Optical encryption by means of the Talbot array illuminator

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Talbot effect for periodical objects limited by finite apertures: a new interpretation

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Abstract

The paper presents a new interpretation of the Talbot effect for periodical objects limited by finite apertures. According to the proposed approach, a self-image of a real, finite object is a superposition of deformed images of an elementary cell. The singular elementary cell image is equivalent to that formed in a proper optical system. Two possible optical arrangements are discussed. The theoretical description makes possible to define a structure of self-images. Particularly, the approach enables a determination of apertures' dimensions, which lead to self-images of a reasonable quality in a desired region of an image plane. The theory is illustrated and verified by numerical simulations.

Keywords: Talbot effect; Optical diffraction; Image forming and processing

1. Introduction

When a periodical object is illuminated by a plane wave, then in the Fresnel region of diffraction exact object's images appear. The images are formed in a free space and are localized in planes periodically situated along the illumination direction [1]. This phenomenon is known as the self-imaging effect or the Talbot effect. According to the theory of the Talbot effect, exact replicas are formed in the diffractive process only when the periodical structure is infinite. This crucial condition cannot be fulfilled in reality. Although many scientific works presented different aspects of the Talbot effect, to our knowledge only a small amount of them were devoted to the analysis of the influence of finite dimensions of real objects [2–5].

This article presents a new interpretation of the Talbot effect in a case of periodical objects limited by apertures of finite dimensions. The approach is based on the theory of the sampling filter [6,7]. According to this theory the sampling filter is equivalent to a sum of mutually shifted lenses. The theory was lately used for an explanation of imaging properties of the Talbot array illuminator [8]. The presented description of the Talbot effect is precise within the Fresnel paraxial approximation. According to our approach, a self-image is equivalent to the superposition of differently spoiled images of the elementary cell of the periodical object. The singular elementary cell image is equivalent to that formed in a proper optical system. The filtering process depending on the aperture function of the object

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deforms the image. Two possible optical arrangements are presented and analysed.

The theory enables determination of the self-image structure in a case of a given elementary cell transmittance and an aperture function limiting a periodical input object. Numerical simulations verify the presented description.

For simplicity all calculations correspond to the one-dimensional case of diffractive gratings. A transition to the two-dimensional case of periodical objects arranged in square arrays is straightforward and requires only an addition of the second variable. All conclusions can be extended onto arbitrary arrays fulfilling the self-imaging condition [9,10] and onto the fractional Talbot effect.

2. Theory

The transmittance of the periodical object limited by a finite aperture can be expressed as follows:

$$T(x) = A(x)\left[\sum_{n=-\infty}^{\infty} t(x-nd)\right] = \frac{1}{d}\,A(x)\left[t(x)\otimes \mathrm{comb}\left(\frac{x}{d}\right)\right], \tag{1}$$

where d is the object period, $t(x)$ defines transmittance of the object elementary cell, $A(x)$ is the aperture function limiting the object dimension, and the symbol $\otimes$ denotes the convolution operator. According to the theory of the sampling filter, the comb function in Eq. (1) can be rewritten with accuracy to a constant in the following equivalent form [6,7]:

$$\mathrm{comb}\left(\frac{x}{d}\right) = \sum_{n=-\infty}^{\infty} \exp\left[-\frac{\mathrm{i}k}{2Z_N}(x-nd-D)^2\right], \tag{2}$$

where $Z_N = Nd^2/\lambda$ $(N = 1, 2, 3\ldots)$, $k = 2\pi/\lambda$, $D = 0$ for N even and $D = d/2$ for N odd, λ is a wavelength of monochromatic light used in an optical system. According to Eqs. (1) and (2), when the infinite object ($A(x) = 1$) with the transmittance $T(x)$ is illuminated by a plane wave of a wavelength λ, then in planes at distances Z_N behind the object its images are formed [7]. Self-images being the exact object images correspond to N even. For N odd the self-image is additionally shifted by $d/2$ in respect to the object. According to the Fresnel paraxial approximation, Eqs. (1) and (2) lead to the following form of the diffractive field in the plane Z_N behind the object:

$$U(x) = \sum_{n=-\infty}^{\infty}\left\{A(x)\left[t(x-nd-D)\otimes \exp\left(-\frac{\mathrm{i}kx^2}{2Z_N}\right)\right]\right\} \times \otimes \exp\left(\frac{\mathrm{i}kx^2}{2Z_N}\right). \tag{3}$$

The non-important constant was omitted in the above formula.

