Analysis of Transportation Networks Subject To Natural Hazards – insights from a Colombian case

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Abstract

This study provides an applied framework to derive the connectivity reliability and vulnerability of inter-urban transportation systems under network disruptions. The proposed model integrates statistical reliability analysis to find the reliability and vulnerability of transportation networks. Most of the modern research in this field has focused on urban transportation networks where the primary concerns are guaranteeing predefined standards of capacity and travel time. However, at a regional and national level, especially in developing countries, the connectivity of remote populations in the case of disaster is of utmost importance. The applicability of the framework is demonstrated with a case study in the state of Antioquia, Colombia, using historical records from the 2010-2011 rainy season, an aspect that stands out and gives additional support compared to previous studies that considers simulated data from assumed distributions. The results provide significant insights to practitioners and researchers for the design and management of transportation systems and route planning strategies under this type of disruptions.

Keywords

Transportation reliability, vulnerability, systems reliability

1. Introduction

Transportation network reliability is a growing field of study [1], [2]. The increase in natural disasters in the last few decades and the strong dependency of developing countries on their transportation systems has increased the need for urban and inter-urban transportation reliability studies. This necessity is mostly attributed to the dependency upon an efficient and reliable transportation network to provide accessibility and promote the safe and efficient movement of people and goods [3].

One of the main research fields in transportation reliability focuses on the study of urban networks and the probability that the system performs within predefined standards of capacity and travel time [4]. Although guaranteeing mobility in contemporary mega-cities is one of the main concerns of modern societies, at a regional and national level, especially in developing countries, the connectivity of remote populations to the main consumption markets are of utmost importance, mostly due to the potentially severe consequences to the community when network degradation occurs. According to Kim & Kang [5], infrastructures such as transportation, water supply, sewers, telecommunications and electrical and gas networks comprise the critical backbone of an urban society and contribute to economic services, activities and quality of life. Nicholson & Du [6] highlight the importance of transportation networks as the most important lifelines in the event of natural disasters because the restoration of other systems depends strongly on the ability to transport people, equipment and basic items to damaged sites [3].

In Colombia, the distances between communities imply that the probability of reaching primary services such as hospitals and health centers in the case of a major natural disaster is low. For this reason, a transportation network reliability study of inter-urban networks attains a higher priority. In this work, a connectivity reliability and vulnerability study of the primary transportation network in the state of Antioquia, Colombia, is developed. Although these types of disruptions can come from different risk sources, this study focuses on the impact of road disruptions in Antioquia, the state most affected by the Colombian rainy season of 2010-2011. The objectives of this paper are as follows: (1) analyze the different risks and threats that can affect the road network performance, (2) develop a quantitative framework that integrates statistical analysis to assess the reliability and vulnerability of a degradable road network and (3) identify the most vulnerable links (critical segments) in the network to evaluate the failure consequences for the whole system.

The rest of the paper is organized as follows. Section 2 reviews the literature related to applications of transportation network reliability, focusing on four main areas: connectivity reliability, performance reliability and vulnerability. Section 3 explains the proposed framework and develops the network reliability and vulnerability model. Section 4 presents an illustrative example applied to a real network in Antioquia, Colombia. The reliability function for each road is depicted along with the identification of the critical links and how their closure affects the network traffic flow. Properties of the developed model are analyzed to find interesting insights. Finally, conclusions and future work in the area are presented in Section 5.

2. Literature Review– Transportation Network Reliability

Transportation network reliability has provided a framework for the strategic planning of urban and inter-urban road systems [2]. Its origins were inspired by the study of natural disasters that affect network connectivity, focusing only on the performance of the transportation system. Its practical applications range from a new evaluation measure for the design and planning of road networks satisfying a certain level of redundancy or connectivity, to its potential role in traffic management systems. As a system wide perspective, reliability can be viewed as the degree of stability of the quality of service that a system offers [7]. Technically, reliability is often defined as the probability that a system will perform its intended function under operating conditions, for a specified period of time [8]. Consequently, a transport network can be termed as reliable when it can perform in good condition and can accommodate different levels of disruption for a certain time [9].

Transportation reliability can be viewed from two angles. In the view of the individual traveller the system is either operative or not, whereas at an aggregate level there will be a portion of travelers finding the system operative, while others will not [10]. Given the transportation system is based on the transportation network (link and node) and is characterized by various traffic parameters, the study of transportation system reliability involves two interrelated factors [11]: the infrastructure, consisting of roads, bridges, tunnels, railway lines, etc, and the behavioral responses of the users; grouping transportation network reliability studies into two major areas: connectivity reliability and performance reliability [12]. Applications of connectivity reliability have focused on inter-urban road networks, while performance reliability has typically been applied to urban networks focusing on the impact of capacity and travel time variability on users' performance [14], [15]. Due to the sole emphasis of connectivity reliability on failure probabilities and the socio-economic cost derived from a degraded transportation network, the concept of vulnerability, which is strongly related to the consequences of link failure, has been the subject of considerable research interest in recent years [16]-[18].

A number of approaches can be undertaken for the assessment of transportation reliability depending on the type of system, the objective of analysis and the information available. Among the most popular approaches are [19] statistical analysis (parametric and non-parametric models, regression analysis, hazard models and accelerated lifetime models), probabilistic modeling (Markov chains, Petri nets, dynamic modeling and Bayesian networks), risk analysis (probabilistic risk assessment, expert judgment and tabular methods), complex network theory (graph theory and network flow theory), simulation (Agent based and Monte Carlo methods), and dynamic control system theory.

Since a thorough review is not within the scope of this research, the reader is referred to Wang et al. [20] and Da-Rong et al. [21] for connectivity reliability; Taylor [22] and Carrion & Levinson [23] for performance reliability; Berdica [24], Wang et al. [25] and Papathoma–Köhle et al. [26] for vulnerability; and Sullivan et al. [27] for general link/network disruption analysis.

Connectivity reliability

Connectivity reliability can be viewed as a pure topological measure that is concerned with the probability that the network nodes remain connected, or the probability that there exist at least one path without disruption or heavy delay to a given destination within a given time period, when one or more links of the network have been removed [4], [7], [9], [12]. As a result, the connectivity reliability of a network depends entirely on link reliability, which is determined by factors such as road structure, roadside condition, and disaster severity [28]. Connectivity reliability is regarded as the most fundamental reliability measure since guaranteeing connectivity to other nodes can be considered as a necessary condition, whereas other functions like accessibility, that indicates the ease with which individuals at a node can pursue an activity, can be considered as a sufficient condition for establishing the robustness of a network [28]. According to Nicholson [13] connectivity reliability may be appropriate when congestion is not an issue, and one is dealing with a relatively sparse network (e.g. an interstate highway system).

