

Minimal scalar sector of 3-3-1 models without exotic electric charges

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We study the minimal set of Higgs scalars, for models based on the local gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$, which do not contain particles with exotic electric charges. We show that only two Higgs $SU(3)_L$ triplets are needed in order to properly break the symmetry. The exact tree-level scalar mass matrices resulting from symmetry breaking are calculated at the minimum of the most general scalar potential, and the gauge bosons are obtained, together with their couplings to the physical scalar fields. We show how the scalar sector introduced is enough to produce masses for fermions in a particular model. By using experimental results we constrain the scale of new physics to be above 1.5 TeV.

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I. INTRODUCTION

The standard model (SM) based on the local gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ [1] can be extended in several different ways: first, by adding new fermion fields (adding a right-handed neutrino field constitutes its simplest extension and has profound consequences, such as, for example, the implementation of the seesaw mechanism and the enlarging of the possible number of local gauge Abelian symmetries that can be gauged simultaneously); second, by augmenting the scalar sector to more than one Higgs representation; and third, by enlarging the local gauge group. In this last direction $SU(3)_L \otimes U(1)_X$ as a flavor group has been studied previously in the literature [2–10] by many authors who have explored possible fermion and Higgs-boson representation assignments. From now on, models based on the local gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ are going to be called 3-3-1 models.

There are in the literature several 3-3-1 models; the most popular one, the Pleitez-Frampton model [2], is far from being the simplest construction. Not only is its scalar sector quite complicated and messy (three triplets and one sextet [3]), but its physical spectrum is plagued with particles with exotic electric charges; namely, double charged gauge and Higgs bosons which induce tree-level flavor changing neutral currents (FCNC) in the lepton sector [4] and exotic quarks with electric charges $5/3$ and $-4/3$. Other 3-3-1 models in the literature are just introduced or merely sketched in a few papers [5–8], with a detailed phenomenological analysis of them still lacking. In particular, there are no published papers related to the study of the scalar sector for models [5–8].

All possible 3-3-1 models without exotic electric charges are presented in Ref. [9] and summarized in the next section. As shown, there are just a few anomaly-free models for one or three families which all share in common the same gauge-boson content and, as we are going to see in this paper, they also may share a common scalar sector (which does not contain scalars with exotic electric charges either).

This paper is organized as follows. In Sec. II we review

all possible 3-3-1 models without exotic electric charges for one and three families. In Sec. III we study the common scalar sector for all those models, including the analysis of its mass spectrum. In Sec. IV we analyze the gauge boson structure common to all the models considered. In Sec. V we present the couplings between the neutral scalar fields in the models and the SM gauge bosons, and in Sec. VI we make general remarks and present the conclusions. An Appendix at the end shows how the Higgs scalars that are used to break the symmetry can also be used to produce a consistent mass spectrum for the fermion fields in the particular model which is an E_6 subgroup [7].

II. A REVIEW OF THE MODEL

Let us start with a summary of Refs. [9] and [10]. First we assume that the electroweak group is $SU(3)_L \otimes U(1)_X \supset SU(2)_L \otimes U(1)_Y$ and that the left-handed quarks and left-handed leptons transform as the two fundamental representations of $SU(3)_L$ (the 3 and 3^*). The right-handed fields are singlets under $SU(3)_L$, and $SU(3)_c$ is vectorlike as in the SM.

The most general electric charge operator in $SU(3)_L \otimes U(1)_X$ is a linear combination of the three diagonal generators of the gauge group

$$Q = aT_{3L} + \frac{2}{\sqrt{3}}bT_{8L} + XI_3, \quad (1)$$

where $T_{iL} = \lambda_{iL}/2$, λ_{iL} is the Gell-Mann matrices for $SU(3)_L$ normalized as usual, $I_3 = Dg(1,1,1)$ is the diagonal 3×3 unit matrix, $a = 1$ gives the usual isospin of the electroweak interactions, and b is a free parameter. The X values are fixed by anomaly cancellation [9,10] and an eventual coefficient for XI_3 can be absorbed in the hypercharge definition.

There are a total of 17 gauge bosons in the gauge group under consideration. They are one gauge field B^μ associated with $U(1)_X$, the 8 gluon fields associated with $SU(3)_c$

TABLE I. Anomalies for S_i .

Anomalies	S_1	S_2	S_3	S_4	S_5	S_6
$[SU(3)_c]^2 U(1)_X$	0	0	0	0	0	0
$[SU(3)_L]^2 U(1)_X$	-2/3	-1/3	1	0	0	-1
$[grav]^2 U(1)_X$	0	0	0	0	0	0
$[U(1)_X]^3$	10/9	8/9	-12/9	-6/9	6/9	12/9
$[SU(3)_L]^3$	1	-1	-3	3	-3	3

which remain massless after breaking the symmetry, and another 8 gauge fields associated with $SU(3)_L$. We may write the latter in the following way:

$$\frac{1}{2} \lambda_{\alpha L} A_{\mu}^{\alpha} = \frac{1}{\sqrt{2}} \begin{pmatrix} D_{1\mu}^0 & W_{\mu}^+ & K_{\mu}^{(1/2+b)} \\ W_{\mu}^- & D_{2\mu}^0 & K_{\mu}^{-(1/2-b)} \\ K_{\mu}^{-(1/2+b)} & \bar{K}_{\mu}^{1/2-b} & D_{3\mu}^0 \end{pmatrix},$$

where $D_{1\mu}^0 = A_{3\mu}/\sqrt{2} + A_{8\mu}/\sqrt{6}$, $D_{2\mu}^0 = -A_{3\mu}/\sqrt{2} + A_{8\mu}/\sqrt{6}$, and $D_{3\mu}^0 = -2A_{8\mu}/\sqrt{6}$. The upper indices of the gauge bosons in the former expression stand for the electric charge of the corresponding particle, some of them are functions of the b parameter, as they should be [10]. Notice that the gauge bosons have integer electric charges only for $b = \pm n/2$, $n = 1, 3, 5, 7, \dots$, and as shown in Refs. [9,10], a negative value for b can be related to a positive value. So $b = 1/2$ avoids exotic electric charges in the gauge sector of the theory.

For the 3-3-1 models both, $SU(3)_L$ and $U(1)_X$ are anomalous [$SU(3)_c$ is vectorlike]. So, special combinations of multiplets must be used in each particular model in order to cancel the several possible anomalies, and end with physical acceptable models. The triangle anomalies we must take care of are as follows: $[SU(3)_L]^3$, $[SU(3)_c]^2 U(1)_X$, $[SU(3)_L]^2 U(1)_X$, $[grav]^2 U(1)_X$ (the gravitational anomaly), and $[U(1)_X]^3$.

