PHYSICAL REVIEW D VOLUME 23, NUMBER 5 1 MARCH 1981

Effective weak Hamiltonian for the $\Delta b = 1$ nonleptonic decays in the six-quark model

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(Received 29 August 1980)

Quantum-chromodynamic corrections and flavor-symmetry-breaking effects are considered in the leading-logarithmic approximation for the calculation of the $\Delta b = 1$ nonleptonic effective weak Hamiltonian in the context of the Kobayashi-Maskawa model. It is found that flavor-symmetry breaking is very important for the cases under consideration here.

I. INTRODUCTION

The discovery of the Υ particle[1] has been interpreted as evidence for a new quark flavor, named the b quark. The simplest way to accommodate this fifth flavor in the standard $SU(2)_L \times U(1)$ model[2] is by postulating a sixth quark (the t quark). The six-quark model, known as the Kobayashi-Maskawa (KM) model,[3] ensures proper Glashow-Iliopoulos-Maiani[4] (GIM) cancellations by describing the mixing between quarks by three Cabibbo-type angles θ_1, θ_2, and θ_3, and by a single phase δ, which results in CP violations.

Although the search for the t quark has been fruitless so far,[5] and there are already exotic and nonexotic models in which there is no t flavor, it is not yet time to abandon the standard KM model. It could be the case that the mass for the t quark is in the range of energy above 30 GeV, and we will have to wait until the next generation of accelerators before the t quark shows up in the laboratory. Because such a wait could take several years, the best way to establish or to eliminate the standard model could be by studying b decays.

In this paper we calculate the effective weak Hamiltonian for the $\Delta b = 1$ nonleptonic decays in the context of the KM model, including quantum-chromodynamics[6] (QCD) renormalization effects, when flavor-symmetry breaking (FSB) is considered. For the b sector the FSB effects enter, not only via the penguin diagrams in Fig. 1 (as originally proposed by Shifman, Vainshtein, and Zakharov[7] for the strange sector), but they also enter via the box diagrams in Fig. 2 as proposed in Ref. 8, and shown in more detail in Appendix A. The presence of FSB makes the whole scheme quite involved and complicated, but the calculations can be carried out in a mathematically well defined way, giving exact results in the leading-logarithmic approximation. Calculations analogous to the present one have been presented in Ref. 9 for the strange sector, where FSB effects enter only via the penguin diagrams, and in Ref. 8 for the Cabibbo-nonsuppressed charm sector, where FSB effects enter only via the box diagrams.

In the KM model, the current $J_\alpha^{\rm wk}$ that couples to the $W^\pm$ bosons with strength f is

$$J_\alpha^{\rm wk} = (\bar{u}, \bar{c}, \bar{t})\Gamma_\alpha R \begin{pmatrix} d \\ s \\ b \end{pmatrix} + \text{H.c.}, \tag{1}$$

where R is the matrix

$$R = \begin{pmatrix} C_1 & S_1C_3 & S_1S_3 \\ -S_1C_2 & C_1C_2C_3 - S_2S_3e^{i\delta} & C_1C_2S_3 + S_2C_3e^{i\delta} \\ S_1S_2 & -C_1S_2C_3 - C_2S_3e^{i\delta} & C_2C_3e^{i\delta} - C_1S_2S_3 \end{pmatrix}, \tag{2}$$

$$\Gamma_\alpha = \gamma_\alpha(1+\gamma_5).$$

C_i (S_i) = $\cos\theta_i$ ($\sin\theta_i$), $i = 1, 2, 3$ are the cosines and sines of the generalized Cabibbo angles, δ is a real parameter that allows for CP violations, and γ are Dirac matrices.

The matrix in (2) must be unitary; as a consequence of its unitarity the columns must be orthonormal. This orthogonality for the different columns is the manifestation of the GIM mechanism for the different cases under consideration

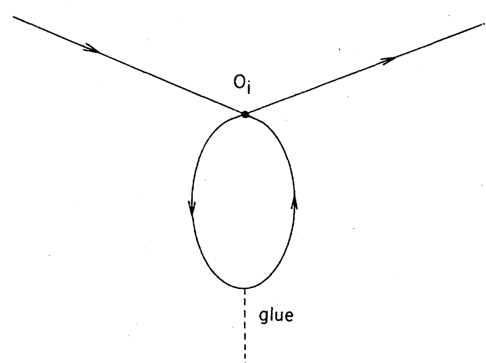

FIG. 1. Lowest-order penguin diagram contributing to the anomalous dimension of O_i.

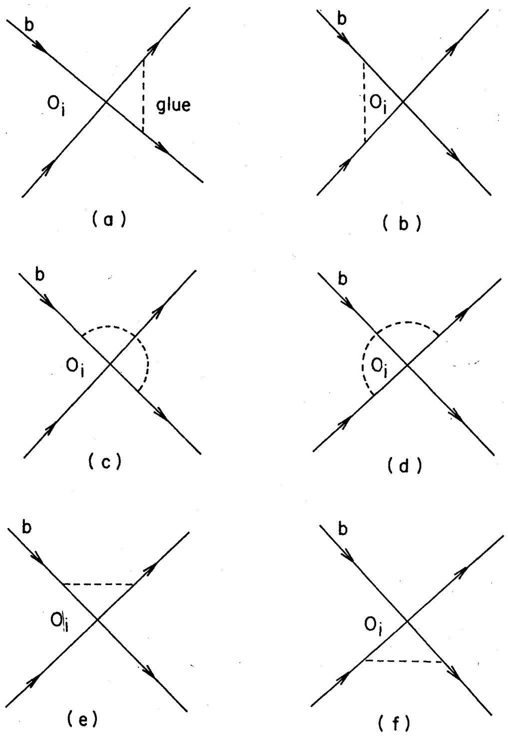

FIG. 2. Lowest-order box diagrams contributing to the anomalous dimension of O_i.

here.

