, 2, 3$ constitute a system of three equations with the unification variables $\alpha$, $M$ and $H$ as the three unknowns (for $F = 3$ families). The system of equations (7) may be solved for these variables as a function of the numerical factors $c_i$ and the experimental values for $\alpha_i$ at the $m_Z$ scale. The solution is unique for each set of values $\{c_1, c_2, c_3\}$ characteristic of each model. For $c_3 = 1$ (and also for $c_3 = \frac{1}{2}$), the solutions to (7) produce three families of curves in the $c_1$-$c_2$ plane defined by the equations $\alpha(c_1, c_2) = c_\alpha$, $H(c_1, c_2) = c_H$ and $M(c_1, c_2) = c_M$, where $c_\alpha, c_H$ and $c_M$ are arbitrary constant values. Each curve in each family is then characterized by the numerical constant $c_\alpha, c_H$ and $c_M$, respectively. As a consequence, for each point in the plane $(c_1, c_2)$ corresponds unique values for the unification variables associated with those curves which intersect at that particular point.

Now, there exist some experimental and theoretical bounds for the possible values of the unification variables. First, the unification scale $M$ must be lower than the Planck scale $M_{\rm P} \sim G_N^{1/2} \sim 10^{19}$ GeV, and also it must be greater than $10^5$ GeV in order to agree with the experimental bounds on FCNC [1]. Also, since some models predict proton decay, and the experimental bound for the proton lifetime $\tau_{\rm p}$ is $\tau_{{\rm p}\to{\rm e}\pi} \sim M^4 > 10^{32}$ y, then $M$ must be greater than $10^{16}$ GeV if the proton is unstable in the model under consideration. Hence, in the analysis we have to consider two different zones in the $c_1$-$c_2$ plane, given by $10^{16}$ GeV $< M < M_{\rm P}$ and $10^5$ GeV $\leq M \leq 10^{16}$ GeV, admitting and not admitting proton decay, respectively. Next, because $b_3 > 0$ and $b_1 < 0$ always, $\alpha_1(m_Z) < \alpha < \alpha_s(m_Z)/c_3$ and thus $\alpha$ is finite. Hence, as $\ln(M/m_Z)$ is also finite, from (7) we deduce that $H$ should be also finite and then there is an upper bound $H_{\max}$ which represents the maximum number of low-energy Higgs doublets allowed. Therefore, $0 \leq H \leq H_{\max}$. These bounds limit the region in the $c_1$-$c_2$ plane where the coupling constant unification is possible and consistent with the experimental data and theoretical requirements. Notice also that $H$ can take only integer values.

The solutions of eqs. (7) for $\alpha, H$ and $M$ are

$$\alpha^{-1} = c_1c_2c_3 \cdot \frac{(\alpha_1^{-1} - \alpha_2^{-1})(99 - 12F) + \alpha_3^{-1}(8F + 66)}{c_1c_2(8F + 66) + c_1(c_1 - c_2)(12F - 99)}, \tag{8}$$

$$H = \frac{2}{3} \cdot \frac{c_2(\alpha_1^{-1}c_1 - \alpha_3^{-1}c_3)(66 - 12F) + c_3(\alpha_1^{-1}c_1 - \alpha_2^{-1}c_2)(12F - 99) + 20c_1(\alpha_2^{-1}c_2 - \alpha_3^{-1}c_3)}{c_1c_2(\alpha_1^{-1} - \alpha_2^{-1}) + \alpha_3^{-1}c_3(c_1 - c_2)}, \tag{9}$$

$$\ln\left(\frac{M}{m_Z}\right) = 18\pi \cdot \frac{c_1c_2(\alpha_1^{-1} - \alpha_2^{-1}) + \alpha_3^{-1}c_3(c_1 - c_2)}{c_1c_2(8F + 66) + c_1(c_1 - c_2)(12F - 99)}. \tag{10}$$

From these expressions, the limited region obtained for values of $c_1$ and $c_2$ that give unification is plotted in fig. 1 for $c_3 = 1$ and in fig. 2 for $c_3 = \frac{1}{2}$, where we used $F = 3$ for three families, and central values for $\alpha_s$, $\alpha_{\rm em}$ and $\sin^2\theta_{\rm W}$. Let us see the consequences of those graphs.

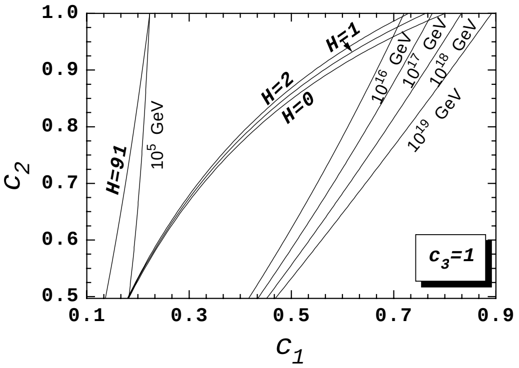


Fig. 1

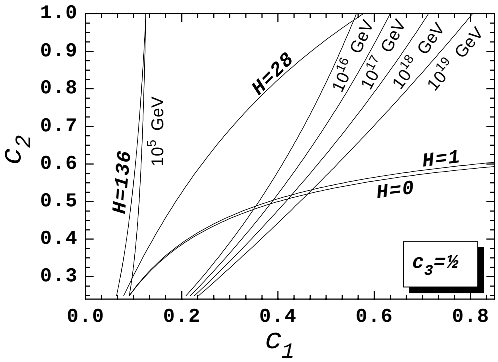


Fig. 2

Fig. 1. – Plots for some values of $H$ and $M$ for the non-chiral color models. The bounds in $c_1$ and $c_2$ impose at once for $\alpha$ the bounds $16.5454 < \alpha^{-1} < 48.4809$.