In order to present specific examples let us see how the charge operator in Eq. (1) acts on the representations 3 and 3^* of $SU(3)_L$:

$$Q[3] = Dg \left(\frac{1}{2} + \frac{b}{3} + X, -\frac{1}{2} + \frac{b}{3} + X, -\frac{2b}{3} + X \right),$$

$$Q[3^*] = Dg \left(-\frac{1}{2} - \frac{b}{3} + X, \frac{1}{2} - \frac{b}{3} + X, \frac{2b}{3} + X \right).$$

Notice from these expressions that if we accommodate the known left-handed quark and lepton isodoublets in the two upper components of 3 and 3^* (or 3^* and 3), and forbid the presence of exotic electric charges in the possible models, then $b = \pm 1/2$ is mandatory.

To see this, let us take $b = 3/2$, then $Q[3] = Dg(1 + X, X, X - 1)$ and $Q[3^*] = Dg(X - 1, X, 1 + X)$. Then the following multiplets are associated with the respective $[SU(3)_c, SU(3)_L, U(1)_X]$ quantum numbers: $(e^-, \nu_e, e^+)_L^T \sim (1, 3^*, 0)$, $(u, d, j)_L^T \sim (3, 3, -1/3)$, and $(d, u, k)_L^T \sim (3, 3^*, 2/3)$, where j and k are isosinglet exotic quarks of

electric charges $-4/3$ and $5/3$, respectively. This multiplet structure is the basis of the Pleitez-Frampton model [2] for which the anomaly-free arrangement for the three families is given by

$$\psi_L^a = (e^a, \nu^a, e^{ca})_L^T \sim (1, 3^*, 0),$$

$$q_L^i = (u^i, d^i, j^i)_L^T \sim (3, 3, -1/3),$$

$$q_L^1 = (d^1, u^1, k)_L^T \sim (3, 3^*, 2/3),$$

$$u_L^{ca} \sim (3^*, 1, -2/3), \quad d_L^{ca} \sim (3^*, 1, 1/3),$$

$$k_L^a \sim (3^*, 1, -5/3), \quad j_L^{ci} \sim (3^*, 1, -4/3),$$

where the upper c symbol stands for charge conjugation, $a = 1, 2, 3$ is a family index, and $i = 2, 3$ is related to two of the three families.

Now, for $a = 1$ and $b = 1/2$ in Eq. (1) let us introduce the following closed sets of fermions (closed in the sense that each one includes the antiparticles of the charged particles):

$S_1 = [(\nu_{\alpha}, \alpha^-, E_{\alpha}^-); \alpha^+; E_{\alpha}^+]$ with quantum numbers $[(1, 3, -2/3); (1, 1, 1); (1, 1, 1)]$.

$S_2 = [(\alpha^-, \nu_{\alpha}, N_{\alpha}^0); \alpha^+]$ with quantum numbers $[(1, 3^*, -1/3); (1, 1, 1)]$.

$S_3 = [(d, u, U); d^c; u^c; U^c]$ with quantum numbers $(3, 3^*, 1/3); (3^*, 1, 1/3); (3^*, 1, -2/3)$; and $(3^*, 1, -2/3)$, respectively.

$S_4 = [(u, d, D); u^c; d^c; D^c]$ with quantum numbers $(3, 3, 0); (3^*, 1, -2/3); (3^*, 1, 1/3)$; and $(3^*, 1, 1/3)$, respectively.

$S_5 = [(e^-, \nu_e, N_1^0); (E^-, N_2^0, N_3^0); (N_4^0, E^+, e^+)]$ with quantum numbers $(1, 3^*, -1/3); (1, 3^*, -1/3)$; and $(1, 3^*, 2/3)$, respectively.

$S_6 = [(\nu_e, e^-, E_1^-); (E_2^+, N_1^0, N_2^0); (N_3^0, E_2^-, E_3^-); e^+; E_1^+; E_3^+]$ with quantum numbers $[(1, 3, -2/3); (1, 3, 1/3); (1, 3, -2/3); (1, 1, 1); (1, 1, 1); (1, 1, 1)]$.

The quantum numbers in parentheses refer to $[SU(3)_c, SU(3)_L, U(1)_X]$ representations. The several anomalies for the former six sets are presented in Table I, which in turn allows us to build anomaly-free models for one or more families.

A. One family models

There are just two anomaly-free one family structures that can be extracted from Table I. They are as follows:

Model A: ($S_4 + S_5$). This model is associated with an E_6 subgroup and has been partially analyzed in Ref. [7] (see also the Appendix at the end of this paper).

Model B: ($S_3 + S_6$). This model is associated with an $SU(6)_L \otimes U(1)_X$ subgroup and has been partially analyzed in Ref. [8].

B. Three family models

Model C: ($3S_2 + S_3 + 2S_4$). This model deals with the following multiplets associated with the given quantum numbers: $(u, d, D)_L^T \sim (3, 3, 0)$, $(e^-, \nu_e, N^0)_L^T \sim (1, 3^*, -1/3)$, and $(d, u, U)_L^T \sim (3, 3^*, 1/3)$, where D and U are exotic quarks

with electric charges $-1/3$ and $2/3$, respectively. With such a gauge structure the three family anomaly-free model is given by

$$\begin{aligned}\psi_L^{\prime a} &= (e^{-a}, \nu^a, N^{0a})_L^T \sim (1, 3^*, -1/3), \\ e_L^{+a} &\sim (1, 1, 1), \\ q_L^{\prime i} &= (u^i, d^i, D^i)_L^T \sim (3, 3, 0), \\ q_L^{\prime 1} &= (d^1, u^1, U)_L^T \sim (3, 3^*, 1/3), \\ u_L^{ca} &\sim (3^*, 1, -2/3), \quad d_L^{ca} \sim (3^*, 1, 1/3), \\ U_L^c &\sim (3^*, 1, -2/3), \quad D_L^{ci} \sim (3^*, 1, 1/3),\end{aligned}$$

where $a=1,2,3$ and $i=1,2$ as above. This model has been analyzed in the literature in Ref. [5]. If needed, the fermion sector can be augmented with an undetermined number of neutral Weyl states $N_L^{0j} \sim (1, 1, 0)$, $j=1,2, \dots$ without violating the anomaly cancellation.

Model D: ($3S_1 + 2S_3 + S_4$). It makes use of the same multiplets used in the previous model arranged in a different way, plus a new lepton multiplet $(\nu_e, e^-, E^-)_L^T \sim (1, 3, -2/3)$. The family structure of this new anomaly-free model is given by

$$\begin{aligned}\psi_L^{\prime a} &= (\nu^a, e^a, E^a)_L^T \sim (1, 3, -2/3), \\ e_L^{ca} &\sim (1, 1, 1), \quad E_L^{ca} \sim (1, 1, 1), \\ q_L^{\prime 1} &= (u^1, d^1, D)_L^T \sim (3, 3, 0), \\ q_L^{\prime i} &= (d^i, u^i, U^i)_L^T \sim (3, 3^*, 1/3), \\ u_L^{ca} &\sim (3^*, 1, -2/3), \quad d_L^{ca} \sim (3^*, 1, 1/3), \\ D_L^c &\sim (3^*, 1, 1/3), \quad U_L^{ci} \sim (3^*, 1, 2/3).\end{aligned}$$

This model has been analyzed in Ref. [6].