The literature related to connectivity reliability can be divided in two groups. The first group considers algorithms to make the calculation of network reliability more computationally efficient. This group generally uses a mixture of statistical analysis, probabilistic modeling and simulation. Since is not possible to expect to find a definition for reliability that accommodates the many real situations, the second group focuses on the formulation of new measures to calculate network reliability using non classical approaches such as complex network theory and risk analysis. Within the first group, the work made by Colbourn [30] showed the complexity of reliability computations by complete state enumeration in practical networks. State of the art algorithms, bounding methods and Monte Carlo strategies are presented as alternatives to calculate network reliability. Wakabayashi and Iida [9] made an essential work using a graph theory approach for assessing the reliability of travel between a pair of nodes in a transportation network. They proposed a method using minimal path and cut sets that converge rapidly towards the exact reliability. Du and Nicholson [31] proposed an integrated equilibrium model using complex theory that allows the network of a transportation system to have different degrees of arc capacity degradation, resulting in different degrees of system performance. The model is applied to a simple network using an exact algorithm to estimate the system reliability.

Referring to the second group, Kondo et al. [28] established a method for evaluating a network in terms of both connectivity and accessibility. The Potential Accessibility Index (CPAI) is proposed measuring the link reliability per unit of distance. For ease of calculation, an algorithm to approximately compute CPAI by applying a crude sampling Monte Carlo method was developed. Qian et al. [32] presented a model applied to the Lanzhou city using complex networks capable of modeling intentional and random attacks, finding the network's poor connection, being stronger under random attacks than under intentional attacks.

Performance reliability

Performance reliability has focused on the development of measures that affect the user's welfare on the transportation network. While connectivity reliability can be viewed from a supply perspective, analyzing the different hazards that can affect the road network, performance reliability can be viewed from the demand side, and the threats that affect the user's travel performance. Performance reliability literature can be divided in two major groups: travel time reliability and capacity reliability. Travel time reliability is defined as the probability of reaching a chosen destination within a given time [24], [12], and is considered to be one of the key measures for the performance of transport systems [33]. Despite being a relatively recent metric, travel time reliability has been widely used to evaluate the performance and service levels in urban networks, frequently combined with vulnerability analysis [4]. Nicholson & Du [6] distinguished between two types of travel time reliability in urban networks, according to the variability source: daily variations in traffic levels and events causing road network degradation (e.g., traffic accidents). Studies made on events concerning daily variations have stressed the necessity of fitting probability distributions from observed data. Some authors have proposed well-known distributions like the Gamma and lognormal distributions [34]-[36]. Others suggested less common distributions to represent the travel time variability of urban roads [37]-[39]. Aron et al. [40] made a comparison of six statistical distributions concluding that the Gamma mixture and the Normal mixture showed the best fit for the variation of travel times. In this research, we use real failure data to find the reliability function of different roads in Colombia proposing the use of the Poisson Zero Inflated Distribution (ZIP), a relatively unknown distribution that to the best of our knowledge has not been used in reliability studies. Several measures have been proposed including standard deviation, coefficient of variation, buffer index and planning time [41]. A review and application of current measures is presented by Lomax et al. [42] and van Lint et al. [43]. An application of travel time reliability is developed by Tu et al. [33] in the urban roads in the southeast of the Netherlands.

Capacity reliability is defined as the probability that a network can successfully accommodate a given level of travel demand [4]. This concept was first introduced by Chen et al. [14] and applied by Yang et al. [44], who suggested how travel time and capacity reliability concepts could be used to evaluate different traffic calming strategies in urban areas and specified how combining these two measures could provide a valuable network design tool. An application of this metric is made by Chen et al. [14] to the Tuen Mun – Kowloon highway in Hong Kong, China, with random arc capacities.

Vulnerability

The definition of road network vulnerability has been a topic of intense debate and a concise definition has not yet been generally accepted. What is apparent, is that vulnerability, unlike reliability, needs to be more than a quantitative probability calculation related to the functioning or non-functioning of a network link [7]. Early interpretations of road network vulnerability describe the term as a kind of "Achilles' heel" - a deadly weakness in spite of an overall strength that can potentially lead to a downfall [25] and "little strokes fell great oaks" - a relatively small incident that can cause a major damage [45]. These interpretations have been used to exhibit its minor origins and long lasting consequences. In an attempt to provide a complete insight of network vulnerability Murray et al. [46] presented a scenario-based typology for identifying important disruptions and evaluating associated network vulnerabilities. Husdal [7] affirmed that although transportation networks are susceptible to a wide range of hazards is possible to categorize them in terms of its structure, nature and traffic related characteristics. Early definitions of road network vulnerability closely related the concept to reduce levels of accessibility/serviceability as a consequence of network failure or degradation [4], [16], [24], [29], [47], [48]. However, several authors have stressed that vulnerability, needs to be more closely aligned with network weakness and the consequences of failure [49], [50]. Whereas probability or predictability is a major concern in network reliability studies, the impacts or consequences of disruptions are the main focus of vulnerability studies [7]. Subsequent researches have attempted to indicate the connection between vulnerability and the concepts of risk (involving both the consequences and the probability of degradation) [10], [17], [18], [51], [52], and the concept of resilience (the ability to anticipate, withstand and recover from a disruption) [53]. Although vulnerability is still in developing stages, it seems to be an accepted metric for use in both urban and interurban road networks subject to different types of disruption or degradation (e.g., recurrent congestion, traffic accidents and construction sites) [10].

A great body of literature related to transportation vulnerability has focused on finding vulnerable links, weak spots or critical infrastructure in the road network to maintain its robustness. Sullivan et al. [27] made a comprehensive review of the literature related to the field of network disruption analysis. They presented a framework that categorizes network disruptions, addressing important concepts and challenges associated with measuring and quantifying the impacts of various types of disruptions in transportation networks. Knoop et al. [54] classified the literature related to link level vulnerability into two groups. The first group of full calculation methods in which the capacity is reduced for each link separately. In order to find the most critical links all possible states in the network are simulated. A second group uses key link measures to guide the search for the most vulnerable links. In the first group Knoop et al. [55] presented a study of the performance reduction caused by blocking one link in a sequential manner. Using a simulation model two situations were addressed, one in which the driver stick to their route and one in which adapt their route to the new situation. The model is tested in the roads around Rotterdam. In a later article the authors modeled the same situations with the introduction of spillbacks to assess the consequences of link closures [56]. Jenelius & Cats [57] presented a methodology for assessing the value of new links for transportation robustness considering disruptions. Berdica and Mattsson [17] made a model-based case study in Stockholm to analyze the city's vulnerability to the failure of critical links, capacity reductions and traffic demand variations. Nicholson and Du [58] argued that vulnerability can be dissociated into one component of probability and one component of consequence and included both of this aspects by stating that a link is "weak" if the probability of degradation is high and "important" if the consequences of degradation are high, with the "critical links" being those that are both weak and important. Jenelius et al. [10] followed this definition to introduce the concepts of "importance" and "exposure". They argued that a reasonable measure of reduced serviceability/operability/accessibility is the increase in generalized cost of travel (time, distance, money, etc). Therefore the term importance is related to the consequences (increased in travel cost) for a collection of sites of a failing link or group of links. On the other hand exposure, which is site-dependent, is used to measure the consequences of link failures for a single or a group of demand nodes. These measures are applied to a road network in northern Sweden. In a later article Jenelius and Mattsson [18] extended the application of site exposure with a methodology for studying road vulnerability under area covering disruptions, where the network is covered by grids of uniformly sized cells.