Fig. 2. – Plots for some values of $H$ and $M$ for GUTs containing the chiral color extension. In this case we have $8.27269 < \alpha^{-1} < 26.1967$.

*Analysis of fig. 1.* – It corresponds to the case of a GUT group which does not include chiral color symmetries. The allowed region of parameters $(c_1, c_2)$ lies inside the lines $M = 10^5$ GeV s, $H = 0$ and $c_2 = 1$. There is a maximum unification mass scale possible given by $M \leq 10^{17.5}$ GeV s $< M_{\rm P}$ and the number of Higgs field doublets allowed is such that $0 < H \leq 91$ in general, but if the proton does decay in the context of the GUT model, then $0 < H \leq 2$. Let us see the implications of this for some specific models:

1) $SU(5)$. For all the models in this group proton decay is always present [4], and $(c_1, c_2) = (\frac{3}{5}, 1)$ which lies inside the allowed zone, but in a region where $M = 10^{13}$ GeV s. Hence the $SU(5)$ GUT scale $M$ is in conflict with the bounds for proton decay. Since $SU(5)$ allows only the one-step symmetry-breaking chain (sbc) $SU(5) \xrightarrow{M} SM$, $SU(5)$ is ruled out in general. That is, the experimental bounds on proton decay rule out not only minimal $SU(5)$ but also all the possible extensions which include arbitrary representations of Higgs field multiplets.

2) $SO(10)$. As for the previous model, proton decay is always present for this group [8] and $(c_1, c_2) = (\frac{3}{5}, 1)$. Therefore the one-step sbc $SO(10) \xrightarrow{M} SM$ is ruled out. From our analysis nothing can be said about the two-stage sbc $SO(10) \xrightarrow{M} SU(4)_c \otimes SU(2)_L \otimes SU(2)_R \xrightarrow{M'} SM$.

3) $SU(4)_{\rm c} \otimes SU(2)_{\rm L} \otimes SU(2)_{\rm R}$. This group can be viewed as a subgroup of $SO(10)$ [8], or as a subgroup of the Pati-Salam model [15], or either as a non-simple unification model by itself. For this group $(c_1, c_2) = (\frac{3}{5}, 1)$ again. In this model, the proton cannot decay via leptoquark gauge bosons (see the first paper in [15]), but it can decay via Higgs fields scalars. So, the one-stage breaking of this model is not ruled out as long as one can break the symmetry using scalars which do not break spontaneously the baryon quantum number $B$.

4) $E_6$. Proton decay is always present for this group [9], and $(c_1, c_2) = (\frac{3}{5}, 1)$ also. So, the one-step sbc $E_6 \xrightarrow{M} SM$ is ruled out. Nothing can be said for the multistage sbc.

5) $SU(3)_{\rm L} \otimes SU(3)_{\rm c} \otimes SU(3)_{\rm R}$. This group can be viewed as a subgroup of $E_6$ [9] or as a unification model by itself (the trinification model of Georgi-Glashow-de Rujula [18]). Again $(c_1, c_2) = (\frac{3}{5}, 1)$ and the proton decay in the model is only Higgs-boson mediated. *The one-stage breaking of this model is not ruled out* as long as one can break the symmetry using scalars which do not break $B$ spontaneously (see the second paper in [18]).

6) $SO(18)$. Proton decay is always present for this group, and $(c_1, c_2) = (\frac{3}{5}, 1)$ [19]. The conclusions are the same as for $E_6$.

7) $[SU(6)]^3 \times Z_3$. The proton is stable in the context of this model [12]. For this group $(c_1, c_2) = (\frac{3}{14}, \frac{1}{3})$, which lies outside the allowed zone, and the one-stage sbc is ruled out (the two-stage sbc for the model is presented in the last paper of ref. [12]).

*Analysis of fig. 2.* – It corresponds to the case of a GUT group which includes chiral color symmetries. The allowed region in the plane $(c_1, c_2)$ lies inside the lines $M = 10^5$ GeV s, $H = 0$, $M = 10^{19}$ GeV s $= M_\mathrm{P}$ and $c_2 = 1$. Therefore, there is no bound for a maximum unification mass scale and the allowed number of Higgs field doublets is such that $0 < H \le 136$ in general, but if the proton does decay in the context of the GUT model, then $0 < H \le 28$. Let us see the implications of this for some specific models:

1) $SU(5) \otimes SU(5)$. Proton decay is mediated via gauge and Higgs bosons for the models in this group [11]. $(c_1, c_2) = (\frac{3}{13}, 1)$, which lies inside the allowed zone but in a region where $M \ll 10^{16}$ GeV s, in serious conflict with bounds for proton decay. The models are all ruled out.

2) $[SU(6)]^4 \times Z_4$. The proton is stable in the context of the model presented in ref. [13]. For this group $(c_1, c_2) = (\frac{3}{19}, \frac{1}{3})$, which lies inside the allowed zone. So, the *one-stage sbc for this model is also possible*, and is presented in ref. [13].

We mention that our analysis has been done assuming non-supersymmetric unification. Also, we have neglected threshold effects which depend on the particular structure of each model, we have not included second-order corrections to the rge which are typically of the order of 1 to 10%, and we have not included the experimental errors of the SM gauge coupling constants.

The previous analysis allows us to conclude that it is indeed possible to achieve the unification of the coupling constants of the SM in one step in a general class of non-supersymmetric models. Two particular models with simple unifying groups were singled out: the trinification model of Georgy-Glashow-de Rujula [18] for GUT groups which do not include chiral color symmetry, and the model in ref. [13] for GUT models with chiral color symmetry.

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