Model E: ($S_1 + S_2 + S_3 + 2S_4 + S_5$).

Model F: ($S_1 + S_2 + 2S_3 + S_4 + S_6$).

Besides the former four three family models, another four, carbon copies of the two one family models *A* and *B* can also be constructed. They are as follows:

Model G: $3(S_4 + S_5)$.

Model H: $3(S_3 + S_6)$.

Model I: $2(S_4 + S_5) + (S_3 + S_6)$.

Model J: $(S_4 + S_5) + 2(S_3 + S_6)$.

There are a total of eight different three-family models, each one with a different fermion field content. Notice in particular that in models *E* and *F* each one of the three families is treated differently. As far as we know the last six models have not yet been studied in the literature.

III. THE SCALAR SECTOR

If we pretend to use the simplest $SU(3)_L$ representations in order to break the symmetry, at least two complex scalar triplets, equivalent to 12 real scalar fields, are required. For

$b=1/2$ there are just two Higgs scalars (together with their complex conjugates) which may develop nonzero vacuum expectation values (VEV); they are $\phi_1(1, 3^*, -1/3)^T = (\phi_1^-, \phi_1^{\prime 0}, \phi_1^0)$ with VEV $\langle \phi_1 \rangle^T = (0, v_1, V)$ and $\phi_2(1, 3^*, 2/3)^T = (\phi_2^0, \phi_2^+, \phi_2^{\prime +})$ with VEV $\langle \phi_2 \rangle^T = (v_2, 0, 0)$. As we will see ahead, to reach consistency with phenomenology we must have the hierarchy $V > v_1 \sim v_2$.

Our aim is to break the symmetry in one single step:

$$SU(3)_c \otimes SU(3)_L \otimes U(1)_X \rightarrow SU(3)_c \otimes U(1)_Q,$$

which implies the existence of eight Goldstone bosons included in the scalar sector of the theory [11]. For the sake of simplicity we assume that the VEV are real. This means that the *CP* violation through the scalar exchange is not considered in this work. Now, for convenience in reading we rewrite the expansion of the scalar fields which acquire VEV as

$$\begin{aligned}\phi_1^0 &= V + \frac{H_{\phi_1}^0 + iA_{\phi_1}^0}{\sqrt{2}}, \\ \phi_1^{\prime 0} &= v_1 + \frac{H_{\phi_1}^{\prime 0} + iA_{\phi_1}^{\prime 0}}{\sqrt{2}}, \\ \phi_2^0 &= v_2 + \frac{H_{\phi_2}^0 + iA_{\phi_2}^0}{\sqrt{2}}.\end{aligned}\tag{2}$$

In the literature, a real part H is called a *CP*-even scalar and an imaginary part A a *CP*-odd scalar or pseudoscalar field.

Now, the most general potential which includes ϕ_1 and ϕ_2 can then be written in the following form:

$$\begin{aligned}V(\phi_1, \phi_2) &= \mu_1^2 \phi_1^\dagger \phi_1 + \mu_2^2 \phi_2^\dagger \phi_2 + \lambda_1 (\phi_1^\dagger \phi_1)^2 + \lambda_2 (\phi_2^\dagger \phi_2)^2 \\ &\quad + \lambda_3 (\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2) + \lambda_4 (\phi_1^\dagger \phi_2) (\phi_2^\dagger \phi_1).\end{aligned}\tag{3}$$

Requiring that in the shifted potential $V(\phi_1, \phi_2)$, the linear terms in fields must be absent, we get in the tree-level approximation the following constraint equations:

$$\begin{aligned}\mu_1^2 + 2\lambda_1(v_1^2 + V^2) + \lambda_3 v_2^2 &= 0, \\ \mu_2^2 + \lambda_3(v_1^2 + V^2) + 2\lambda_2 v_2^2 &= 0.\end{aligned}\tag{4}$$

The analysis to the former equations shows that they are related to a minimum of the scalar potential with the value

$$\begin{aligned}V_{min} &= -v_2^4 \lambda_2 - (v_1^2 + V^2) [(v_1^2 + V^2) \lambda_1 + v_2^2 \lambda_3] \\ &= V(v_1, v_2, V),\end{aligned}$$

where $V(v_1=0, v_2, V) > V(v_1 \neq 0, v_2, V)$, implying that $v_1 \neq 0$ is preferred. Substituting Eqs. (2) and (4) in Eq. (3) we get the following mass matrices.

A. Spectrum in the scalar neutral sector

In the $(H_{\phi_1}^0, H_{\phi_2}^0, H_{\phi_1}'^0)$ basis, the square mass matrix can be calculated using $M_{ij}^2 = 2[\partial^2 V(\phi_1 \phi_2) / \partial H_{\phi_i}^0 \partial H_{\phi_j}^0]$. After imposing the constraints in Eq. (4) we get

$$M_H^2 = 2 \begin{pmatrix} 2\lambda_1 V^2 & \lambda_3 v_2 V & 2\lambda_1 v_1 V \\ \lambda_3 v_2 V & 2\lambda_2 v_2^2 & \lambda_3 v_1 v_2 \\ 2\lambda_1 v_1 V & \lambda_3 v_1 v_2 & 2\lambda_1 v_1^2 \end{pmatrix}, \quad (5)$$

which has zero determinant, providing us with a Goldstone boson G_1 and two physical massive neutral scalar fields H_1 and H_2 with masses

$$M_{H_1, H_2}^2 = 2(v_1^2 + V^2)\lambda_1 + 2v_2^2\lambda_2 \pm 2\sqrt{[(v_1^2 + V^2)\lambda_1 + v_2^2\lambda_2]^2 + v_2^2(v_1^2 + V^2)(\lambda_3^2 - 4\lambda_1\lambda_2)},$$

where real lambdas produce positive masses for the scalars only if $\lambda_1 > 0$ and $4\lambda_1\lambda_2 > \lambda_3^2$ (which implies $\lambda_2 > 0$).

We may see from the former equations that in the limit $V > v_1 \sim v_2$, and for lambdas of order one, there is a neutral Higgs scalar with a mass of order V and another one with a mass of the order of $v_1 \sim v_2$, which may be identified with the SM scalar as we will see ahead.

