Eur. Phys. J. C (2009) 63: 461–475
DOI 10.1140/epjc/s10052-009-1101-4

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Stability of the scalar potential and symmetry breaking in the economical 3-3-1 model

Yithsbey Giraldo[1,2,a], **William A. Ponce**[1], **Luis A. Sánchez**[3]

[1]Instituto de Física, Universidad de Antioquia, A.A. 1226, Medellín, Colombia
[2]Departamento de Física, Universidad de Nariño, A.A. 1175, Pasto, Colombia
[3]Eseuela de Física, Universidad Nacional de Colombia, A.A. 3840, Medellín, Colombia

Received: 22 May 2009 / Published online: 31 July 2009

Abstract A detailed study of the criteria for stability of the scalar potential and the proper electroweak symmetry breaking pattern in the economical 3-3-1 model, is presented. For the analysis we use and improve a method previously developed to study the scalar potential in the two-Higgs-doublet extension of the standard model. A new theorem related to the stability of the potential is stated. As a consequence of this study, the consistency of the economical 3-3-1 model emerges.

1 Introduction

Extensions of the standard model (SM) based on the local gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ [1–18] (called hereafter 3-3-1 for short) contain, in general, a scalar sector quite complicated to be analyzed in detail. For this type of models, three Higgs triplets, and in some cases one additional Higgs sextet are used, in order to break the symmetry and provide at the same time with masses to the fermion fields of each model [19–25].

Among the 3-3-1 models with the simplest scalar sector are the ones proposed for the first time in Ref. [26] and further analyzed in Refs. [27–34] (they make use of only two scalar Higgs field triplets). This class of models include eight different three-family models where the Higgs scalar fields, the gauge-boson sector and the fermion field representations are restricted to particles without exotic electric charges [17, 18, 26]. Because of their minimal content of Higgs scalar fields they are named in the literature "economical 3-3-1 models".

A simple extension of the SM consists of adding to the model a second Higgs scalar doublet [35–39], defining in this way the so-called two-Higgs-doublet model (THDM). The different ways how the two Higgs scalar doublets couple to the fermion sector define the several versions of this extension [35–45]. Many gauge group extensions of the SM have the THDM as an effective low energy theory (in this regard see the papers in [40–45] and references therein). In these extensions one of the first steps in the symmetry breaking chain leads to the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge theory with two Higss doublets in one of its several versions.

A novel method for a detailed analysis of the scalar potential in the most general THDM was presented in Refs. [46, 47] where, by using powerful algebraic techniques, the authors studied in detail the stationary points of the scalar potential. This allowed them to give, in a very concise way, clear criteria for the stability of the scalar potential and for the correct electroweak symmetry breaking pattern. In the present work we use this approach to analyse the scalar sector of the economical 3-3-1 model. No relevant new additional conditions are necessary to be imposed in order to implement the method in this last case.

One important advantage of the economical 3-3-1 model, compared with the THDM, concerns the Higgs potential. The 14 parameters required to describe the most general potential for the second case, should be compared with the six parameters required in the economical 3-3-1 model. For the THDM this is associated to the fact that the two Higgs doublets have the same $U(1)$ hypercharge [35–45]. In the economical 3-3-1 model, by contrast, the two scalar triplets have different $U(1)_X$ hypercharges so that the most general Higgs potential shows itself in a very simple form.

In this work we deduce constraints on the parameters of the economical 3-3-1 scalar potential coming from the stability and from the electroweak symmetry breaking conditions. The stability of an scalar potential at the classical level, which is fulfilled when it is bounded from below, is a necessary condition in order to have a sound theory. The global minimum of the potential is found by determining its stationary points. Some of our results agree with those

[a] e-mail: yithsbey@gmail.com

already presented in Refs. [26–34]. Our study extends thus the method proposed in Refs. [46–48] to the economical 3-3-1 model, where the results are very concise and should, in principle, be used as a guide in order to extend the method to other situations.

This paper is organized as follows: in Sect. 2 we briefly review the mathematical formalism in order to make this work self-contained; in Sect. 3 we apply the method to the scalar sector of the economical 3-3-1 model, which is followed in Sect. 4 by the introduction of new parameterizations. In Sect. 5 we derive expressions for the masses of the scalar fields, and our conclusions are presented in Sect. 6. In Appendix A a new theorem that facilitates the stability criteria is proved. In Appendix B two exceptional solutions for the global minimum of the potential are analyzed. Finally, in Appendix C, it is verified that if only one scalar triplet acquires a nonzero Vacuum Expectation Value (VEV), the economical 3-3-1 model is inconsistent.