In the second group different authors have calculated the vulnerability of a road network using accessibility-based indicators. Tampére et al. [59] presented a methodology capable of rapidly scanning a large network for the most vulnerable sections. The measures developed represented different traffic dynamics to find the most vulnerable points in the network. Knoop et al. [54] made a comparison of different criteria to indicate the most vulnerable links applied to urban networks in the cities of Delft and Rotterdam. They concluded that link based indicators are suitable for indicating the vulnerability of the traffic flow on specific links. However, when capturing all network effects, the indicators proved to be insufficient. Several works have analyzed the vulnerability of Australia's main highway network. Taylor and Susilawati [48] used the ARIA remoteness index seeking to determine the most critical location in rural and remote areas. A similar work is developed by Taylor et al. [29] and Taylor & D'Este [16], who applied the ARIA and Hansen integral accessibility index to determine the vulnerability based on the identification of possible weak spots. Scott et al. [60] used the same measure as Jenelius et al. [10] (travel time cost increase) for evaluating the critical importance of a given highway segment to the overall system. They proposed the Network Robustness Index (NRI) that compares the travel time cost of removing a link to the system, with the travel time cost incurred when all links are in the network. The results showed that the NRI yields different highway solutions than the traditional Volume/Capacity ratio. Sullivan et al. [61] also used the term robustness for identifying critical road segments in transportation networks advancing the work made by Scott [60]. Snelder et al. [62] presented a framework for analyzing network robustness, defining its main elements and the connection with reliability.

The work presented in this paper can be classify within the first research group (full calculation methods) since we analyze the flow reduction for all disruption states in the network, finding the most critical links. The propose measure bears a close resemblance with the work of Nicholson and Du [58], having one component of probability and one component of consequence. However, unlike Jenelius et al. [10] we use a combined measure to express both the failure probability and the disruption consequences; calculating the expected value of flow reduction when a specific link fails, compared to the flow when all links are functioning, an aspect that concurs well with the work made by Scott et al. [60].

Despite most of the literature can be classified into the two groups mentioned above, novel approaches using game theory have been emerging. Bell [11] and Bell and Cassir [66] uses game theory to study a two player non-cooperative game between the network user that tries to minimize the expected trip cost and an evil entity that tries to maximize it. Murray-Tuite and Mahmassani [67] presented a bilevel formulation that can be viewed as a game between an “evil entity” and the traffic management agency. A measure for the importance of a specific link to the connectivity of an origin destination pair is used as vulnerability index.

Figure 1 classifies relevant theoretical models and applications of transportation network reliability that bear a close resemblance to this research. The sunburst diagram classifies the literature according to the three major groups stated above (connectivity, performance and vulnerability), the corresponding divisions among every group, the method applied and the research made. Table 1 shows additional characteristics.

Figure 1. Applications of transportation network reliability by metrics and methodology applied

Table 1. Reviewed literature on transportation network reliability

3. Connectivity Reliability

The Connectivity reliability concept use in this work concerns the probability of having at least one failure in a specific day of operation. Therefore, the model is based on the following assumptions that reflect to a greater extend the conditions of inter-urban transportation networks in Colombia: (i) total interruption of the traffic flow over the disruptive link; (ii) in the case of a road closure, the user is willing to wait the necessary time until the network is repaired; (iii) congestion in the network is not considered; and (iv) at most, there is only one interruption in every link at a time. A random variable $\mathcal{X}(t),\ \ t \geq 0$, represents the number of failures over a specific link of the network in a period t or time interval $[0, \tau]$ and it can be considered to come from a Poisson process. The cumulative probabilities of the reliability function $\mathcal{R}(\tau)$ and the failure function $\mathcal{F}(\tau)$ are shown in equation (1). Events $\{\mathcal{X}(t) \in \mathcal{D}, 0 \leq t \leq \tau\}$ and $\{\mathcal{X}(t) \in \mathcal{D}, 0 \leq t \leq \tau\}^{\mathcal{C}}$ refer to the system reliability and the failure probability, respectively [83].

$$\mathcal{R}(\tau) = \sum_{n=0}^{\infty} p_{r,n} \mathcal{P}(\mathcal{N}(\tau) = n); \qquad \mathcal{F}(\tau) = 1 - \mathcal{R}(\tau) \tag{1}$$

Where $\{\mathcal{P}(\mathcal{N}(\tau) = n)\}$ refers to the cumulative probability of the random variable, $\mathcal{N}(\tau)$ is the number of failures in the analysis interval $[0, \tau]$ and $p_{r,n}$ is the probability that $\{\mathcal{X}(t) \in \mathcal{D}, 0 \leq t \leq \tau\}$ in the event $\{\mathcal{N}(\tau) = n\}$. The reliability always decreases over time; thus, the longer the operation time is, the lower the probability that the system works correctly. Along with the concept of reliability is the concept of hazard rate $h(x)$, which is the conditional probability that the system shows one or more failures because x failures have occurred in the past. This definition is shown in equation (2).

$$h(x) = f(x)/\mathcal{R}(x); \qquad \mathcal{H}(x) = \int h(x) \tag{2}$$

3.1 Zero Inflated Poisson distribution (ZIP)

Classical Poisson processes describe random variables that represent the total number of occurrences of a given phenomenon during a fixed time interval. However, in some situations, despite the random variable has a Poisson distribution, a wide dispersion is obtained due to the presence of a greater number of null counts. In these cases, is necessary to apply alternative models for data analysis with excessive amounts of null entries and long right tails. Ridout et al. [83] presented a survey of models for count data with excess zeros in disciplines including agriculture, econometrics, patent applications, medicine and recreational facility use. One of these models is the the Zero-Inflated Poisson distribution (ZIP) presented by Lambert [84], which is a method of modeling this type of data and assumes that the null entries come from two different sources considered as a combination of two stochastic processes: one that generates null entries equal to zero and another that generates null entries and non-null entries. A Bernoulli random variable is used to determine whether the result of an individual count comes from the process that generates (or not) only zeros. For example, for an insurance company excess zeros may arise when claims near the deductible are not reported, thus inflating the number of zero policies when compared to the predictions of a Poisson or Negative Binomial distribution [85]. This is the distinction between structural zeros, which are inevitable, and sampling zeros, which occur by chance [83]. The Zero-Inflated Poisson Distribution (ZIP) is a generalization of the Poisson distribution and can be expressed as in equation (2):

$$f(x, p, \lambda) = \begin{cases} p + (1-p) \cdot \exp(-\lambda) & if \;\; x = 0 \\ (1-p) \cdot f(x, \lambda) & if \;\; x > 0 \end{cases} \tag{3}$$

where $f(x, \lambda)$ is the Probability density function (*p.d.f.*) of the Poisson distribution and refers to the probability of obtaining x failures in a specific analysis period, λ is the average number of failures in a specific day and p is the probability of obtaining extra zeros. For the ZIP model, it is possible to verify that the expected value of the random variable can be expressed as: $E[\mathcal{X}] = p\lambda$, and its variance as: $V[\mathcal{X}] = p\lambda + p\lambda^2(1-p)$, knowing that $\{x_1, x_2, x_{3,} \dots, x_n\}$ is the set of observations where n is the sample size; n_i is the counting number i in the sample (e.g., n_0 is the number of zeros in the sample, which, for this study, is the number of days with zero road closures).