The physical fields are related to the scalars in the weak basis by the lineal transformation:

$$\begin{pmatrix} H_{\phi_1}^0 \\ H_{\phi_2}^0 \\ H_{\phi_1}'^0 \end{pmatrix} = \begin{pmatrix} \frac{v_2 V}{S_1} & \frac{v_2 V}{S_2} & -\frac{v_1}{\sqrt{v_1^2 + V^2}} \\ \frac{M_{H_1}^2 - 4(v_1^2 + V^2)\lambda_1}{2S_1\lambda_3} & \frac{(M_{H_1}^2 - 4v_2^2\lambda_2)}{2S_2\lambda_3} & 0 \\ \frac{v_1 v_2}{S_1} & \frac{v_1 v_2}{S_2} & \frac{V}{\sqrt{v_1^2 + V^2}} \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \\ G_1 \end{pmatrix}, \quad (6)$$

where we have defined

$$S_1 = \sqrt{v_2^2(v_1^2 + V^2) + [M_{H_1}^2 - 4(v_1^2 + V^2)\lambda_1]^2 / 4\lambda_3^2},$$

$$S_2 = \sqrt{v_2^2(v_1^2 + V^2) + (M_{H_1}^2 - 4v_2^2\lambda_2)^2 / 4\lambda_3^2}.$$

B. Spectrum in the pseudoscalar neutral sector

The analysis shows that $V(\phi_1, \phi_2)$ in Eq. (3), when expanded around the most general vacuum given by Eqs. (2) and using the constraints in Eq. (4), does not contain pseudoscalar fields $A_{\phi_i}^0$. This allows us to identify another three Goldstone bosons $G_2 = A_{\phi_1}^0$, $G_3 = A_{\phi_2}^0$, and $G_4 = A_{\phi_1}'^0$.

C. Spectrum in the charged scalar sector

In the basis $(\phi_1^+, \phi_2^+, \phi_2'^+)$ the square mass matrix is given by

$$M_+^2 = 2\lambda_4 \begin{pmatrix} v_2^2 & v_1 v_2 & v_2 V \\ v_1 v_2 & v_1^2 & v_1 V \\ v_2 V & v_1 V & V^2 \end{pmatrix}, \quad (7)$$

which has two eigenvalues equal to zero equivalent to four

Goldstone bosons (G_5^\pm, G_6^\pm) and two physical charged Higgs scalars with large masses given by $\lambda_4(v_1^2 + v_2^2 + V^2)$, with the new constraint $\lambda_4 > 0$.

Our analysis shows that after symmetry breaking, the original 12 degrees of freedom in the scalar sector have become eight Goldstone bosons (four electrically neutral and four charged), and four physical scalar fields, two neutrals (one of them the SM Higgs scalar) and two charged ones. The eight Goldstone bosons must be swallowed up by eight gauge fields as we will see in the next section.

IV. THE GAUGE BOSON SECTOR

For $b = 1/2$ the nine gauge bosons in $SU(3)_L \otimes U(1)_X$, when acting on left-handed triplets, can be arranged in the following convenient way:

$$\begin{aligned} A_\mu &= \frac{1}{2} g \lambda_{\alpha L} A_\mu^\alpha + g' X B_\mu I_3 \\ &= \frac{g}{\sqrt{2}} \begin{pmatrix} Y_{1\mu}^0 & W_\mu^+ & K_\mu^+ \\ W_\mu^- & Y_{2\mu}^0 & K_\mu^0 \\ K_\mu^- & \bar{K}_\mu^0 & Y_{3\mu}^0 \end{pmatrix}, \end{aligned}$$

where $Y_{1\mu}^0 = A_{3\mu} / \sqrt{2} + A_{8\mu} / \sqrt{6} + \sqrt{2}(g'/g)XB_\mu$, $Y_{2\mu}^0 = -A_{3\mu} / \sqrt{2} + A_{8\mu} / \sqrt{6} + \sqrt{2}(g'/g)XB_\mu$, and $Y_{3\mu}^0 = -2A_{8\mu} /$

$\sqrt{6} + \sqrt{2}(g'/g)XB_\mu$. X is the hypercharge value of the given left-handed triplet, and g and g' are the gauge coupling constants for $SU(3)_L$ and $U(1)_X$, respectively. After breaking the symmetry with $\langle \phi_i \rangle$, $i=1,2$, and using for the covariant derivative for triplets $D^\mu = \partial^\mu - iA^\mu$, we get the following mass terms in the gauge boson sector.

A. Spectrum in the charged gauge boson sector

In the basis (K_μ^\pm, W_μ^\pm) the square mass matrix produced is

$$M_\pm^2 = \frac{g^2}{2} \begin{pmatrix} (V^2 + v_2^2) & v_1 V \\ v_1 V & (v_1^2 + v_2^2) \end{pmatrix}. \quad (8)$$

The former symmetric matrix give us the masses $M_{W'}^2 = g^2 v_2^2/2$ and $M_{K'}^2 = g^2(v_1^2 + v_2^2 + V^2)/2$, related to the physi-

cal fields $W'_\mu = \eta(v_1 K_\mu - VW_\mu)$, and $K'_\mu = \eta(VK_\mu + v_1 W_\mu)$ associated with the known charged weak current W'^\pm_μ , and with a new one K'^\pm_μ predicted in the context of this model ($\eta^{-2} = v_1^2 + V^2$ is a normalization factor). From the experimental value $M_{W'} = 80.419 \pm 0.056$ GeV [12] we obtain $v_2 \approx 174$ GeV as in the SM.

B. Spectrum in the neutral gauge boson sector

For the five electrically neutral gauge bosons we get first that the imaginary part of $K_\mu^0 = (K_{\mu R}^0 + iK_{\mu I}^0)/\sqrt{2}$ decouples from the other four electrically neutral gauge bosons, acquiring a mass $M_{K_I^0}^2 = g^2(v_1^2 + V^2)/2$. Then, in the basis $(B^\mu, A_3^\mu, A_8^\mu, K_R^{0\mu})$, the following squared mass matrix is obtained:

$$M_0^2 = \begin{pmatrix} \frac{g'^2}{9}(v_1^2 + V^2 + 4v_2^2) & -\frac{gg'}{6}(v_1^2 + 2v_2^2) & -\frac{gg'}{3\sqrt{3}}(V^2 + v_2^2 - v_1^2/2) & gg'v_1V/3 \\ -\frac{gg'}{6}(v_1^2 + 2v_2^2) & g^2(v_1^2 + v_2^2)/4 & \frac{g^2}{4\sqrt{3}}(v_2^2 - v_1^2) & -g^2v_1V/4 \\ -\frac{gg'}{3\sqrt{3}}(V^2 + v_2^2 - v_1^2/2) & \frac{g^2}{4\sqrt{3}}(v_2^2 - v_1^2) & \frac{g^2}{12}(v_1^2 + v_2^2 + 4V^2) & -g^2v_1V/(4\sqrt{3}) \\ gg'v_1V/3 & -g^2v_1V/4 & -g^2v_1V/(4\sqrt{3}) & g^2(v_1^2 + V^2)/4 \end{pmatrix}. \quad (9)$$