In the absence of the strong-interaction renormalization effects, and according to the Wilson-expansion technique,[10] the $\Delta b = 1$ Hamiltonian

$$H_{\rm wk} = -f^2 \int d^4x\, D(x, M_{\rm W}^2)\, T(J_\alpha^{\rm wk\dagger}(x/2) J_{\rm wk}^{\alpha}(-x/2)) \tag{3}$$

has six different pieces:

(a) $\Delta s = 0,\ \Delta b = -\Delta c = 1$:

$$H_1 = \frac{G}{\sqrt{2}} R_{11}^* R_{23}\, \bar{d} u\, \bar{c} b\,; \tag{4a}$$

(b) $\Delta c = 0,\ \Delta b = \Delta s = 1$:

$$H_2 = \frac{G}{\sqrt{2}} (R_{12}^* R_{13}\, \bar{s} u\, \bar{u} b + R_{22}^* R_{23}\, \bar{s} c\, \bar{c} b + R_{32}^* R_{33}\, \bar{s} t\, \bar{t} b)\,; \tag{4b}$$

(c) $\Delta c = \Delta s = 0,\ \Delta b = 1$:

$$H_3 = \frac{G}{\sqrt{2}} (R_{11}^* R_{13}\, \bar{d} u\, \bar{u} b + R_{21}^* R_{23}\, \bar{d} c\, \bar{c} b + R_{31}^* R_{33}\, \bar{d} t\, \bar{t} b)\,; \tag{4c}$$

(d) $\Delta b = \Delta s = -\Delta c = 1$:

$$H_4 = \frac{G}{\sqrt{2}} R_{12}^* R_{23}\, \bar{s} u\, \bar{c} b\,; \tag{4d}$$

(e) $\Delta b = \Delta c = \Delta s = 1$:

$$H_5 = \frac{G}{\sqrt{2}} R_{13}^* R_{22}\, \bar{s} c\, \bar{u} b\,; \tag{4e}$$

(f) $\Delta s = 0,\ \Delta b = \Delta c = 1$:

$$H_6 = \frac{G}{\sqrt{2}} R_{21}^* R_{13}\, \bar{d} c\, \bar{u} b\,, \tag{4f}$$

where R_{ij} is the ij element of R in Eq. (2), $G/\sqrt{2} = f^2/M_{\rm W}^2$ is the Fermi weak coupling constant, $M_{\rm W}$ is the mass of the intermediate vector boson, $\bar{a} b\, \bar{c} d$ refers to the color-singlet structure $\bar{a}_i \Gamma^\alpha b_i\, \bar{c}_j \Gamma_\alpha d_j$ (sum over i and j understood), and u, d, s, c, b, and t refer to quark fields.

II. THE $\Delta s = 0$, $\Delta b = -\Delta c = 1$ EFFECTIVE HAMILTONIAN

When the strong interactions are present, the Wilson expansion for the Hamiltonian in Eq. (3) produces different results than the results presented above. Only at very high energies, and due to the asymptotic-freedom property of QCD, do we recover the results in Eq. (4). For the weak interactions there is a natural threshold for what high energies mean[11]; it means energies in the range $\infty > \kappa > M_{\rm W}$, i.e., for energies above $M_{\rm W}$, the strong interactions are negligible and the effective weak Hamiltonian for the $\Delta b = 1$ sector is properly described by Eq. (4).

For energies below $M_{\rm W}$, the strong-interaction renormalization effects will change drastically the weak Hamiltonian. Not only do the operators in (4) pick up an extra coefficient, but new operators come into play with coefficients that are calculated as solutions to renormalization-group equations (RGE's), i.e.,

$$H_{\rm wk}^{\rm eff} = \sum_i A_i O_i\,, \tag{5}$$

where the expansion in (5) is infinite in principle, but for short distances it is expected to converge very rapidly, and only the lowest-dimension operators O_i, which do not renormalize away, are of relevance here. The coefficients A_i are functions of $M_{\rm W}$, of the subtraction point μ_0 of the theory, and for the case of FSB they are also functions of the heavy quark masses m_t, m_b, m_c.

The coefficients A_i in Eq. (5) are solutions to the RGE:

$$\left(\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} + \sum_i m_i \delta_i(g) \frac{\partial}{m_i} + \gamma_J + \gamma_{J^\dagger} - \gamma_{O_i}\right) A_i = 0\,, \tag{6}$$

where $\beta(g) = \partial g/\partial \mu$, $\delta_i = (\mu/m_i)\partial m_i/\partial \mu$, γ_J is the anomalous dimension for the weak current, γ_{O_i} is the anomalous dimension for the operator O_i, and g is the strength of the strong interaction.

In this paper we are interested in solutions to the RGE in the energy range $m_i < \kappa < m_j$, where m_i and m_j are mass scales of the theory. For a detailed solution to Eq. (6) for such energy ranges see, for example, Ref. 9 and especially Ref. 12. Following Ref. 12, let us define

$$\begin{aligned} \kappa_1 &= 1 - 2b_1 g^2 \ln (m_c/\mu_0), \\ \kappa_2 &= 1 - 2b_2 \frac{g^2}{\kappa_1} \ln (m_b/m_c), \\ \kappa_3 &= 1 - 2b_3 \frac{g^2}{\kappa_1\kappa_2} \ln (m_t/m_b), \\ \kappa_4 &= 1 - 2b_4 \frac{g^2}{\kappa_1\kappa_2\kappa_3} \ln (M_W/m_t), \end{aligned} \qquad (7)$$

where

$$b_i = (\tfrac{2}{3} n_i - 11)/16\pi^2,$$

$$n_1 = 3,\ n_2 = 4,\ n_3 = 5,\ n_4 = 6.$$

As mentioned above, for the energy range $\kappa > M_W$, the effective weak Hamiltonian for the $\Delta s = 0$, $\Delta b = -\Delta c = 1$ sector is given by Eq. (4a).