2.1. The virtual optical system

According to Eq. (3) the complex amplitude $U(x)$ is equivalent to the field formed in the virtual optical system. The creation of the output field $U(x)$ can be divided into the following four steps:

(1) The backward propagation of the fields defined by the elementary cells transmittances $t(x)$ along the distance Z_N:

$$U_1(x) = t(x)\otimes \exp\left(-\frac{\mathrm{i}kx^2}{2Z_N}\right). \tag{4}$$

(2) Filtering of the shifted virtual Fresnel fields $U_1(x)$ by the aperture function $A(x)$:

$$U_{2n}(x-nd-D) = A(x)U_1(x-nd-D). \tag{5}$$

(3) The forward propagation of the fields $U_{2n}(x-nd-D)$ along the distance Z_N:

$$U_n(x-nd-D) = U_{2n}(x-nd-D)\otimes \exp\left(\frac{\mathrm{i}kx^2}{2Z_N}\right). \tag{6}$$

(4) The superposition of the fields $U_n(x-nd-D)$:

$$U(x) = \sum_{n=-\infty}^{\infty} U_n(x-nd-D). \tag{7}$$

The above simple model distinguishes a role of the finite object aperture in the self-image formation. The output field $U(x)$ is a superposition of differently spoiled images of elementary cells. The virtual Fresnel field corresponding to the single elementary cell is filtered by the object aperture after the backward propagation (Eqs. (4) and (5)). Then the elementary cell image in a self-image plane is deformed after the second, forward propagation (Eq. (6)). Because of the nature of the light propagation in the Fresnel region, this deformation depends strictly on the distance Z_N and the localization of the elementary cell in respect to the object aperture. The fixed elementary cell can be reconstructed with a good quality only when the corresponding Fresnel field is not strongly affected by the filtering process.

Although the discussed optical system includes the virtual backward propagation, it can be useful for numerical simulations. Using the above simple model it is possible to determine the dimension of the object's aperture leading to the high-quality object reconstruction within a desired region of the self-image plane.

2.2. The real imaging system with a lens

For the further calculations it is convenient to introduce new coordinate systems x_1 whose centres coincide with those of the elementary cells. In the case of the fixed index n, we have the relation $x_1 = x - nd - D$ and the term under the sum in Eq. (7) can be rewritten in the following way:

$$U_n(x_1) = \left\{ A(x_1 + nd + D)\left[t(x_1) \otimes \exp\left(-\frac{ikx_1^2}{2Z_N} \right) \right] \right\} \times \otimes \exp\left(\frac{ikx_1^2}{2Z_N} \right). \quad (8)$$

After the elementary calculations we obtain the identity:

$$t(x_1) \otimes \exp\left(-\frac{ikx_1^2}{2Z_N} \right) = \left\{ \left[t(-x_1) \exp\left(-\frac{ikx_1^2}{Z_N} \right) \right] \times \otimes \exp\left(\frac{ikx_1^2}{2Z_N} \right) \right\} \times \exp\left(-\frac{ikx_1^2}{Z_N} \right). \quad (9)$$

Then Eq. (8) can be expressed as follows:

$$U_n(x_1) = \left\{ \left\{ \left[t(-x_1) \exp\left(-\frac{ikx_1^2}{Z_N} \right) \right] \otimes \exp\left(\frac{ikx_1^2}{2Z_N} \right) \right\} \times \exp\left(-\frac{ikx_1^2}{Z_N} \right) A(x_1 + nd + D) \right\} \times \otimes \exp\left(\frac{ikx_1^2}{2Z_N} \right). \quad (10)$$

According to the theory of the thin lens and the Fresnel paraxial approximation [11], the elementary cell image described by the function $U_n(x_1)$ is exactly the same as the real image created by a thin lens in the optical arrangement shown in Fig. 1. The lens has a focal length $Z_N/2$ and is limited by the aperture function $A(x_1 + nd + D)$. The object and image planes are distant by Z_N from the lens so the magnification is equal to one. The input object is an inverted elementary cell illuminated by a convergent wave of the curvature radius $Z_N/2$. The physical meaning of this description coincides with the former one given for the virtual optical system. The self-image is a superposition of the spoiled elementary cell images. Using the previous coordinate system x we obtain the Eq. (7) for the output field in the self-image plane. The singular image of the elementary cell is deformed because of the pupil function of the lens $A(x_1 + nd + D)$. A limited aperture of the lens filters the diffractive Fresnel field behind the object.

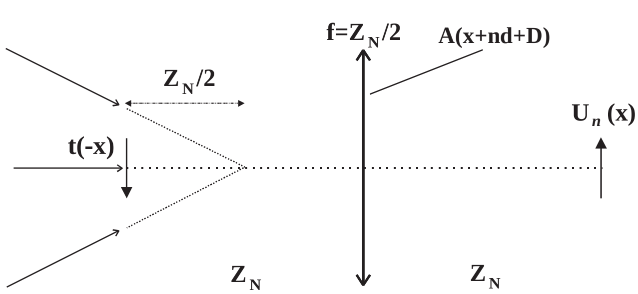

Fig. 1. A scheme of the imaging system with a lens. The convergent wave of a curvature radius $Z_N/2$ illuminates the inverted elementary cell with a transmittance $t(-x)$. The lens with a focal length $Z_N/2$ and the aperture function $A(x + nd + D)$ forms the cell's image corresponding to the complex amplitude $U_n(x)$.