This matrix has a determinant equal to zero, which implies that there is a zero eigenvalue associated with the photon field with the eigenvector

$$A^\mu = S_W A_3^\mu + C_W \left[\frac{T_W}{\sqrt{3}} A_8^\mu + (1 - T_W^2/3)^{1/2} B^\mu \right], \quad (10)$$

where $S_W = \sqrt{3}g'/\sqrt{3g^2 + 4g'^2}$ and C_W are the sine and cosine of the electroweak mixing angle ($T_W = S_W/C_W$). Orthogonal to the photon field A^μ we may define another two fields

$$Z^\mu = C_W A_3^\mu - S_W \left[\frac{T_W}{\sqrt{3}} A_8^\mu + (1 - T_W^2/3)^{1/2} B^\mu \right],$$

$$Z'^\mu = -(1 - T_W^2/3)^{1/2} A_8^\mu + \frac{T_W}{\sqrt{3}} B^\mu, \quad (11)$$

where Z^μ corresponds to the neutral current of the SM and Z'^μ is a new weak neutral current predicted for these models.

We may also identify the gauge boson Y^μ associated with the SM hypercharge in $U(1)_Y$ as

$$Y^\mu = \left[\frac{T_W}{\sqrt{3}} A_8^\mu + (1 - T_W^2/3)^{1/2} B^\mu \right].$$

In the basis $(Z'^\mu, Z^\mu, K_R^{0\mu})$ the mass matrix for the neutral sector reduces to

$$\frac{g^2}{4C_W^2} \begin{pmatrix} \delta^2(v_1^2 C_{2W}^2 + v_2^2 + 4V^2 C_W^4) & \delta(v_1^2 C_{2W} - v_2^2) & \delta C_W v_1 V \\ \delta(v_1^2 C_{2W} - v_2^2) & v_1^2 + v_2^2 & -C_W v_1 V \\ \delta C_W v_1 V & -C_W v_1 V & C_W^2(v_1^2 + V^2) \end{pmatrix}, \quad (12)$$

where $C_{2W} = C_W^2 - S_W^2$ and $\delta = (4C_W^2 - 1)^{-1/2}$. The eigenvectors and eigenvalues of this matrix are the physical fields and their masses. In the approximation $v_1 = v_2 \equiv v \ll V$ and using $q \equiv v^2/V^2$ as an expansion parameter we get, up to first order in q , the following eigenvalues:

$$M_{Z_1}^2 \approx \frac{1}{2} g^2 C_W^{-2} v^2 (1 - q T_W^4),$$

$$M_{Z_2}^2 \approx \frac{g^2 V^2}{1 + 2C_{2W}} [1 + C_{2W} - q(S_{2W}^2 + C_{2W}^{-1})/2C_{2W}^2],$$

$$M_{K_{R'}^0}^2 \approx g^2 V^2 [1 + q(1 + C_{2W}^{-1})].$$

So we have a neutral current associated with a gauge boson Z_1^0 , related to a mass scale $v \approx 174$ GeV, which may be identified with the known experimental neutral current as we will see in what follows, and two new electrically neutral currents associated with a large mass scale $V \gg v$.

The former is the way the eight would-be Goldstone bosons are absorbed by the longitudinal components of the eight massive gauge bosons (W'^{\pm} , K'^{\pm} , K_I^0 , K_R^0 , Z_1^0 , and Z_2^0) as expected.

Using the expressions for $M_{W'}$ and M_{Z_1} we obtain $\rho_0 = M_{W'}^2/(M_{Z_1}^2 C_W^2) \approx 1 + T_W^4 q^2$, and the global fit for $\rho_0 = 1.0012_{-0.0014}^{+0.0023}$ [13] provides us with the lower limit $V \geq 1.5$ TeV (where we are using for $S_W^2 = 0.23113$ [12]). This result justifies the existence of the expansion parameter $q \leq 0.01$ which sets the scale of new physics, together with the hierarchy $V > v_1 \sim v_2$.

V. HIGGS-SM GAUGE BOSON COUPLINGS

In order to identify the considered above Higgs bosons with the one in the SM, in this section we present the couplings of the two neutral scalar fields H_1 and H_2 from Sec. III with the physical gauge bosons W'^{\pm} and Z_1^0 ; then we take the limit $V \gg v = v_1 = v_2$ which produces the couplings of the physical scalars H_1 and H_2 with the SM gauge bosons W^{\pm} and Z^0 .

When the algebra gets done we obtain the following trilinear couplings, provided $\lambda_3 < 0$:

$$g(W'W'H_1) = \frac{g^2 v_2 [M_{H_1}^2 - 4(v_1^2 + V^2)\lambda_1]_{V \gg v}}{2\sqrt{2}S_1\lambda_3} \rightarrow \frac{g^2 v_2^2 \lambda_3}{2\sqrt{2}\lambda_1 V},$$

$$g(W'W'H_2) = \frac{g^2 v_2 (4v_2^2 \lambda_2 - M_{H_1}^2)_{V \gg v}}{2\sqrt{2}S_2\lambda_3} \rightarrow \frac{g^2 v_2}{\sqrt{2}},$$

$$g(Z_1^0 Z_1^0 H_1) = \frac{g^2 v_1}{S_1} \left[\frac{M_{H_1}^2 - 4(v_1^2 + V^2)\lambda_1}{4\sqrt{2}C_W^2 \lambda_3} + q \frac{v_1^2 (\lambda_1 - \lambda_3 S_W^2)}{8\sqrt{2}C_W^2 \lambda_1} + \dots \right]_{V \gg v} \rightarrow \frac{g^2 v_1^2 \lambda_3}{4\sqrt{2}\lambda_1 C_W^2 V},$$

$$g(Z_1^0 Z_1^0 H_2) = \frac{g^2 v_1 V^2}{S_2} \left[-\frac{\lambda_1}{\sqrt{2}\lambda_3 C_W^2} + q \frac{4\lambda_1 \lambda_2 - \lambda_3^2 + 2\lambda_1^2 (T_W^2 C_W^{-2} - 2)}{4\sqrt{2}C_W^2 \lambda_1 \lambda_3} + \dots \right]_{V \gg v} \rightarrow \frac{g^2 v_1}{2\sqrt{2}C_W^2},$$

where $g(W'W'H_i^0)$, $i=1,2$ are exact expressions and $g(Z_1^0 Z_1^0 H_i)$ are expansions in the parameter q up to first order.