For $\kappa > m_t$, the RGE must be solved for the interval $M_W > \kappa > m_t$ and the solution at $\kappa = M_W$ must be matched with Eq. (4a). According to the Appelquist-Carazzone theorem,[13] the six quarks are present in this interval, and according to the RGE techniques, the operators that renormalize multiplicatively are

$$O_\pm = \bar{d}u\bar{c}b \pm \bar{d}b\bar{c}u .$$

The anomalous dimensions for the weak currents are zero. The six diagrams in Fig. 2 do contribute to the anomalous dimension for $O_\pm$, but the penguin diagrams do not enter at all for the sector under consideration here. After the algebra is done, the anomalous dimensions for the operators $O_\pm$ are $\gamma_\pm = c_\pm g^2$, where

$$\begin{aligned} c_+ &= 4/16\pi^2, \\ c_- &= -8/16\pi^2 . \end{aligned} \qquad (9)$$

$\delta_i = 0$ for the six flavors, and

$$H_1^{\text{eff}} (\kappa > m_t) = a_1 (O_- \kappa_4{}^{c_-/2b_4} + O_+ \kappa_4{}^{c_+/2b_4}) \qquad (10)$$

where $a_1 = R_{11}^* R_{23} (G/2\sqrt{2})$. Notice that in the limit when $\kappa = M_W$, $\kappa_4 = 1$ and Eq. (10) reduces to Eq. (4a) as it should.

For $\kappa > m_b$, the RGE must be solved in the interval $m_t > \kappa > m_b$ and the solution at $\kappa = m_t$ must be matched with Eq. (10). The situation here is very similar to the case above; the only difference is that only five flavors are present (the t quark is too heavy and does not enter into the solution to the RGE according to the Appelquist-Carazzone theorem[13]). Because of this $\delta_t = -8g^2/16\pi^2$, δ_i = zero for $i \neq t$. Thus, up to this point we have

$$H_1^{\text{eff}} (\kappa > m_b) = a_1 (O_- \kappa_4{}^{c_-/2b_4} \kappa_3{}^{c_-/2b_3} + O_+ \kappa_4{}^{c_+/2b_4} \kappa_3{}^{c_+/2b_3}) . \qquad (11)$$

So far the FSB effects have not been present and the Hamiltonian in Eq. (11) coincides with previous calculated values.[14]

For $\kappa > m_c$, the RGE must be solved in the interval $m_b > \kappa > m_c$. Here the FSB effects are present via the box diagrams, and only the diagrams in Figs. 2(a), 2(d), and 2(f) contribute to the anomalous dimension for $O_\pm$. The fact that the other three diagrams in Fig. 2 do not contribute is explained in Ref. 8 and also in the Appendix at the end of this paper. In this interval only four quarks are present,

$$\delta_t = \delta_b = -8g^2/16\pi^2 ,$$

$$\delta_i = \text{zero} \quad \text{for } i \neq t, b .$$

$\gamma_{\bar{d}u} = 0$ because both quarks, d and u, are light, but $\gamma_{\bar{c}b} = g^2/12\pi^2$ because, for the interval under consideration here, one of the quarks is heavy and the other one is light. Again $O_\pm$ renormalize multiplicatively, their anomalous dimensions being

$$\begin{aligned} c'_+ &= 2/16\pi^2 , \\ c'_- &= -4/16\pi^2 , \end{aligned} \qquad (12)$$

and

$$H_1^{\text{eff}} (\kappa > m_c) = a_1 (\epsilon O_- + \epsilon^{-1/2} O_+) , \qquad (13)$$

where

$$\epsilon = \kappa_4{}^{c_-/2b_4} \kappa_3{}^{c_-/2b_3} \kappa_2{}^{c'_-/2b_2} .$$

If the subtraction point μ_0 is in the neighborhood of m_c, then the Hamiltonian in Eq. (13) is the final expression we were looking for. But if $\mu_0 \ll m_c$, then one more interval must be considered: $m_c > \kappa > \mu_0$. For this interval

$$\delta_t = \delta_b = \delta_c = -8g^2/16\pi^2,$$

$$\delta_s = \delta_d = \delta_u = 0 ,$$

the anomalous dimensions for both currents are zero. For the operator $\bar{d}u\bar{c}b$ only the diagram in Fig. 1(f) contributes, and for the operator $\bar{d}b\bar{c}u$ the diagram in Fig. 1(d) does. For this interval the operators $O_\pm$ do not renormalize multiplicatively, the mixing matrix being

$$\begin{pmatrix} O_- \\ O_+ \end{pmatrix} \to \left[1 + 2 \begin{pmatrix} 1 & -\frac{1}{2} \\ -1 & \frac{1}{2} \end{pmatrix} \frac{g^2}{16\pi^2} \ln (m_c/\mu_0) \right] \begin{pmatrix} O_- \\ O_+ \end{pmatrix} .$$

The final expression for the Hamiltonian is

$$H_1^{\text{eff}}(\kappa>\mu_0)=a_1\{O_-[\tfrac{1}{3}\epsilon(1+2\kappa_1{}^{c/2b_1}) + \tfrac{2}{3}\epsilon^{-1/2}(1-\kappa_1{}^{c/2b_1})] + O_+[\tfrac{1}{3}\epsilon(1-\kappa_1{}^{c/2b_1}) + \tfrac{1}{3}\epsilon^{-1/2}(2+\kappa_1{}^{c/2b_1})]\}, \quad (14)$$

where $c=3/16\pi^2$.