4. Network Vulnerability

According to Murray et al. [46] the use of multiple methodologies for evaluating the vulnerability of critical infrastructure, particularly networks, is essential for deepening the understanding of the implications of unplanned events. The concept of network vulnerability can have many interpretations and several methodologies have been proposed, still, two main components made the foundations of most network vulnerability studies: probability and consequence. These concepts along with the identification of weak spots - critical infrastructure - in the transportation network represent some of the most important applications of network vulnerability. Examples are the methodologies proposed by Nicholson and du [58] for the identification of critical links and the concepts of importance and exposure used by Jenelius et al. [10]. The network vulnerability approach propose in this research focus on the identification of the most critical links or group of links whose removal from the network result in the greatest damage in terms of decrease in traffic flow.

This research applies concepts of connectivity reliability and vulnerability to a road network degraded by natural disasters, using a probability and graph theory approach. A full state enumeration method is used in order to make a complete search of all sub-graphs in the network. Although the number of sub-graphs increases very rapidly with the number of arcs making the calculation of all possible states computationally inefficient; for small networks this method has significant advantages over more algorithmic approaches due to the exploration of all failure possibilities. An early description of this method is presented by Ahmad [86]. In order to find the vulnerability of the network we propose a measure that calculates the expected decrease in traffic flow among all the states, combining both the failure probabilities and the disruption consequences.

Road networks are usually represented as a graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$, where $\mathcal{N}$ is the set of nodes and $\mathcal{A}$ the set of arcs. A node $n \in \mathcal{N}$ represents a city or a transfer node. The arcs $a \in \mathcal{A}$ represent the road network connecting the nodes. Every arc $a \in \mathcal{A}$ has assigned a traffic flow parameter f_{ij} from an origin-destination (OD) matrix, where i and j are the starting and end nodes, respectively. For simplicity suppose that $f_{ij} = f_{ji}$ making the OD matrix to be symmetrical. Similarly, every arc can fail independently with failure probability p_{ij} and has associated a stochastic binary variable x_{ij} with the following static distribution: $P(X_{ij} = 0) = p_{ij}$ representing the failure probability of the arc (as given by the network reliability study) and $P(x_{ij} = 1) = 1 - p_{ij} = q_{ij}$ representing the functioning probability. In order to compute all the possible failure states in the network with $|\mathcal{A}|$ stochastic variables is necessary to consider $S = 2^{|\mathcal{A}|}$ elementary events, where each event corresponds to a sub-graph $g \subset \mathcal{G}$. For instance, consider the network shown in Figure 2(a). The event $s = \{x_{12} = 0, x_{13} = 1, x_{24} =$

$1, x_{34} = 1,\}$ corresponds to the sub-graph shown in Figure 2(b). Since all variables are independent by assumption, the event probability for event s can be computed as:

$$P(s) = \prod_{i=1,j=1|i\neq j}^{|\mathcal{N}|} [x_{ij}.p_{ij} + (1 - x_{ij})\, q_{ij}] = q_{12}.p_{13}.q_{23}.p_{24}.p_{34} \tag{4}$$

And the total flow in the network is:

$$F_s = \sum_{i=1,j=1|i=j}^{|\mathcal{N}|} (f_{ij}.x_{ij}) = f_{13} + f_{24} + f_{34} \tag{5}$$

Figure 2. Sample network

4.1 Measuring link criticality

In order to find the most critical links in the network we consider the concept of flow decrement reliability for each sub-network proposed by Du and Nicholson [31] defined as the reduction in flow (between the OD pair connected to the sub-network) when a link is degraded, as a proportion of the flow when there is no degradation. However, this concept only includes the consequences of link degradation independent of their failure probability. Hence, we also apply the concept of link criticality defined by Nicholson and Du [58] which comprises both “weakness” as the probability of degradation (for our case the failure probability found with the connectivity reliability study); and “important” as the consequences of degradation (traffic flow decrement). As a result, the link criticality measure propose in this research compute the expected value of the traffic flow decrease across all failure events when a specific link fails, and is given by:

$$E\left(Flow\middle|x_{ij} = 0\right) = \textstyle\sum_{s|x_{ij}=0}^{S}[(F_{cn} - F_s)P(s)] \tag{6}$$

Where F_{cn} is the total flow when all arcs are functioning, F_s is the flow for a specific event in which arc ij has fail and $P(s)$ is the probability of that event. This measure represents the vulnerability of the network or the expected traffic flow that the network can expect to lose when a specific link fails ($x_{ij} = 0$). If equation 5 is divided by the total flow of the network when all arcs are functioning, the link criticality or link vulnerability measure can be expressed as a percentage of the total flow as:

$$E\left(Flow\middle|x_{ij} = 0\right) = {\left[\textstyle\sum_{s|x_{ij}=0}^{S}(F_{cn} - F_s)P(s)\right]}\Big/{F_{cn}} \tag{7}$$

This measure has a close relation with the concepts of “importance” and “exposure” proposed by Jenelius [10], however, both the user and the topological perspectives are combined in a single measure. Besides, we present the consequences of link degradation as the decrease in traffic flow and not as the increase in total travel cost.

The algorithm that illustrates the procedure to find the connectivity reliability and vulnerability can be summarized as follows: (1) Generate all sub-graphs $\mathscr{g} \subset \mathcal{G}$ by changing the values in the binary stochastic variable x_{ij} from 0 to $2^{|\mathcal{A}|} - 1$ one at a time. (2) For each sub-graph compute the "link weakness" as the event probability $P(s)$ of event s; and the "link importance" as the average traffic flow F_s. (3) Using the results from (1) and (2) compute the expected traffic flow decrease for the closure of each link, identifying the most vulnerable sections.

5. Case study – Transportation Network in Antioquia, Colombia

Antioquia, Colombia, is one of the most industrialized states in the country with a population of around six and a half million people. Is located in a zone where special geomorphological, anthropic and weather factors converge, creating the perfect conditions for debris and landslides [87]. This situation increased in the country's 2010-2011 rainy season, causing major floods to many areas and 1,984 cumulative days of road closures, 13.7% over the national average and the highest in the country [88]. The primary road network in Antioquia is composed of seven main roads forming a serial system, with very limited redundant roads for full-truckload vehicles, increasing the fragility of the road network in the case of major disruptions. Approximately 20% of the state's Gross Domestic Product (GDP) comes from industrial activities (e.g., manufacturing in the main city and agriculture and cattle farming in distant populations) [89]. Moreover, 70% of the cargo (e.g., raw materials, finished products, and livestock) is transported using full-truckload vehicles that depend on good network conditions to make their journeys to the country's cities and seaports.