The quartic couplings are determined to be

$$g(W'W'H_1 H_1) = \frac{g^2 [M_{H_1}^2 - 4(v_1^2 + V^2)\lambda_1]_{V \gg v}^2}{16S_1^2 \lambda_3^2} \rightarrow \frac{g^2 v_2^2 \lambda_3^2}{16\lambda_1^2 V^2},$$

$$g(W'W'H_2 H_2) = \frac{g^2 (M_{H_1}^2 - 4v_2^2 \lambda_2)_{V \gg v}^2}{16S_2^2 \lambda_3^2} \rightarrow \frac{g^2}{4},$$

$$g(Z_1^0 Z_1^0 H_1 H_1) = \frac{g^2}{S_1^2} \left[\frac{[M_{H_1}^2 - 4(v_1^2 + V^2)\lambda_1]^2}{32C_W^2 \lambda_3^2} + q \frac{2v_1^4 \lambda_3^2 - S_W^2 [M_{H_1}^2 - 4(v_1^2 + V^2)\lambda_1]^2}{64C_W^6 \lambda_3^2} + \dots \right]_{V \gg v} \rightarrow \frac{g^2 v_1^2 \lambda_3^2}{32\lambda_1^2 C_W^2 V^2},$$

$$g(Z_1^0 Z_1^0 H_2 H_2) = \frac{g^2 V^4}{S_2^2} \left[\frac{\lambda_1^2}{2\lambda_3^2 C_W^2} + q \frac{\lambda_3^2 - 4\lambda_1 \lambda_2 + \lambda_1^2 (4 - C_W^{-2} T_W^2)}{4C_W^2 \lambda_3^2} + \dots \right]_{V \gg v} \rightarrow \frac{g^2}{8C_W^2},$$

where as before $g(W'W'H_i^0 H_i^0)$, $i=1,2$ are exact expressions and $g(Z_1^0 Z_1^0 H_i^0 H_i^0)$ are expansions in the parameter q up to first order.

As can be seen, in the limit $V > v_1 \sim v_2$, the couplings $g(W'W'H_2)$, $g(Z_1^0 Z_1^0 H_2)$, $g(W'W'H_2 H_2)$, and $g(Z_1^0 Z_1^0 H_2 H_2)$ coincide with those in the SM as far as $\lambda_3 < 0$. This gives additional support for the hierarchy $V > v_1 \sim v_2$.

Summarizing, from the couplings of the SM gauge bosons with the physical Higgs scalars we can conclude, as anticipated before, that the scalar H_2 can be identified with the SM neutral Higgs particle, and that Z_1^0 can be associated with the

known neutral current of the SM (more support for this last statement is presented in the Appendix).

VI. GENERAL REMARKS AND CONCLUSIONS

In this paper we have studied in detail the minimal scalar sector of some models based on the local gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$. By restricting the field representations to particles without exotic electric charges we end up with ten different models, two one family models and eight models for three families. The two one family models are studied in the papers in Refs. [7,8], but not enough attention was paid to the scalar sector in the analysis that was done. As far as we know, most of the three family models are new in the literature, except models *C* and *D*, which have been partially analyzed in Refs. [5] and [6], respectively.

We have also considered the mass spectrum eigenstates of the most general scalar potential specialized for the 3-3-1 models without exotic electric charges, with two Higgs triplets with the most general VEV possible. It is shown that in the considered models there is just one light neutral Higgs scalar which can be identified with the SM Higgs scalar; there are also three more heavy scalars, one charged, its charge conjugate, and one extra neutral one.

The two triplets of $SU(3)_L$ scalars with the most general VEV possible produces a consistent fermion mass spectrum at least for one of the models in the literature, and the scale of the new physics predicted by the class of models analyzed in this paper lies above 1.5 TeV as shown in the main text. This scale is consistent with the analysis done in other papers [7,8] using a different phenomenological analysis.

Our analysis allows us to constrain all the parameters in the scalar potential; that is, our model is a consistent one as far as $\lambda_1 > 0$, $\lambda_2 > 0$, $4\lambda_1\lambda_2 > \lambda_3^2$, $\lambda_3 < 0$, and $\lambda_4 > 0$.

Even though we have used in our analysis the most general VEV for the two scalar triplets, it is clear that taking the particular value $v_1 = 0$ will give a consistent two step symmetry breaking pattern of the form

$$\begin{aligned} \langle \phi_1 \rangle \\ SU(3)_c \otimes SU(3)_L \otimes U(1)_X &\rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \\ \langle \phi_2 \rangle \\ &\rightarrow SU(3)_c \otimes U(1)_Q, \end{aligned}$$

with the gauge bosons (K^+ , K^0) and Z' acquiring masses of order V (a detailed analysis shows also that this two step symmetry breaking pattern produces, for all the charged exotic fermions, masses of order V in all the models discussed in Sec. II). The gauge boson W^\pm get a square mass equal to $g^2 v_2^2/2$ (as in the general case), with the extra bonus that no mixing is present between W^\pm and K^\pm . But two particular problems appear when $v_1 = 0$:

(1) Several ordinary fermions will remain massless even after radiative corrections are included (see the analysis done in the Appendix where the mixing coefficients ϵ and ϵ' in Fig. 1 are proportional to v_1).

(2) The coupling $g(Z_1^0 Z_1^0 H_2)$ will be zero and will not coincide with the coupling obtained in the SM (this is due to

the fact that for $SU(2)_L$ the fundamental representation 2 is equivalent to 2^* , which is not the case for $SU(3)_L$).

There are two different ways to cure these two problems: (a) To introduce a third Higgs scalar $\phi_3 \sim (1, 3^*, -1/3)$ with VEV $\langle \phi_3 \rangle^T = (0, v_3, 0)$, where $v_3 \sim v_2$, as it is done, for example, in Refs. [7,8]. For this case the mixing between W^\pm and K^\pm is absent, but the price we pay is to have a very complicated scalar potential. (b) To make $v_1 \sim v_2 \sim 175$ GeV, as it is done in this paper. For this case the scalar potential analysis is as simple as presented in Sec. III, but at the price of generating the mixing obtained in Sec. IV A. This mixing is well under control due to the fact that the physical W'^\pm is mainly the W^\pm in the weak basis, with a small component of K^\pm of the order of v_1/V which will contaminate tree-level physical processes at the order of 1% (by the way, such a mixing can contribute to the $\Delta I = 1/2$ enhancement in nonleptonic weak processes).

Except for the one family models *A* and *B* (and their simplest carbon copy replicas *G* and *H*) where all the families are universal as in the SM, the other models suffer from potentially large tree-level FCNC, and even though a detailed analysis is model dependent, some general remarks are worth mentioning here.

First let us notice that none of our closed sets of fermion fields S_i , $i = 1, 2, \dots, 6$, in Sec. II contain a particle and its antiparticle in the same representation of $SU(3)_L$ (except maybe for the neutral sectors in some of them). So, contrary to what happens in the Pleitez-Frampton model [4], lepton number violation is not present at tree level, due to the fact that our gauge bosons, especially (K^\pm, K^0), do not carry an explicit lepton number. So, FCNC induced by lepton number violation at tree level are absent in all of our models.