In order to get numerical results, let us use the following set of parameters: $g^2(\mu_0)/4\pi=1$, $M_W = 100$ geV, $m_t=30$ GeV, $m_b=5$ GeV, $m_c=2$ GeV, and $\mu_0=1$ GeV. For this particular set of parameters we have

$$H_1^{\text{eff}}=a_1(1.60O_- + 0.81O_+) \text{ for } \kappa>\mu_0 \text{ and } \mu_0\ll m_c,$$

$$H_1^{\text{eff}}(\kappa>m_c\approx\mu_0)=a_1(1.67O_- + 0.77O_+).$$

III. THE $\Delta c=0$, $\Delta b=\Delta s=1$ EFFECTIVE HAMILTONIAN

For this sector the FSB effects enter via the box and the penguin diagrams. For the energy range $\kappa>M_W$, the effective weak Hamiltonian is given by Eq. (4b). Using the property $R_{12}^*R_{13}+R_{22}^*R_{23}+R_{32}^*R_{33}=0$, we can write

$$\begin{aligned}H_2&=-\frac{G}{\sqrt{2}}R_{22}^*R_{23}(\bar{s}u\bar{u}b-\bar{s}c\bar{c}b)\\&\quad+R_{32}^*R_{33}(\bar{s}u\bar{u}b-\bar{s}t\bar{t}b)\\&=a_2(O_{c+}+O_{c-})+a_2'(O_{t+}+O_{t-}),\end{aligned} \quad (15)$$

where

$$O_{q\pm}=\bar{s}u\bar{u}b\pm\bar{s}b\bar{u}u-(u\to q),$$

$$a_2=-GR_{22}^*R_{23}/2\sqrt{2},$$

and

$$a_2'=-GR_{32}^*R_{33}/2\sqrt{2}.$$

For the interval $\kappa>m_t$, the situation here is similar to the situation in the same interval of the previous section. The operators $O_{c\pm}$ and $O_{t\pm}$ renormalize multiplicatively and the effective weak Hamiltonian is

$$H_2^{\text{eff}}(\kappa>m_t)=\kappa_4{}^{c_+/2b_4}(a_2O_{c+}+a_2'O_{t+}) + \kappa_4{}^{c_-/2b_4}(a_2O_{c-}+a_2'O_{t-}), \quad (16)$$

where $c_\pm$ are as given in Eq. (9).

For $\kappa>m_b$, let us divide H_2 in Eq. (16) into three pieces:

$$H_{2a}=-a_2'(\xi_1\bar{s}b\bar{t}t+\xi_2\bar{s}t\bar{t}b),$$

$$H_{2b}=a_2(\kappa_4{}^{c_+/2b_4}O_{c+}+\kappa_4{}^{c_-/2b_4}O_{c-}),$$

$$H_{2c}=a_2'(\xi_1\bar{s}b\bar{u}u+\xi_2\bar{s}u\bar{u}b),$$

where

$$\xi_1=\kappa_4{}^{c_+/2b_4}-\kappa_4{}^{c_-/2b_4},$$

$$\xi_2=\kappa_4{}^{c_+/2b_4}+\kappa_4{}^{c_-/2b_4}.$$

If hadrons are made mostly of valence quarks, H_{2a} is not important in the $\Delta b=1$ sector, and it will not be considered here further; however, H_{2a} has to be considered in the decays for the t quark. Let us call H_{2a} the induced Hamiltonian via penguin diagrams in the t sector, from the $\Delta b=1$ effective nonleptonic weak Hamiltonian. As a matter of fact, there are also induced Hamiltonians in the b sector from the $\Delta c=1$ and $\Delta s=1$ effective Hamiltonians. Such pieces are not considered here. In the energy range $m_t>\kappa>m_b$, H_{2b} renormalizes multiplicatively, and no penguin diagrams are involved due to the GIM cancellation between the u and c quark. The effective Hamiltonian for H_{2b} will have the form

$$H_{2b}^{\text{eff}}(\kappa>m_b)=a_2(\kappa_4{}^{c_+/2b_4}\kappa_3{}^{c_+/2b_3}O_{c+} + \kappa_4{}^{c_-/2b_4}\kappa_3{}^{c_-/2b_3}O_{c-}). \quad (17)$$

For H_{2c}, the GIM cancellation between the t and u quark is not effective any longer, and the FSB effects enter via the penguin diagrams. For the renormalization of H_{2c} let us define, following Ref. 9, the six operators:

$$\begin{aligned}O_1&=\bar{s}b\bar{u}u,\\O_2&=\bar{s}u\bar{u}b,\\O_3&=\bar{s}b(\bar{u}u+\bar{d}d+\bar{s}s+\bar{c}c+\bar{b}b),\\O_4&=\bar{s}(u\bar{u}+d\bar{d}+s\bar{s}+c\bar{c}+b\bar{b})b,\\O_5&=\bar{s}b(\bar{u}'u'+\bar{d}'d'+\bar{s}'s'+\bar{c}'c'+\bar{b}'b'),\\O_6&=\bar{s}_ib_j(\bar{u}_j'u_i'+\bar{d}_j'd_i'+\bar{s}_j's_i'+\bar{c}_j'c_i'+\bar{b}_j'b_i'),\end{aligned} \quad (18)$$

where the current $\bar{q}'q'$ refers to the right-handed structure $\bar{q}\gamma^\alpha(1-\gamma_5)q$. The mixing matrix for $O_1\cdots O_6$ is given by (B1) in Appendix B.