Figure 3 shows Antioquia's political map along with the primary and secondary road corridors, the population density and the location of the main community centers. The state cartography and the highway system were obtained from the System for Geographical Information and Territorial Planning – SIGOT.

Figure 3. General description of the roads, population density and main city centers in the state of Antioquia, Colombia

5.1 Hazard Identification

According to D'Este & Taylor [4], reliability studies based exclusively on probabilities and absolute connectivity may obscure potential network problems because the standard measures of network reliability might underestimate small failure probabilities even if the economic and social consequences of a link failure are extremely high. However, Dalziell [90] presents the conditions that must be met to ensure the study's validity: (1) a reliable record of adequate length must be available, describing what has occurred in the past, and (2) the conditions under which the data were gathered must not have changed significantly during that period or be expected to change in the future. In this study, both conditions are fulfilled because the analysis was based on 3,688 road closures provided by the national road institute (INVIAS – Instituto Nacional de Vías de Colombia), an aspect that stands out and gives additional support compared to previous studies that considers simulated data from assumed distributions. According to Jenelius and Mattsson [18] when dealing with precipitation phenomena detailed historical or modeled future data on both average and extreme levels are often available. In Colombia, the rainy season is associated with a cyclic phenomenon known as ENSO (El Niño Southern Oscillation), which has a steady seasonality with available historical records and can be forecasted with relatively accuracy.

The data were gathered during the 2010-2011 rainy season in Colombia. Twenty different failure types were identified in the road network (e.g., road sinking - 40.7%, landslide - 24.4%, and asphalt layer loss - 6.7%). A functional tree was created to understand the effects of failures in the road network and to identify the primary and secondary components and the functions they fulfill within the network system. Each of the different failure types was later classified according to its frequency of occurrence and its capacity to damage the system's functions (Figure 4).

Figure 4. Functional tree

The total number of road closures in the state of Antioquia during the rainy season is shown in Figure 5, divided by road. The red bubbles show the spots where the disruptions occurred, while their sizes represent the total failure occurrences. Roads such as Medellín – Caucasia show relatively uniform failures over their entire lengths, while Medellín – Puerto Berrío, Medellín – Puerto Triunfo and Medellín – Uraba show a great amount of road failures in the segments close to the capital city (Medellín). This behavior is mostly attributed to the geography of the zone, which lies at the ramification of the central and western ranges of the Andes, the area most affected by the rainy season.

Figure 5. Road closures during the 2010-2011 rainy season in Antioquia, Colombia

5.2 Road closure analysis

This section analyzes the number of road closures that occurred daily in the road network of the state of Antioquia, Colombia. Because there is a large number of days with null entries (zero road closures), the distribution that shows the best fit is the ZIP distribution [84]. The frequency histograms are shown in Figure 6.

Figure 6. Histograms of road closures in Antioquia's main roads

The fitted values of road closures for the ZIP distribution are shown in Table 2. All the road failures in the state of Antioquia come from a ZIP distribution, with the exception of the Medellín – Puerto Triunfo road, which shows a better fit with a Poisson distribution. The most frequent result is at least one failure per day of operation. The parameter λ indicates the average number of days per failure, $1/\lambda$ the number of failures per day, and p the probability of obtaining extra zeros.

Table 2. Distribution parameters

Figure 7 shows the behavior of the reliability and the hazard functions. Equation (3) was used to measure the probability of having zero road closures P_0. With this probability the reliability and hazard functions were calculated for a one year period ($0 \leq t \leq 365$). The reliability function can be expressed using the exponential cumulative density function $\mathrm{F}(t) = 1 - e^{-P_0 t}$, as $\mathcal{R}(t) = 1 - \mathrm{F}(t) = e^{-P_0 t}$ and the hazard function as $\mathcal{H}(t) = e^{-(1-P_0)t}$. In general, the reliability of the roads of Antioquia is very low with all roads having reliability close to cero after the seventh day of operation. For a valid analysis, it is necessary to combine the reliability and the hazard functions with the results shown in Table 2. For example, from the reliability function, it is possible to observe that the Medellín - Sonsón road exhibits the lowest reliability in the highway system, however, the failure risk is relatively constant over time with a gradual slope. The number of failure per day is equal to 0.03 (33 days per failure), meaning that the road will face around 11 failures in a yearly period. The Medellín – Urabá (0.29 failures per day) and Medellín – Pintada (0.29 failures per day) roads have a much steeper failure risk and a low reliability, meaning that these roads are very unreliable and will tend to fail relatively quickly. The Medellín – Puerto Berrío road shows the highest reliability, starting at around 55%; however, the failure rate is very steeply meaning that the road will have a great amount of failures during the first days of operation and then this failure rate will tend to stabilize. This road has also the highest number of failures per day with 0.45 (2.22 days per failure). The Medellín – Puerto Triunfo (1.32 failures per day), Medellín – Caucasia (0.06 failures per day) and Primavera – Mansa (0.44 failures per day) roads exhibit the same behavior.

5.3 Vulnerability calculations

Since the road network under analysis is composed by seven major roads, a total of $2^7 = 128$ different failure events were analyzed using the algorithm proposed in section 3. The traffic flows were taken from the results of the 2014 OD survey made by the state government where around 50.000 heavy cargo individual trips where registered. Using the $p.d.f$ of the ZIP distribution we find the probability of encounter one failure for every road and by applying equation (7) it was possible to identify the road segments which subject to failure will cause the greater expected traffic flow decrease to the road network. These results are shown in Table 3. The weakest links can be identified by the highest failure probabilities and the important links the ones with the highest possible consequences, in this case the average percentage in traffic flow throughout the network. Almost all roads in the network have significant loss decrements values indicating the system vulnerability to this type of hazard; the roads Medellín – Puerto Berrío, Medellín – Puerto Triunfo, Primavera – La Mansa and Medellín - Caucasia can be categorized as the most critical links in the network. These roads have the biggest failure probabilities; though not always the greatest decreases in traffic flow. Figure 8 shows the empirical cumulative probability function depicted from the 128 individual events, versus the traffic flow. This function shows that the probability of obtaining higher traffic flow decrements increase rapidly. For example, there are around 10% probability of obtaining a flow less than or equal than 30% and for flow decrements less than or equal to 47% the probability is close to 30%, showing the vulnerability of the road network.

Table 3. Failure probability and link criticality

Figure 8. Cumulative density function

6. Conclusions

The development of models using network reliability theory to tackle network disruptions is a relatively recent field of study, with a tendency to increase in the coming years. Connectivity reliability has mostly been applied to urban networks and the consequences of a disruption to the user's mobility and much of the recent research has focused on urban road networks, where the main concerns are traffic congestion, travel time variability, and the probability that the network will deliver a required standard of performance. In contrast, the connectivity reliability and vulnerability research on inter-urban road networks is a primary consideration in emerging countries and the development of mathematical models that include network disruptions is an area of utmost economic and social importance and highly relevant within the academic field.