The dimension of the elementary cell is limited by the period d. Hence

$$\frac{k}{Z_N}\left(\frac{d}{2}\right)^2 = \frac{\pi}{2N}$$

defines the maximal phase's module of the illuminating cylindrical wave within the input object's region corresponding to the range

$$x_1 \in \left(-\frac{d}{2}, \frac{d}{2} \right).$$

This module never overcomes $\pi/2$. Therefore in many cases seems to be reasonable to neglect the cylindrical factor in the product

$$t(-x_1) \exp\left(-\frac{ikx_1^2}{Z_N} \right).$$

According to this approximation the self-image consists of images of inverted elementary cells illuminated by plane waves. Lenses with different pupil functions $A(x_1 + nd + D)$ form singular images. The theory of the thin lens and especially the frequency analysis of coherent optical imaging systems [11] allow to predict the quality of images.

3. Numerical simulations

Numerical simulations have been performed in order to verify the presented description. The simulations were conducted for the wavelength of He–Ne laser ($\lambda = 632.8$ nm) using a diffractive modelling package working according to the modified convolution approach [12] on a matrix 4096×4096 points with a sampling interval of 0.5 μm covering an area in a form of a square of the width about 2.05 mm.The numerical results corresponding to the described virtual optical system and the real imaging system with a lens are shown in Figs. 2 and 3, respectively. The performed simulations compare adequate elementary cell images formed in the self-imaging process and created in the optical arrangements described in the Sections 2.1 and 2.2. In all cases the

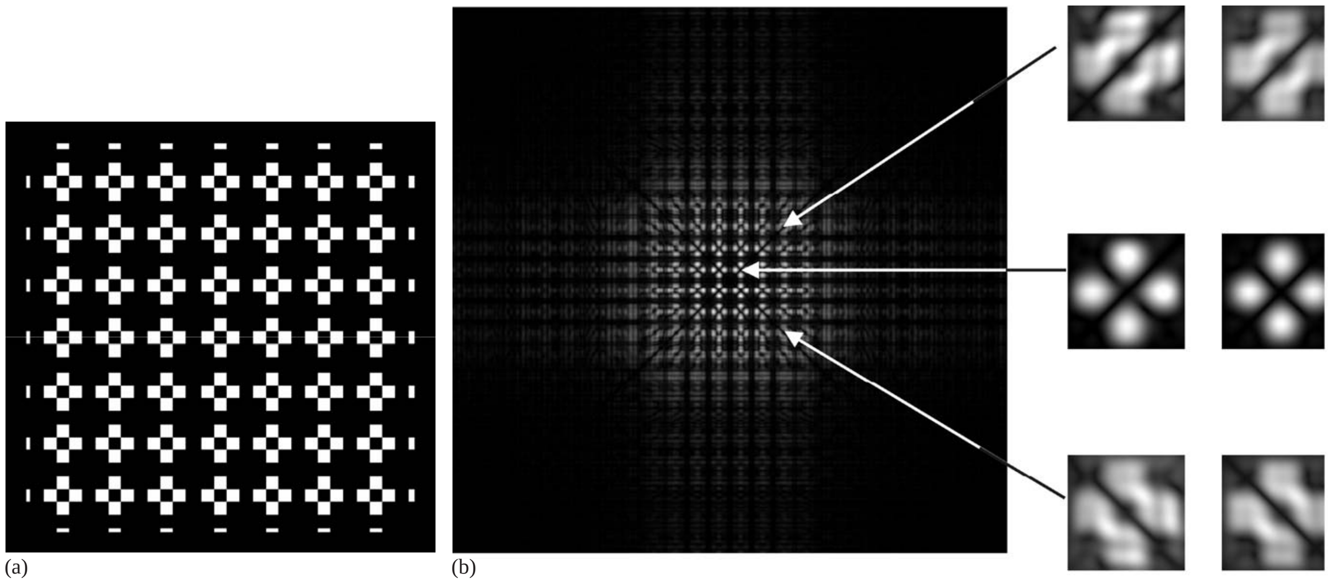

Fig. 2. (a) The periodical object with a period 40 μm limited by a square aperture of a width 300 μm. (b) The intensity distribution of the diffractive field behind the object corresponding to the distance $Z_1 = 2.528$ mm. The arrows indicate magnified fragments of the intensity distribution corresponding to images of selected elementary cells. On the right-hand side of the images there are shown their counterparts obtained in the described virtual optical system.