The safest models as far as FCNC are concerned are models *C* and *D* in which the leptons are generation universal and couple diagonal to Z' ; thus a Z' FCNC is present only in the hadronic sector for those models. Since both models have four or even five up and down type quarks, unknown mixing parameters beyond the ordinary Cabibbo-Kobayashi-Maskawa ones prevent us from making quantitative statements about a lower bound on the mass of the heavy neutral current beyond that obtained from the ρ parameter. Then, theoretical upper bounds on the Z' mass may be used to restrict the unknown mixing parameters.

So, the only relevant FCNC effects for models *C* and *D* may be those coming from the gauge boson doublet (K^\pm, K^0), both in the quark and lepton sectors. However, with only ordinary external fermions, these gauge boson effects first show up at loop level, and the analysis, which is model dependent, may be used to set bounds on the mixing angle between Z and Z' and on the mass scale of the heavy neutral current.

For the other models (*E*, *F*, *I*, and *J*), the analysis in the hadronic sector is similar to the previous one, but since universality is lost for the lepton families, there exist extremely dangerous tree-level FCNC induced by Z' , and even if we restrict the Yukawa sector by an appropriate discrete symmetry in a particular model in order to diminish the tree-level effects, we do not believe that the one-loop radiative corrections give values smaller than the very stringent experimen-

tal constraints on FCNC for leptons, for a value of V in the TeV scale. So, when studying the structures E , F , I , and J , special attention must be paid to FCNC effects in order to decide the fate of these models.

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APPENDIX

In this appendix we show how the fermion fields of a particular model acquire masses with the Higgs scalars and VEV introduced in the main text. The analysis is model dependent, so let us use the one family model A , for which the fermion multiplets are [7] $\chi_L^T = (u, d, D)_L \sim (3, 3, 0)$; $u_L^c \sim (3^*, 1, -2/3)$; $d_L^c \sim (3^*, 1, 1/3)$, $D_L^c \sim (3^*, 1, 1/3)$; $\psi_{1L}^T = (e^-, \nu_e, N_1^0)_L \sim (1, 3^*, -1/3)$, $\psi_{2L}^T = (E^-, N_2^0, N_3^0)_L \sim (1, 3^*, -1/3)$, and $\psi_{3L}^T = (N_4^0, E^+, e^+)_L \sim (1, 3^*, 2/3)$. As shown in Ref. [7], this structure corresponds to an E_6 subgroup.

1. Bare masses for fermion fields

The most general Yukawa Lagrangian that the Higgs scalars in Sec. III produce for the fermion fields in this model can be written as $\mathcal{L}_Y = \mathcal{L}_Y^Q + \mathcal{L}_Y^l$, with

$$\mathcal{L}_Y^Q = \chi_L^T C (h_u \phi_2 u_L^c + h_D \phi_1 D_L^c + h_d \phi_1 d_L^c) + \text{H.c.},$$

$$\mathcal{L}_Y^l = \epsilon_{abc} [\psi_{1L}^a C (h_1 \psi_{2L}^b \phi_2^c + h_2 \psi_{3L}^b \phi_1^c) + \psi_{2L}^a C h_3 \psi_{3L}^b \phi_1^c] + \text{H.c.},$$

where h_η , $\eta = u, d, D, 1, 2, 3$ are Yukawa couplings of order one, a, b, c are $SU(3)_L$ tensor indices, and C is the charge conjugation operator.

Using for $\langle \phi_i \rangle$, $i = 1, 2$, the VEV in Sec. III we get $m_u = h_u v_2$ for the up-type quark mass and for the down sector in the basis (d, D) we get the mass matrix

$$M_d = \begin{pmatrix} h_d v_1 & h_d V \\ h_D v_1 & h_D V \end{pmatrix}. \quad (\text{A1})$$

Now, looking for the eigenvalues of $M_d M_d^\dagger$, we get $\sqrt{(h_d^2 + h_D^2)(v_1^2 + V^2)}$ and zero. Notice that for $h_u = 1$ and assuming, for example, that we are referring to the third family, we obtain the correct mass for the top quark (remember from Sec. IV that $v_2 \approx 174$ GeV), the bottom quark remains massless at zero level, and there is an exotic bottom quark with a very large mass. Since there is no way to distinguish between d_L^c and D_L^c in the Yukawa Lagrangian it is just natural to impose the discrete symmetry $h_d = h_D \equiv h$.

For the charged lepton sector the mass eigenvalues are 0 and $\sqrt{(h_2^2 + h_3^2)(v_1^2 + V^2)}$, with similar consequences as in the down quark sector, where again it is natural to impose the symmetry $h_2 = h_3 \equiv h'$.

The analysis of the neutral lepton sector is more elaborated; at zero level and in the basis $(\nu, N_1, N_2, N_3, N_4)$ we get the mass matrix

$$M_N = \begin{pmatrix} 0 & 0 & 0 & h_1 v_2 & -h_2 V \\ 0 & 0 & -h_1 v_2 & 0 & h_2 v_1 \\ 0 & -h_1 v_2 & 0 & 0 & -h_3 V \\ h_1 v_2 & 0 & 0 & 0 & h_3 v_1 \\ -h_2 V & h_2 v_1 & -h_3 V & h_3 v_1 & 0 \end{pmatrix}, \quad (\text{A2})$$

with eigenvalues 0, $\pm h_1 v_2$, and $\pm \sqrt{h_1^2 v_2^2 + (h_2^2 + h_3^2)(V^2 + v_1^2)}$, which implies a Majorana neutrino of zero mass and two Dirac neutral particles with masses one of them at the electroweak mass scale and the other one at the TeV scale.

So, at zero level the charged exotic particles get large masses of order $V > 1.5$ TeV; the top quark and a Dirac neutral particle get masses of order $v_2 \sim 174$ GeV; there is a Dirac neutral particle with a mass of order V ; and the bottom quark, charged lepton, and a Majorana neutrino remain massless. In what follows we will see that they pick up a radiative mass in the context of the model studied here.

2. Currents

The interactions among the charged gauge fields in Sec. IV with the fermions of *model A* are [7]

$$\begin{aligned} H^{CC} = & \frac{g}{\sqrt{2}} [W_\mu^+ (\bar{u}_L \gamma^\mu d_L - \bar{\nu}_e \gamma^\mu e_L - \bar{N}_{2L}^0 \gamma^\mu E_L^- - \bar{E}_L^+ \gamma^\mu N_{4L}^0) \\ & + K_\mu^+ (\bar{u}_L \gamma^\mu D_L - \bar{N}_{1L}^0 \gamma^\mu e_L - \bar{N}_{3L}^0 \gamma^\mu E_L^- - \bar{e}_L^+ \gamma^\mu N_{4L}^0) \\ & + K_\mu^0 (\bar{d}_L \gamma^\mu D_L - \bar{N}_{1L}^0 \gamma^\mu \nu_e - \bar{N}_{3L}^0 \gamma^\mu N_{2L}^0 - \bar{e}_L^+ \gamma^\mu E_L^+)] \\ & + \text{H.c.}, \end{aligned}$$

where the first two terms constitute the charged weak current of the SM, and K^\pm , K^0 , and \bar{K}^0 are related to new charged currents which violate weak isospin.