This work presents the development and application of a mathematical model to diagnose the connectivity reliability and vulnerability of the primary inter-urban road network in Antioquia, Colombia, one of the most industrialized states in the country. The analysis identified the most vulnerable (critical road segments) where network degradation has severe consequences on the system's overall performance. The reliability study made with real failure data clearly reflects the conditions caused by the rainy season, turning Antioquia's primary road network into one of the most unreliable road systems in the country. In general, the system's capacity to absorb failures is extremely low or decreases abruptly after the second failure. These results allow practitioners to take decisive action when making transportation decisions and designing route planning strategies. Similarly, the results can provide government agencies in charge of the state infrastructure an indication of the current state of the road system.

The measure of vulnerability proposed involved the concepts of weakness and importance in order to calculate the criticality of a specific link, showing the results of the possible impacts of network degradation in terms of traffic flow reduction to remote rural areas as a result of major disruptions in the road network. The complete state enumeration procedure, although inefficient for larger networks, has an extremely short computation time for the system evaluated, guaranteeing exploration of all failure possibilities and the convergence to the true value of the link criticality measure. Although Antioquia's road network was used as a case study, this measure has the potential to be applied to more complex networks (e.g., at a national level). Similarly, the results are not limited to the transportation reliability field but have practical applications to emergency response and humanitarian systems, for the planning and control of road maintenance activities and within the supply chain field regarding the supply and distribution of goods under conditions of high uncertainty caused by a rainy season. More specifically, the reliability function depicted in this study gives a clear and concise sample for the behavior of the Colombian national grid subject to disruptions.

While this study provides a comprehensive framework for understanding network disruption risk caused by natural disasters, it is clear that additional work is necessary. Therefore, it is important to mention the most relevant research paths that can contribute to its development. In this study, the methods for analyzing the reliability of the network consider the links to have a probabilistic or binary state: normal operation and failure. However, natural hazards can cause not only total road failure but also different levels of degradation, partially limiting the transit capacity of the link. This characteristic can be modeled by establishing different levels of road performance and presents a possible extension of the current work.

The availability of accurate information about network disruptions when natural disasters strike is very limited, making the development of future policies for optimal preparedness for these types of disruptions extremely difficult. Therefore, it is necessary to collect and gather disaster information from both government and non-government agencies and to make it available to the academic field to increase the practicality and accuracy of the current research. The results can help road users to improve route decision-making in future natural disasters affecting the road network. Additionally, the information gathered could be used as a new evaluation measure for designing road networks and traffic management systems, satisfying a certain level of redundancy or connectivity within a limited budget.

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References

[1] W. H. K. Lam, "Editorial Special Issue: Network Reliability and Transport Modelling" J. Adv. Transp., vol. 33, no. 2, pp. 121–123, 2010. DOI: 10.1002/atr.5670330202.

[2] M. G. H. Bell and C. Cassir, Reliability of transport networks. Baldock, Hertfordshire: Research Studies Press, 2000. ISBN: 9780863802607.

[3] A. Chen, H. Yang, H. K. Lo, and W. H. Tang, "Capacity reliability of a road network: an assessment methodology and numerical results" Transp. Res. Part B Methodol., vol. 36, no. 3, pp. 225–252, 2002. DOI: 10.1016/S0191-2615(00)00048-5.

[4] G. M. D'Este and M. A. P. Taylor, "Network vulnerability: An approach to reliability analysis at the level of national strategic transport networks" in The Network Reliability of transport - Proceedings of the first international symposium of transportation network reliability (INSTR), 2003.

[5] Y. Kim and W.-H. Kang, "Network reliability analysis of complex systems using a non-simulation-based method" Reliab. Eng. Syst. Saf., vol. 110, pp. 80–88, 2013. DOI: 10.1016/j.ress.2012.09.012.

[6] A. Nicholson and Z. Du, "Degradable transportation systems: An integrated equilibrium model" Transp. Res. Part B Methodol., vol. 31, no. 3, pp. 209–223, 1997. DOI: 10.1016/S0191-2615(96)00022-7.

[7] J. Husdal, "Reliability and Vulnerability versus Costs and Benefits" In: Nicholson, A., Dantas, A., (Eds.), Proceedings of the Second International Symposium on Transportation Network Reliability (INSTR). Christchurch, New Zealand, pp. 182–188

[8] W. Q. Meeker and L. A. Escobar, "Statistical Methods for Reliability Data." John wiley and Sons, 1998. ISBN 0471143286.

[9] H. Wakabayashi and Y. Iida, "Upper and lower bounds of terminal reliability in road networks: an efficient method with Boolean algebra" J. Nat. Disaster Sci., vol. 14, no. 1, pp. 29–44, 1992.

[10] E. Jenelius, T. Petersen, and L.-G. Mattsson, "Importance and exposure in road network vulnerability

analysis" Transp. Res. Part A Policy Pract., vol. 40, no. 7, pp. 537–560, 2005. DOI: 10.1016/j.tra.2005.11.003.

[11] M. G.H. Bell, "A game theory approach to measuring the performance reliability of transport networks" Transp. Res. Part B Methodol., vol. 34, no. 6, pp. 533–545, 2000. DOI: 10.1016/S0191-2615(99)00042-9.

[12] Y. Iida, "Basic concepts and future directions of road network reliability analysis" J. Adv. Transp., vol. 33, no. 2, pp. 125–134, 1999. DOI: 10.1002/atr.5670330203.

[13] A. Nicholson, "Transport Network Reability Measurement and Analysis" Rev. Transp., vol. v. XI, pp. 49–62, 2003.

[14] A. Chen, H. Yang, H. K. Lo, and W. Tang, "A Capacity Related Reliability For Transportation Networks" J. Adv. Transp., vol. 33, no. 2, pp. 183–200, 1999. DOI: 10.1002/atr.5670330207.

[15] M. G. H. Bell and Y. Iida, Transportation network analysis. Wiley, 1997. ISBN 978-0-471-96493-3.

[16] M. A. P. Taylor and G. M. D'Este, "Transport network vulnerability: A method for diagnosis of critical locations in transport infrastructure systems" " in Critical Infrastructure - Reliability and Vulnerability, A. T. Murray and T. H. Grubesic, Eds. pp. 9–30, 2007. DOI: 10.1007/978-3-540-68056-7_9.

[17] K. Berdica and L. Mattsson, "Vulnerability: A model-based case study of the road network in Stockholm" in Critical Infrastructure - Reliability and Vulnerability, A. T. Murray and T. H. Grubesic, Eds. pp. 9–30, 2007. DOI: 10.1007/978-3-540-68056-7_9.

[18] E. Jenelius and L.-G. Mattsson, "Road network vulnerability analysis of area-covering disruptions: A grid-based approach with case study" Transp. Res. Part A Policy Pract., vol. 46, no. 5, pp. 746–760, 2012. DOI: 10.1016/j.tra.2012.02.003.

[19] W. Kröger and E. Zio, Vulnerable systems. Springer, 2011. ISBN 085729654X.