2 A review of the method

In this section, and following Refs. [46–48], we review a new algebraic approach used to determine the global minimum of the Higgs scalar potential, its stability, and the spontaneous symmetry breaking from $SU(2)_L \otimes U(1)_Y$ down to $U(1)_{em}$, in the extension of the SM known as the THDM, where φ_1 and φ_2 stand for two Higgs scalar field doublets with identical quantum numbers

Stability and the stationary points of the potential can be analyzed in terms of four real constants given by

$$K_0 = \sum_{i=1,2} \varphi_i^\dagger \varphi_i, \qquad K_a = \sum_{i,j=1,2} (\varphi_i^\dagger \varphi_j)\sigma_{ij}^a \quad (a=1,2,3), \tag{1}$$

where $\sigma^a (a=1,2,3)$ are the Pauli spin matrices. The four vector $(K_0, \boldsymbol{K})$ must lie on or inside the forward *light cone*, that is

$$K_0 \geq 0, \qquad K_0^2 - \boldsymbol{K}^2 \geq 0. \tag{2}$$

Then the positive and hermitian 2×2 matrix

$$\underline{K} = \begin{pmatrix} \varphi_1^\dagger\varphi_1 & \varphi_2^\dagger\varphi_1 \\ \varphi_1^\dagger\varphi_2 & \varphi_2^\dagger\varphi_2 \end{pmatrix} \tag{3}$$

may be written as

$$\underline{K}_{ij} = \frac{1}{2}\left(K_0\delta_{ij} + K_a\sigma_{ij}^a\right). \tag{4}$$

Inverting (1) we obtain

$$\varphi_1^\dagger\varphi_1 = (K_0+K_3)/2, \qquad \varphi_1^\dagger\varphi_2 = (K_1+iK_2)/2, \qquad \varphi_2^\dagger\varphi_2 = (K_0-K_3)/2, \qquad \varphi_2^\dagger\varphi_1 = (K_1-iK_2)/2. \tag{5}$$

The most general $SU(2)_L \otimes U(1)_Y$ invariant Higgs scalar potential can thus be expressed as

$$V(\varphi_1,\varphi_2) = V_2 + V_4, \tag{6a}$$

$$V_2 = \xi_0 K_0 + \xi_a K_a, \tag{6b}$$

$$V_4 = \eta_{00}K_0^2 + 2K_0\eta_a K_a + K_a\eta_{ab}K_b, \tag{6c}$$

where the 14 independent parameters ξ_0, ξ_a, η_{00}, η_a and $\eta_{ab} = \eta_{ba}$ are real. Subsequently, we define $\boldsymbol{K} = (K_a)$, $\boldsymbol{\xi} = (\xi_a)$, $\boldsymbol{\eta} = (\eta_a)$ and $E = (\eta_{ab})$.

2.1 Stability

From (6), for $K_0 > 0$ and defining $\boldsymbol{k} = \boldsymbol{K}/K_0$, it is obtained

$$V_2 = K_0\,J_2(\boldsymbol{k}), \qquad J_2(\boldsymbol{k}) := \xi_0 + \boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{k}, \tag{7}$$

$$V_4 = K_0^2\,J_4(\boldsymbol{k}), \qquad J_4(\boldsymbol{k}) := \eta_{00} + 2\boldsymbol{\eta}^{\mathrm{T}}\boldsymbol{k} + \boldsymbol{k}^{\mathrm{T}}E\boldsymbol{k}, \tag{8}$$

where the functions $J_2(\boldsymbol{k})$ and $J_4(\boldsymbol{k})$ on the domain $|\boldsymbol{k}| \leq 1$ have been introduced. For the potential to be stable, it must be bounded from below. The stability is determined by the behavior of V in the limit $K_0 \to \infty$, and hence by the signs of $J_4(\boldsymbol{k})$ and $J_2(\boldsymbol{k})$ in (7) and (8). In this analysis only the *strong* criterion for stability is considered, that is, the stability is determined solely by the V quartic terms

$$J_4(\boldsymbol{k}) > 0 \quad \text{for all } |\boldsymbol{k}| \leq 1. \tag{9}$$

To assure that $J_4(\boldsymbol{k})$ is always positive, it is sufficient to consider its value for all its stationary points on the domain $|\boldsymbol{k}| < 1$, and for all the stationary points on the boundary $|\boldsymbol{k}| = 1$. This leads to bounds on η_{00}, η_a and η_{ab}, which parameterize the quartic term V_4 of the potential.

The regular solutions for the two cases $|\boldsymbol{k}| < 1$ and $|\boldsymbol{k}| = 1$ lead to

$$f(u) = u + \eta_{00} - \boldsymbol{\eta}^{\mathrm{T}}(E-u)^{-1}\boldsymbol{\eta}, \tag{10}$$

$$f'(u) = 1 - \boldsymbol{\eta}^{\mathrm{T}}(E-u)^{-2}\boldsymbol{\eta}, \tag{11}$$

so that for all "regular" stationary points $\boldsymbol{k}$ of $J_4(\boldsymbol{k})$ both

$$f(u) = J_4(\boldsymbol{k})|_{\mathrm{stat}}, \quad \text{and} \tag{12}$$

$$f'(u) = 1 - \boldsymbol{k}^2 \tag{13}$$

hold, where $u = 0$ must be set for the solution with $|\boldsymbol{k}| < 1$. There are stationary points of $J_4(\boldsymbol{k})$ with $|\boldsymbol{k}| < 1$ and $|\boldsymbol{k}| = 1$ exactly if $f'(0) > 0$ and $f'(u) = 0$, respectively, and the value of $J_4(\boldsymbol{k})$ is then given by $f(u)$.