The algebra also shows that the neutral currents $J_\mu(EM)$, $J_\mu(Z)$, and $J_\mu(Z')$, associated with the Hamiltonian $H^0 = eA^\mu J_\mu(EM) + (g/C_W)Z^\mu J_\mu(Z) + (g'/\sqrt{3})Z'^\mu J_\mu(Z')$ [where A_μ is the photon field in Eq. (10) and Z_μ and Z'_μ are the neutral gauge bosons introduced in Eq. (11)] are

$$\begin{aligned} J_\mu(EM) = & \frac{2}{3} \bar{u} \gamma_\mu u - \frac{1}{3} (\bar{d} \gamma_\mu d + \bar{D} \gamma_\mu D) - \bar{e}^- \gamma_\mu e^- \\ & - \bar{E}^- \gamma_\mu E^-, \\ J_\mu(Z) = & J_{\mu,L}(Z) - S_W^2 J_\mu(EM), \end{aligned} \quad (\text{A3})$$

$$J_\mu(Z') = T_W J_\mu(EM) - J_{\mu,L}(Z'), \quad (\text{A4})$$

where $e = gS_W = g' C_W \sqrt{(1 - T_W^2/3)} > 0$ is the electric charge, $J_\mu(EM)$ is the (vectorlike) electromagnetic current, and the two neutral left-handed currents are given by

$$\begin{aligned} J_{\mu,L}(Z) &= \frac{1}{2} (\bar{u}_L \gamma_\mu u_L - \bar{d}_L \gamma_\mu d_L + \bar{\nu}_{eL} \gamma_\mu \nu_{eL} - \bar{e}_L^- \gamma_\mu e_L^- \\ &\quad + \bar{N}_2^0 \gamma_\mu N_2^0 - \bar{E}^- \gamma_\mu E^-) \\ &= \sum_f T_{3f} \bar{f}_L \gamma_\mu f_L, \end{aligned} \quad (\text{A5})$$

$$\begin{aligned} J_{\mu,L}(Z') &= S_{2W}^{-1} (\bar{u}_L \gamma_\mu u_L - \bar{e}_L^- \gamma_\mu e_L^- - \bar{E}_L^- \gamma_\mu E_L^- \\ &\quad - \bar{N}_{4L}^0 \gamma_\mu N_{4L}^0) + T_{2W}^{-1} (\bar{d}_L \gamma_\mu d_L - \bar{E}_L^+ \gamma_\mu E_L^+ \\ &\quad - \bar{\nu}_{eL} \gamma_\mu \nu_{eL} - \bar{N}_{2L}^0 \gamma_\mu N_{2L}^0) - T_W^{-1} (\bar{D}_L \gamma_\mu D_L \\ &\quad - \bar{e}_L^+ \gamma_\mu e_L^+ - \bar{N}_{1L}^0 \gamma_\mu N_{1L}^0 - \bar{N}_{3L}^0 \gamma_\mu N_{3L}^0), \end{aligned}$$

where

$$S_{2W} = 2S_W C_W, \quad T_{2W} = S_{2W}/C_{2W},$$

$$\begin{aligned} \bar{N}_{2L}^0 \gamma_\mu N_{2L}^0 &= \bar{N}_{2L}^0 \gamma_\mu N_{2L}^0 + \bar{N}_{2R}^0 \gamma_\mu N_{2R}^0 \\ &= \bar{N}_{2L}^0 \gamma_\mu N_{2L}^0 - \bar{N}_{2L}^{0c} \gamma_\mu N_{2L}^{0c} \\ &= \bar{N}_{2L}^0 \gamma_\mu N_{2L}^0 - \bar{N}_{4L}^0 \gamma_\mu N_{4L}^0, \end{aligned}$$

similarly $\bar{E}^- \gamma_\mu E^- = \bar{E}_L^- \gamma_\mu E_L^- - \bar{E}_L^+ \gamma_\mu E_L^+$ and $T_{3f} = Dg(1/2, -1/2, 0)$ is the third component of the weak isospin acting on representation 3 of $SU(3)_L$ (the negative when acting on 3^*). Notice that $J_\mu(EM)$ and $J_\mu(Z)$ are just the generalization of the electromagnetic and neutral weak currents of the SM, as they should be, implying that Z_μ can be identified as the neutral gauge boson of the SM.

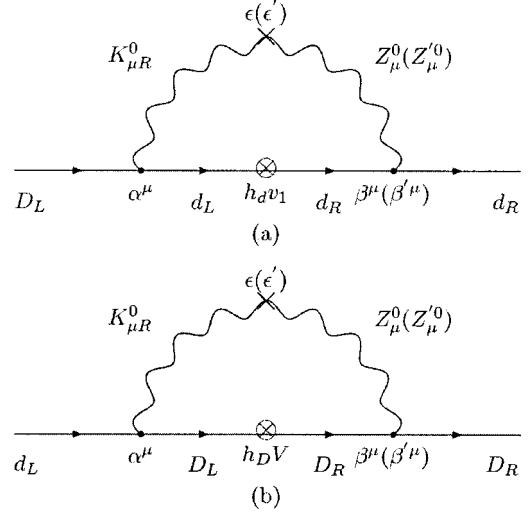


FIG. 1. Four one-loop diagrams contributing to the radiative generation of the ordinary down quark mass. For the meaning of α , β , and ϵ see the main text.

3. Radiative masses for fermion fields

Using the currents in the previous section and the off-diagonal entries in the matrix in Eq. (12), we may draw the four diagrams in Fig. 1 which allow for nondiagonal entries in the mass matrix for the down quark sector of the form $(\Delta_D D_L d_R + \text{H.c.})$ and $(\Delta_d d_L D_R + \text{H.c.})$, respectively, which in turn produce a radiative mass for the ordinary down quark. Notice that due to the presence of $K_R^{0\mu}$ in the graphs, mass entries of the form $d_L d_R$ and $D_L D_R$ are not present.

The equations in this paper imply for the diagrams in Fig. 1 that $\alpha_\mu = g \gamma_\mu / 2$, $\beta_\mu = g \gamma_\mu S_W T_W / 3$, $\beta'_\mu = -g' \gamma_\mu T_W / \sqrt{27}$, $\epsilon = -C_W v_1 V$, and $\epsilon' = C_W v_1 V / \sqrt{4C_W^2 - 1}$.

In a similar way we achieve radiative masses for the charged lepton and for the Majorana neutrino. The detailed analysis for these leptons will be presented elsewhere.

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