[20] D. Wang, H. Qi, and C. Xu, "Reviewing Traffic Reliability Research" J. Transp. Syst. Eng. Inf. Technol., vol. 10, no. 5, pp. 12–21, 2010. DOI: 10.1016/S1570-6672(09)60060-X.

[21] H. Da-rong, S. Li-bing, Z. Ling, and S. Jun, "Review on road network reliability and trends in the information model" 25th Chinese Control Decis. Conf., pp. 4885–4891, 2013. DOI: 10.1109/CCDC.2013.6561819.

[22] M. A. P. Taylor, "Travel through time" Transp. B, vol. 1, no. 3, pp. 174–194, 2013. DOI:10.1080/21680566.2013.859107.

[23] C. Carrion and D. Levinson, "Value of travel time reliability: A review of current evidence" Transp. Res. Part A Policy Pract., vol. 46, no. 4, pp. 720–741, 2012. DOI: 10.1016/j.tra.2012.01.003.

[24] K. Berdica, "An introduction to road vulnerability: What has been done, is done and should be done" Transp. Policy, vol. 9, no. 2, pp. 117–127, 2002. DOI: 10.1016/S0967-070X(02)00011-2.

[25] Z. Wang, A. P. C. Chan, J. Yuan, B. Xia, M. Skitmore, and Q. Li, "Recent Advances in Modeling the Vulnerability of Transportation Networks" J. Infrastruct. Syst., Vol. 21, no.2, October 2015, pp. 1–9, 2014. DOI 1943-555X.0000232.

[26] M. Papathoma-Köhle, M. Kappes, M. Keiler, and T. Glade, "Physical vulnerability assessment for alpine hazards: State of the art and future needs" Nat. Hazards, vol. 58, no. 2, pp. 645–680, 2010. DOI: 10.1007/s11069-010-9632-4.

[27] J. L. Sullivan, L. Aultman-Hall, and D. C. Novak, "A review of current practice in network disruption analysis and an assessment of the ability to account for isolating links in transportation networks" Transp. Lett. Int. J. Transp. Res., vol. 1, no. 4, pp. 271–280, 2009. DOI: 10.3328/TL.2009.01.04.271-280.

[28] R. Kondo, Y. Shiomi, and N. Uno, "Network evaluation based on connectivity reliability and accessibility" in Network Reliability in Practice - Selected Papers from the Fourth International Symposium on Transportation Network Reliability, D. Levinson, H. Liu, M.G.H. Bell 2012. DOI: 10.1007/978-1-4614-0947-2_9.

[29] M. A. P. Taylor, S. V. C. Sekhar, and G. M. D'Este, "Application of Accessibility Based Methods for Vulnerability Analysis of Strategic Road Networks" Networks Spat. Econ., vol. 6, no. 3–4, pp. 267–291, 2006. DOI: 10.1007/s11067-006-9284-9.

[30] C. Colbourn, "Combinatorial aspects of network reliability" Ann. Oper. Res., vol. 33, pp. 3–15, 1991. DOI 10.1007/BF02061656.

[31] Z. Du and A. Nicholson, "Degradable transportation systems: sensitivity and reliability analysis" Transp. Res. Part B Methodol., vol. 31, no. 3, pp. 225–231, 1997. DOI: 10.1016/S0191-2615(96)00022-7.

[32] Y.-S. Qian, M. Wang, H.-X. Kang, J.-W. Zeng, and Y.-F. Liu, "Study on the Road Network Connectivity Reliability of Valley City Based on Complex Network" Math. Probl. Eng., vol. 2012, pp. 1–14, 2012. DOI:10.1155/2012/430785.

[33] H. Tu, H. Li, H. Van Lint, and H. van Zuylen, "Modeling travel time reliability of freeways using risk assessment techniques" Transp. Res. Part A Policy Pract., vol. 46, no. 10, pp. 1528–1540, 2012. DOI: 10.1016/j.tra.2012.07.009.

[34] R. Herman and T. Lam, "Trip time characteristics of journeys to and from work" Transportation and traffic theory, vol. 6. pp. 57–86, 1974.

[35] A. Polus, "A study of travel time and reliability on arterial routes" Transportation, vol. 8. pp. 141–151, 1979. DOI: 10.1007/BF00167196.

[36] M. Arezoumandi, "Estimation of Travel Time Reliability for Freeways Using Mean and Standard Deviation of Travel Time" J. Transp. Syst. Eng. Inf. Technol., vol. 11, no. 6, pp. 74–84, 2011. DOI:10.1016/S1570-6672(10)60149-3.

[37] S. Susilawati, M. A. P. Taylor and S. V. C. Somenhalli, "Distributions of travel time variability on urban roads" Journal of Advanced Transportation, vol. 47, no.8, pp. 720-736, 2013. DOI: 10.1002/atr.192.

[38] M. Zhenliang, M. Ferreira, L. Mesbah, and S. Zhu, "Modeling distributions of travel time variability for bus operations" Journal of Advanced Transportation, vol. 45. pp. 117–128, 2015. DOI: 10.1002/atr.1314.

[39] M. A. P. Taylor, "Modelling Travel Time Reliability with the Burr Distribution" Procedia - Soc. Behav. Sci., vol. 54, pp. 75–83, 2012. DOI:10.1016/j.sbspro.2012.09.727.

[40] M. Aron, N. Bhouri, and Y. Guessous, "Estimating Travel Time Distribution for Reliability Analysis" in Transport Research Arena 2014, Paris, 2014. DOI: 10.1016/j.trpro.2014.10.014.

[41] N. Bhouri, M. Aron, and J. Kauppila, "Relevance of Travel Time Reliability Indicators: A Managed Lanes Case Study" Procedia - Soc. Behav. Sci., vol. 54, pp. 450–459, 2012. DOI:10.1016/j.sbspro.2012.09.763.

[42] T. Lomax, D. Schrank, S. Turner, and R. Margiotta, "Selecting travel reliability measures" Working Report, Texas Transp. Inst., 2003.

[43] J. W. C. van Lint, H. J. van Zuylen, and H. Tu, "Travel time unreliability on freeways: Why measures based on variance tell only half the story" Transp. Res. Part A Policy Pract., vol. 42, no. 1, pp. 258–277, 2008. DOI:10.1016/j.tra.2007.08.008.

[44] H. Yang, M. G. H. Bell, and Q. Meng, "Modeling the capacity and level of service of urban transportation networks" Transp. Res. Part B Methodol., vol. 34, no. 4, pp. 255–275, 2000. DOI:10.1016/S0191-2615(99)00024-7.

[45] T. Abrahamsson, "Characterization of vulnerability in the road transport system" Working Report, 1997.

[46] A. T. Murray, T. C. Matisziw, and T. H. Grubesic, "A Methodological Overview of Network Vulnerability Analysis" Growth Change, vol. 39, no. 4, pp. 573–592, 2008. DOI: 10.1111/j.1468-2257.2008.00447.x.