After the algebra is done we have for H_{2c}^{eff} the following expression:

$$H_{2c}^{\text{eff}}(\kappa>m_b)=a_2'\sum_{l=1}^{2}\sum_{i,j=1}^{6}\xi_lR^{-1}{}_{li}K_iR_{ij}O_j, \quad (19)$$

where R is given by Eq. (B3), K_i is the six-component column vector

$$K_i=\kappa^{V_i/16\pi^2b_3},$$

and V_i is given by Eq. (B2). A numerical evaluation of the coefficients gives

$$H_2^{\rm eff}(\kappa > m_b) = a_2(1.489O_{c-} + 0.820O_{c+}) + a_2'(-0.239O_1 + 2.222O_2 + 0.020O_3 - 0.047O_4 + 0.014O_5 - 0.056O_6) \quad (20)$$

for the set of parameters quoted in the former section.

For $\kappa > m_c$, we solve the RGE for the operators in H_{2b} and H_{2c} in the energy range $m_b > \kappa > m_c$. For H_{2b}, the GIM cancellation between the u and c quarks is still in effect and no penguin diagrams are involved, but the FSB effects are present via the box diagrams, due to the fact that in the four quark operators there is one heavy quark (the b quark) and three light quarks. Still $O_{c\pm}$ renormalize multiplicatively, but their anomalous dimensions are the $c_\pm'$ of Eq. (12). So we have

$$H_{2b}^{\rm eff}(\kappa > m_c) = a_2(\epsilon O_{c-} + \epsilon^{-1/2}O_{c+}), \quad (21)$$

where ϵ is as defined in Eq. (13).

For the renormalization of H_{2c} in the energy range $m_b > \kappa > m_c$, it is convenient to decompose it in two pieces:

$$H_{2c'}^{\rm eff}(\kappa > m_b) = a_2' \sum_{l=1}^{2} \sum_{i,j=1}^{6} \xi_l R^{-1}{}_{li} K_i R_{ij} O_j',$$

$$H_{2d}^{\rm eff}(\kappa > m_b) = a_2' \sum_{l=1}^{2} \sum_{i=1}^{6} \sum_{j=3}^{6} \xi_l R^{-1}{}_{li} K_i R_{ij} \theta_j',$$

where

$$\begin{aligned} &O_1' = \bar{s}b\bar{u}u, \quad O_2' = \bar{s}u\bar{u}b, \\ &O_3' = \bar{s}b(\bar{u}u + \bar{d}d + \bar{s}s + \bar{c}c), \\ &O_4' = \bar{s}(u\bar{u} + d\bar{d} + s\bar{s} + c\bar{c})b, \\ &O_5' = \bar{s}b(\bar{u}'u' + \bar{d}'d' + \bar{s}'s' + \bar{c}'c'), \\ &O_6' = \bar{s}_i b_j(\bar{u}_j'u_i' + \bar{d}_j'd_i' + \bar{s}_j's_i' + \bar{c}_j'c_i'), \\ &\theta_3' = \theta_4' = \bar{s}b\bar{b}b, \quad \theta_5' = \bar{s}b\bar{b}'b', \quad \theta_6' = \bar{s}_i b_j \bar{b}_j' b_i'. \end{aligned} \quad (22)$$

Because H_{2d} has three heavy quarks now, it does not renormalize any further. For the renormalization of $H_{2c'}$, the operator basic O_1', O_2', O_3', O_4', O_5', O_6', and O_7' must be considered, where

$$\begin{aligned} O_7' &= \bar{s}_i \Gamma^\mu \lambda_{ij}^A b_j \bar{b}_k \gamma_\mu \lambda_{kl}^A b_l \\ &= \tfrac{1}{2}\bar{s}\lambda^A b\bar{b}\lambda^A b + \tfrac{1}{2}\bar{s}\lambda^A b\bar{b}'\lambda^A b'. \end{aligned} \quad (23)$$

For the quark operators in Eqs. (22) and (23) only the b flavor is heavy for the energy range under consideration here. Because O_7' has three heavy quarks, its anomalous dimension is zero and it does not mix with the other operators. Unfortunately in the analysis we can not separate O_7' from the other six operators, because due to the presence of the penguin diagrams, operators $O_1' \cdots O_6'$ mix triangularly with O_7'.

The mixing matrix for the operators $O_1' \cdots O_7'$ is given by (B5), and we get for the effective Hamiltonian

$$H_{2c'}^{\rm eff}(\kappa > m_c) = a_2' \sum_{l=1}^{2} \sum_{i,j=1}^{6} \sum_{s,t=1}^{7} \xi_l R^{-1}{}_{li} K_i R_{ij} R'{}_{js}^{-1} K_s' R_{st}' O_t', \quad (24)$$

where R' is given by Eq. (B7) and K_s' is defined as

$$K_s' = \kappa_2^{V_s'/16\pi^2 b_2},$$

where V_s' is presented in (B6). The numerical evaluation of the Hamiltonians in (21) and (22) gives

$$\begin{aligned} H_2^{\rm eff}(\kappa > m_c) = a_2(1.669O_{c-} + 0.774O_{c+}) &+ a_2'(-0.443O_1' + 2.316O_2' + 0.037O_3' \\ &- 0.084O_4' + 0.025O_5' - 0.105O_6' \\ &- 0.049\theta_3' + 0.025\theta_5' - 0.089\theta_6'), \end{aligned} \quad (25)$$

where we have used

$$O_7' = \tfrac{2}{3}\theta_3' - \tfrac{1}{3}\theta_5' + \theta_6'.$$

Again, if $\mu_0 \approx m_c$, then Eq. (25) is the final expression we are looking for. However, if $\mu_0 \ll m_c$, then the interval $m_c > \kappa > \mu_0$ must be considered. For this interval O_{c+} do not renormalize multiplicatively any longer, and the penguin cancellation between the u and c quark is no longer effective (c is a heavy quark now). H_{2b} and $H_{2c'}$ must be treated on the same basis.