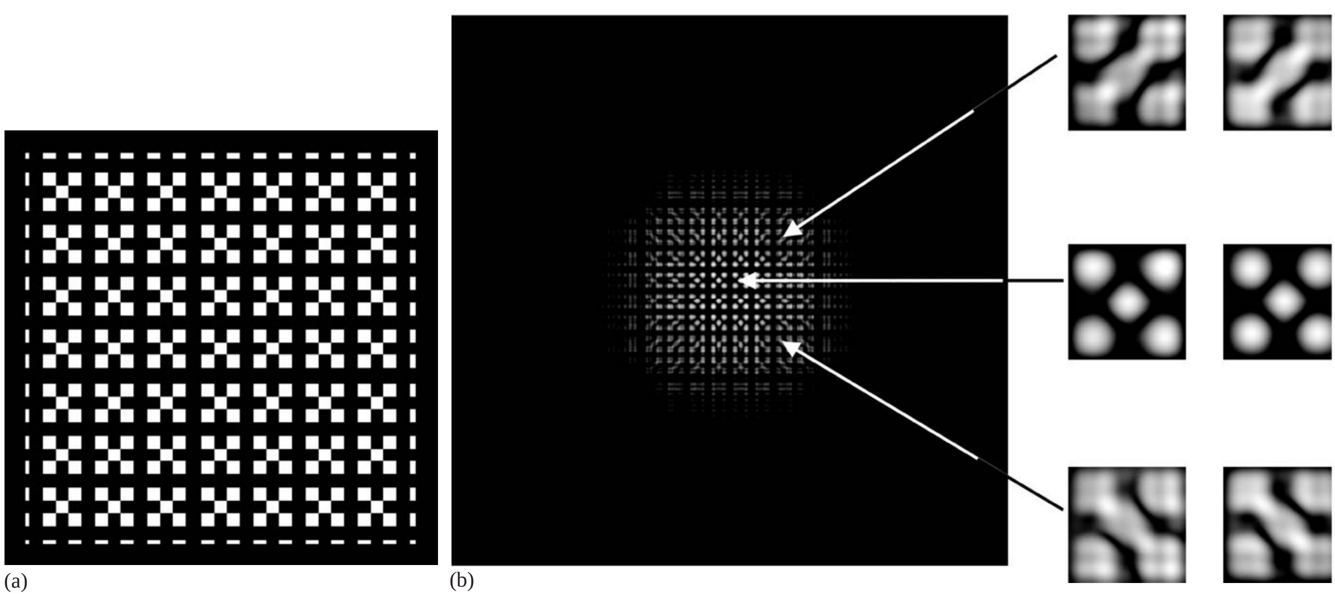

Fig. 3. (a) The periodical object with a period 40 μm limited by a square aperture of a width 300 μm. (b) The intensity distribution of the diffractive field behind the object corresponding to the distance $Z_1 = 2.528$ mm. The arrows indicate magnified fragments of the intensity distribution corresponding to images of selected elementary cells. On the right-hand side of the images there are shown their counterparts obtained in the optical arrangement with a lens presented in Fig. 1.

compared images are very similar. The slight difference is caused by the additional interference between neighbouring elementary cell images what occurs in the self-imaging process. Singular images of elementary cells created in the optical systems are not modified by the above interference. Nevertheless the compared images have congruent structures. The obtained results confirm the theoretical approach. By means of numerical simulations one can evaluate a quality of self-images corresponding to limited periodical objects.

4. Conclusions

The paper describes the Talbot effect in a case of periodical objects limited by finite apertures. According to the proposed interpretation based on the theory of the sampling filter [6,7], a self-image is a superposition of differently spoiled images of elementary cells of a periodical object. An image of a singular elementary cell is equivalent to that formed in a proper optical arrangement. Two possible optical systems are presented and discussed. In the case of the virtual optical system described in the Section 2.1 the virtual diffractive Fresnel field corresponding to the elementary cell is filtered by the object's aperture. In turn, in the case of the real optical system presented in the Section 2.2, the real diffractive Fresnel field behind the elementary cell is filtered by the proper aperture of the lens. The filtering process causes an image deformation and depends on elementary cell localization within the object's aperture. The presented approach makes possible to evaluate aperture's dimensions, which lead to a high self-image quality within a desired region of a self-image plane. This evaluation requires an analysis of a structure of the diffractive field corresponding to the elementary cell. An influence of the Fresnel field filtration should be negligible for proper elementary cells. Especially, in the case of the optical system with the lens, the investigation of the diffractive field's filtration can be performed by means of the well-known frequency analysis of coherent optical imaging systems [11].

A self-image is a sum of complex amplitudes corresponding to elementary cells' images created in the described optical systems. Therefore, one can determine a quality of self-images of a given limited object. Appropriate numerical simulations confirming the presented theory were performed. The structures of elementary cell images formed in the optical systems coincide with structures of their counterparts in self-image planes.

Acknowledgments

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