For this interval the following six operators are needed:

$$\begin{aligned} &O_1'' = \bar{s}b\bar{u}u, \quad O_2'' = \bar{s}u\bar{u}b, \\ &O_3'' = \bar{s}b(\bar{u}u + \bar{d}d + \bar{s}s), \\ &O_5'' = \bar{s}b(\bar{u}'u' + \bar{d}'d' + \bar{s}'s'), \\ &O_6'' = \bar{s}_i b_j(\bar{u}_j'u_i' + \bar{d}_j'd_i' + \bar{s}_j's_i'), \\ &O_7'' = \bar{s}_i \Gamma^\mu \lambda_{ij}^A b_j(\bar{c}_K \gamma_\mu \lambda_{Kl}^A c_l + \bar{b}_K \gamma_\mu \lambda_{Kl}^A b_l). \end{aligned} \quad (26)$$

One operator such as $O_4'' = \bar{s}(u\bar{u} + d\bar{d} + s\bar{s})b$ is not needed here because[9] $O_4'' = -O_1'' + O_2'' + O_3''$. Again O_7'' has three heavy flavors and its anomalous dimension is zero, but it mixes triangularly with the other five operators due to the penguin diagrams.

The mixing matrix for $O_1'' \cdots O_7''$ is given by M'' in (B8), and the anomalous dimension vector is proportional to V'' in (B9). For the renormalization of H_{2b} let us divide it into two pieces:

$$H^{\rm eff}_{2b'}(\kappa>m_c)=a_2[\bar s b\bar u u(\epsilon^{-1/2}-\epsilon)+\bar s u\bar u b(\epsilon^{-1/2}+\epsilon)]\,,$$

$$H^{\rm eff}_{2e}(\kappa>m_c)=-a_2[\bar s b\bar c c(\epsilon^{-1/2}-\epsilon)+\bar s c\bar c b(\epsilon^{-1/2}+\epsilon)]\,.$$

Because H_{2e} has three heavy quarks now, it does not renormalize any further. For the renormalization of $H_{2b'}$ the FSB effects enter via the box and the penguin diagrams simultaneously. After the algebra is done we get

$$H^{\rm eff}_{2b'}(\kappa>\mu_0)=a_2\left[(\epsilon^{-1/2}-\epsilon){\sum_{i,j}}' R''^{-1}{}_{1i}K''_i R''_{ij}O''_j + (\epsilon^{-1/2}+\epsilon){\sum_{i,j}}' R''^{-1}{}_{2i}K''_i R''_{ij}O''_j\right]\,, \qquad (27)$$

where R'' is given by (B10), and K''_i is defined by

$$K''_i=\kappa_1{}^{V''_i/16\pi^2 b_1}\,. \qquad (28)$$

${\sum}'_{i,j}$ means i,j runs over 1, 2, 3, 5, 6, 7 (4 excluded), and V''_i is given by (B9). For the renormalization of $H_{2c'}$ in Eq. (24), several things must be done. First, O'_i must be decomposed in $O''_i+\theta''_i$, where θ''_i are operators containing three heavy quarks in the interval $m_c>\kappa>\mu_0$ (as for example $\bar s b\bar c c$, $\bar s c\bar c b$, etc.). Second, O''_4 must be eliminated in favor of O''_1, O''_2, and O''_3. Finally, for the remaining operators make the replacement

$$O''_n\to{\sum_{i,j}}' R''^{-1}{}_{ni}K''_i R''_{ij}O''_j\,,$$

where the parameters are defined as in (27) and (28) above. The final analytic expression gets prohibitively large. Let us quote only the final numerical results for $H_{2b'}$ and $H_{2c'}$:

$$\begin{aligned}H^{\rm eff}_2(\kappa>\mu_0)=&-O''_1(1.164a_2+0.602a'_2)+O''_2(2.598a_2+2.380a'_2)-O''_3(0.039a_2+0.072a'_2)\\&+O''_5(0.020a_2+0.038a'_2)-O''_6(0.070a_2+0.191a'_2)+\bar s b\bar c c(0.914a_2+0.052a'_2)\\&-\bar s c\bar c b(2.504a_2+0.128a'_2)+\bar s b\bar c'c'(0.020a_2+0.039a'_2)-\bar s_i b_j\bar c'_j c'_i(0.061a_2+0.149a'_2)\\&-\bar s b\bar b b(0.041a_2+0.079a'_2)+\bar s b\bar b'b'(0.020a_2+0.040a'_2)-\bar s_i b_j\bar b'_j b'_i(0.061a_2+0.134a'_2)\,.\end{aligned} \qquad (29)$$

IV. THE OTHER SECTORS

For the renormalization of the other four sectors in Eq. (4), most of the equations derived in the last two sections may be used.

(i) The $\Delta b=\Delta s=-\Delta c=1$ sector. For this sector we may write Eq. (4d) as

$$H_4(\kappa>M_W)=a_4(O_{4-}+O_{4+})\,,$$

where

$$a_4=\frac{G}{2\sqrt{2}}R^*_{12}R_{23}$$

and

$$O_{4\pm}=\bar s u\bar c b\pm\bar s b\bar c u\,.$$

The analysis for this sector follows exactly that of Sec. II, and Eqs. (10), (11), (13), and (14) follow exactly just by changing $H_1\to H_4$, $a_1\to a_4$, $O_\pm\to O_{4\pm}$.

(ii) The $\Delta b=\Delta c=\Delta s=1$ sector. For this sector we may write Eq. (4e) as

$$H_5(\kappa>M_W)=a_5(O_{5-}+O_{5+})\,,$$

where

$$a_5=\frac{G}{2\sqrt{2}}R^*_{13}R_{22}$$

and

$$O_{5\pm}=\bar s c\bar u b\pm\bar s b\bar u c\,.$$

Here the situation is slightly different from the case presented in Sec. II because the original currents are $J_5=\bar u b$ and $J_5^+=\bar s c$ (instead of $J_1=\bar c b$ and $J_1^+=\bar d u$ of Sec. II). The present current structure gives different anomalous dimensions for the weak currents in the interval $m_c>\kappa>\mu_0$, compared with the analysis presented for this interval in Sec. II. Because for the other intervals, nothing changes; Eqs. (10), (11), and (13) follow exactly just by changing $H_1\to H_5$, $a_1\to a_5$, $O_\pm\to O_{5\pm}$.

For the interval $m_c>\kappa>\mu_0$, the operators $O_{5\pm}$ renormalize multiplicatively, in contrast to $O_\pm$ which do not. So, instead of one equation such as (14), we have

$$H^{\rm eff}_5(\kappa>\mu_0)=a_5(\epsilon'O_{5-}+\epsilon'^{-1/2}O_{5+})\,, \qquad (30)$$

where

$$\epsilon'=\epsilon\kappa_1{}^{c''_-/2b_1},\quad c''_-=-1/3\pi^2\,.$$

(iii) The $\Delta s=0$, $\Delta b=\Delta c=1$ sector. For this sector we may write Eq. (4*f*) as

$$H_6(\kappa > M_w) = a_6(O_{6-} + O_{6+}),$$

where

$$a_6 = \frac{G}{2\sqrt{2}} R^*_{21} R_{13}$$

and

$$O_{6\pm} = \bar{d}c\bar{u}b \pm \bar{d}b\bar{u}c \ .$$

The situation here is entirely analogous to the case presented just above, so the effective Hamiltonians are given by Eqs. (10), (11), and (13) by replacing $H_1 \to H_6$, $a_1 \to a_6$, and $O_\pm \to O_{6\pm}$, and by Eq. (30) by replacing $H_5 \to H_6$, $a_5 \to a_6$, and $O_{5\pm} \to O_{6\pm}$.

(iv) The $\Delta c = \Delta s = 0$, $\Delta b = 1$ *sector.* For this sector we may write the Hamiltonian H_3 in (4c) as

$$H_3(\kappa > M_w) = a_3(O_{3c+} + O_{3c-}) + a_3'(O_{3t+} + O_{3t-}) \ ,$$

where

$$O_{3q\pm} = \bar{d}u\bar{u}b \pm \bar{d}b\bar{u}u - (u \to q) \ ,$$

$$a_3 = -\frac{G}{2\sqrt{2}} R^*_{21} R_{33}, \quad a_3' = -\frac{G}{2\sqrt{2}} R^*_{31} R_{33} \ .$$

Here the penguin diagrams are present again, and the analysis for this sector follows exactly that of Sec. III, just by making the appropriate changes of subscripts and of quark fields in the operator basis. As a matter of fact, Eqs. (16), (17), and (21) follow exactly just by changing $H_2 \to H_3$, $a_2 \to a_3$, $a_2' \to a_3'$, and $O_{q\pm} \to O_{3q\pm}$. Equations (19), (20), (24), (25), (27), and (29) also follow by changing $H_2 \to H_3$, $a_2 \to a_3$, $a_2' \to a_3'$, and the replacement of the s quark field, in all the places where it occurs in all the operators, for the d quark field.

V. CONCLUSIONS

In this paper we have calculated the effective weak Hamiltonian for the nonleptonic $\Delta b = 1$ decays in the context of the KM model, including QCD corrections and FSB effects. The induced Hamiltonian in the b sector from the $\Delta c = 1$ and $\Delta s = 1$ effective weak Hamiltonian are not considered here, nor are possible magnetic moment terms proportional to quark masses. As can be seen from the analysis, the FSB effects are very important here.

The final expressions we get are very strongly Cabibbo dependent, especially the expressions where penguin diagrams are involved in the derivations. For this reason, it is very difficult to predict, just by looking at the Hamiltonian, the favored channels in the decays of b-flavored particles. A better knowledge of the three Cabibbo-type angles, as well as of some other parameters in the theory, must be acquired before final conclusions may be reached.

ACKNOWLEDGMENTS

I want to thank the International Centre for Theoretical Physics in Trieste for hospitality during the initial stages of the calculation presented here. I also want to thank Colciencias for partial financial support.

APPENDIX A

In calculating the diagrams in Figs. 1 and 2, we encounter a typical integral of the form

$$I = \int \frac{d^4\kappa}{(2\pi)^4} \frac{1}{\kappa^2} \frac{1}{(\kappa - P)^2} \ , \tag{A1}$$

where κ^0 runs over $\mu < \kappa^0 < \Lambda$, P is the four-momentum of one of the quarks in an external leg such that $P^2 = m^2$, and Λ is some gauge-invariant cutoff. If $m \ll \mu$, then we can write

$$\frac{1}{(\kappa - P)^2} = \frac{1}{\kappa^2}\left(1 + \frac{\kappa \cdot P}{\kappa^2} - \frac{m^2}{\kappa^2} + \cdots\right) \tag{A2}$$

and the integral in (A1) can be evaluated as a series, where the leading term is a leading logarithm, proportional to $\ln(\Lambda/\mu)$, and the other terms do not include logarithms at all.

If $\Lambda \gg m \gg \mu$, the series expansion in (A2) does not converge, and instead of that expansion we must use

$$\frac{1}{(\kappa - P)^2} = \frac{1}{(\kappa^2 + m^2)}\left[1 + \frac{\kappa \cdot P}{(\kappa^2 + m^2)} + \cdots\right] \ . \tag{A3}$$

Upon calculating the integral in (A1) as a series, by using (A3), we get for the leading term an expression proportional to $\ln(\Lambda/m)$, that is independent of μ due to the fact that $m + \mu \approx m$.

Because in calculating the anomalous dimension for the different operators we look for the coefficient of $-\ln(\mu^2)$ after evaluating the integral in (A1), we may conclude that, when $m \ll \mu$, the integral (A1) contributes to said anomalous dimension, but when $m \gg \mu$ it does not contribute.

APPENDIX B

In this section we present the mixing matrices, their eigenvalues, their eigenvectors, and the procedure followed in order to diagonalize them, for the different sets of operators which appear in Sec. III.

The transpose of the mixing matrix for $O_1 \cdots O_6$ is

$$M=\begin{pmatrix} -1 & 3 & 0 & 0 & 0 & 0 \\ 3 & -1 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{9} & -\frac{11}{9} & \frac{22}{9} & 0 & -\frac{5}{9} \\ 0 & \frac{1}{3} & \frac{11}{3} & \frac{2}{3} & 0 & \frac{5}{3} \\ 0 & -\frac{1}{9} & -\frac{2}{9} & -\frac{5}{9} & 1 & -\frac{5}{9} \\ 0 & \frac{1}{3} & \frac{2}{3} & \frac{5}{3} & -3 & -\frac{19}{3} \end{pmatrix} . \tag{B1}$$

This matrix is the transpose of Eq. (A7) in Ref. 9 (notice that there is a misprint in the fifth row for this matrix in the above-mentioned reference).

The eigenvalues for (B1) are given by the six-component vector

$$V=(2,\ -4,\ -6.8954,\ -3.2429,\ 1.1166,\ 3.1327). \tag{B2}$$

These eigenvalues are related to the anomalous dimension of the linear combination of operators $O_1 \cdots O_6$ that renormalize multiplicatively.

In order to diagonalize M, given by (B1), we need the following matrix:

$$R=\begin{pmatrix} 1 & 1 & -\frac{1}{7} & -\frac{1}{7} & 0 & 0 \\ -1 & 1 & \frac{1}{3} & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & -1.3888 & 0.2190 & 4.1010 \\ 0 & 0 & -1.1556 & 1 & 0.0962 & 0.1967 \\ 0 & 0 & 0.1776 & 0.0864 & 1 & -0.3675 \\ 0 & 0 & 1.5219 & 2.9386 & -1.1845 & 1 \end{pmatrix} . \tag{B3}$$

The rows in (B3) are the eigenvectors for (B1), i.e., they define the linear combination of operators O_i that renormalize multiplicatively with anomalous dimensions V_i. The diagonalization of M is described by

$$\sum_{l,k=1}^{6} (R^T)^{-1}{}_{il} M_{lk} R^T{}_{kj} = \delta_{ij} V_i \ , \tag{B4}$$

where R^T stands for the transpose of (B3).

For the operators $O_1' \cdots O_7'$ in the energy range $m_b > \kappa > m_c$, the transpose of the mixing matrix is

$$M'=\begin{pmatrix} -\frac{1}{2} & \frac{3}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{3}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{9} & -\frac{13}{18} & \frac{19}{18} & 0 & -\frac{4}{9} & 0 \\ 0 & \frac{1}{3} & \frac{13}{6} & \frac{5}{6} & 0 & \frac{4}{3} & 0 \\ 0 & -\frac{1}{9} & -\frac{2}{9} & -\frac{4}{9} & \frac{1}{2} & -\frac{4}{9} & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & \frac{4}{3} & -\frac{3}{2} & -\frac{8}{3} & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & \frac{4}{3} & 0 & \frac{4}{3} & 0 \end{pmatrix} . \tag{B5}$$

The eigenvalues for (B5) are

$$V'=(1,\ -2,\ 0.5699,\ 2.1020,\ -3.2382,\ -1.4893,\ 0). \tag{B6}$$

The matrix R', the rows of which are the eigenvectors for M', is

$$R'=\begin{pmatrix} 1 & 1 & -\frac{1}{6} & -\frac{1}{6} & 0 & 0 & 0 \\ 1 & -1 & -\frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0.4985 & 4.6064 & -1.7235 & -1.6962 \\ 0 & 0 & 0.3010 & 1 & -0.4473 & 0.4623 & 1.0230 \\ 0 & 0 & 5.1593 & -6.9218 & 1 & 12.7532 & -3.4632 \\ 0 & 0 & -4.8267 & 3.9285 & 0.5619 & 1 & -2.2518 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} . \tag{B7}$$

For the operators O_1'', O_2'', O_3'', O_5'', O_6'', and O_7'' in the energy range $m_c > \kappa > \mu_0$ the transpose of the mixing matrix is

$$M'' = \begin{pmatrix} -\frac{1}{2} & \frac{7}{6} & -\frac{13}{6} & 0 & -1 & 0 \\ \frac{3}{2} & -\frac{1}{6} & \frac{13}{6} & 0 & 1 & 0 \\ 0 & \frac{2}{9} & \frac{13}{9} & 0 & \frac{2}{3} & 0 \\ 0 & -\frac{1}{9} & -\frac{2}{9} & \frac{1}{2} & -\frac{1}{3} & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & -\frac{3}{2} & -3 & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 & 1 & 0 \end{pmatrix}. \tag{B8}$$

The eigenvalues for (B8) are

$$V'' = (1,\ 1.7915,\ -1.7412,\ 0.5564,\ -3.3289,\ 0). \tag{B9}$$

The matrix R'', the rows of which are the eigenvectors for M'', is

$$R'' = \begin{pmatrix} 1 & \frac{2}{3} & -\frac{1}{3} & 0 & 0 & 0 \\ -1 & 1 & 1.4109 & -0.4324 & 0.4012 & 0.9351 \\ 10.0929 & -10.0929 & 1 & -0.6115 & -1.4143 & 2.3615 \\ -0.0830 & 0.0830 & 0.2542 & 1 & -0.3663 & -0.3040 \\ 0.4467 & -0.4467 & -0.1189 & 0.0672 & 1 & -0.2319 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{B10}$$

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