[47] A. Chen, C. Yang, S. Kongsomsaksakul, and M. Lee, "Network-based Accessibility Measures for Vulnerability Analysis of Degradable Transportation Networks" Networks Spat. Econ., vol. 7, no. 3, pp. 241–256, 2006. DOI: 10.1007/s11067-006-9012-5.

[48] M. A. P. Taylor and S. Susilawati, "Remoteness and accessibility in the vulnerability analysis of regional road networks" Transp. Res. Part A Policy Pract., vol. 46, no. 5, pp. 761–771, 2012. DOI:10.1016/j.tra.2012.02.008.

[49] F. Kurauchi, N. Uno, A. Sumalee, and Y. Seto, "Network Evaluation Based on Connectivity Vulnerability" in Transportation and Traffic Theory, pp. 637–649, 2009. DOI: 10.1007/978-1-4419-0820-9_31.

[50] E. Jenelius, "Incorporating Dynamics and Information in a Consequence Model for Road Network Vulnerability Analysis" Third International Symposium on Transportation Network Reliability (INSTR), The Hague, The Netherlands, 2007.

[51] J. Sohn, "Evaluating the significance of highway network links under the flood damage: An accessibility approach" Transp. Res. Part A Policy Pract., vol. 40, no. 6, pp. 491–506, 2006. DOI:10.1016/j.tra.2005.08.006.

[52] E. Dalziell and A. Nicholson, "Risk and impact of natural hazards on a road network" J. Transp. Eng., Vol 127, no. 2, 2001. DOI: 10.1061/(ASCE)0733-947X(2001)127:2(159).

[53] L.-G. Mattsson and E. Jenelius, "Vulnerability and resilience of transport systems – A discussion

of recent research" Transp. Res. Part A Policy Pract., vol. 81, pp. 16–34, 2015. DOI:10.1016/j.tra.2015.06.002.

[54] V. L. Knoop, M. Snelder, H. J. van Zuylen, and S. P. Hoogendoorn, "Link-level vulnerability indicators for real-world networks" Transp. Res. Part A Policy Pract., vol. 46, no. 5, pp. 843–854, 2012. DOI:10.1016/j.tra.2012.02.004.

[55] V. L. Knoop, S. P. Hoogendoorn, and H. J. Van Zuylen, "Approach to Critical Link Analysis of Robustness for Dynamical Road Networks" in Traffic and Granular Flow'05, A. Schadschneider, T. Pöschel, R. Kühne, M. Schreckenberg, and D. E. Wolf, pp. 1–10, 2007. DOI: 10.1007/978-3-540-47641-2_35.

[56] V. Knoop, H. J. Van Zuylen and S. Hoogendoorn, "The Influence of Spillback Modelling when Assessing Consequences of Blockings in a Road Network" Eur. J. Transp. Infrastruct. Res., vol. 8, no. 8, pp. 287–300, 2008. ISSN: 1567-7141.

[57] E. Jenelius and O. Cats, "The value of new public transport links for network robustness and redundancy" Transp. A Transp. Sci., vol. 11, no. 9, pp. 819–835, 2015. DOI:10.1080/23249935.2015.1087232

[58] A. Nicholson and Z. Du, "Improving network reliability: a framework" in 17th proceedings of the Australian Research Board Conference, 1994, p. 17.

[59] C. M. J. Tampère, J. Stada, B. Immers, E. Peetermans, and K. Organe, "Methodology for Identifying Vulnerable Sections in a National Road Network" Transp. Res. Rec., vol. 2012, no. -1, pp. 1–10, 2008. DOI: 10.3141/2012-01.

[60] D. M. Scott, D. C. Novak, L. Aultman-Hall, and F. Guo, "Network Robustness Index: A new method for identifying critical links and evaluating the performance of transportation networks" J. Transp. Geogr., vol. 14, no. 3, pp. 215–227, 2006. DOI:10.1016/j.jtrangeo.2005.10.003.

[61] J. L. Sullivan, D. C. Novak, L. Aultman-Hall, and D. M. Scott, "Identifying critical road segments and measuring system-wide robustness in transportation networks with isolating links: A link-based capacity-reduction approach" Transp. Res. Part A Policy Pract., vol. 44, no. 5, pp. 323–336, 2010. DOI:10.1016/j.tra.2010.02.003.

[62] M. Snelder, H.J. van Zuylen, L.H. Immers, "A framework for robustness analysis of road networks for short term variations in supply" Transp. Res. Part A Policy Pract., vol. 46, pp. 828–842, 2012. DOI:10.1016/j.tra.2012.02.007.

[63] M. Kraan, M. Schereuder, and G. Tamminga, "Vulnerability of a National Road Network" in Proceedings of the 87th Annual Meeting of the Transportation Research Board, 2008.

[64] Y. Chen, M. Bell, and I. Kaparias, "Reliability analysis of road networks and preplanning of emergency rescue paths" in Critical Infrastructure - Reliability and Vulnerability, A. T. Murray and T. H. Grubesic, Eds. pp. 9–30, 2007. DOI: 10.1007/978-3-540-68056-7_9.

[65] W. Burgholzer, G. Bauer, M. Posset, and W. Jammernegg, "Analysing the impact of disruptions in intermodal transport networks: A micro simulation-based model" Decis. Support Syst., vol. 54, no. 4, pp. 1580–1586, 2013. DOI: 10.1016/j.dss.2012.05.060.

[66] M. G. H. Bell and C. Cassir, "Risk-averse user equilibrium traffic assignment: An application of game theory" Transp. Res. Part B Methodol., vol. 36, no. 8, pp. 671–681, 2002. DOI:10.1016/S0191-2615(01)00022-4.

[67] P. Murray-Tuite and H. Mahmassani, "Methodology for determining vulnerable links in a transportation network" Transp. Res. Rec., vol. 1882, pp. 1–67, 2014. DOI: 10.3141/1882-11.

[68] L. Issacharoff, S. Lämmer, V. Rosato, and D. Helbing, "Critical Infrastructures Vulnerability: The Highway Networks" in Managing Complexity: Insights, Concepts, Applications, D. Helbing, Eds., Springer, 2008. DOI: 10.1007/978-3-540-75261-5_9.

[69] H. Yin and L. Xu, "Measuring the structural vulnerability of road network: A network efficiency perspective" J. Shanghai Jiaotong Univ., vol. 15, no. 6, pp. 736–742, 2010. DOI: 10.1007/s12204-010-1078-z.

[70] I. Das, G. Kumar, A. Stein, A. Bagchi, and V. K. Dadhwal, "Stochastic landslide vulnerability modeling in space and time in a part of the northern Himalayas, India." Environ. Monit. Assess., vol. 178, no. 1–4, pp. 25–37, 2011. DOI: 10.1007/s10661-010-1668-0.

[71] P. Luathep, A. Sumalee, H. W. Ho, and F. Kurauchi, "Large-scale road network vulnerability analysis: a sensitivity analysis based approach" Transportation (Amst)., vol. 38, no. 5, pp. 799–817, 2011. DOI:

10.1007/s11116-011-9350-0.

[72] L. Masiero and R. Maggi, “Estimation of indirect cost and evaluation of protective measures for infrastructure vulnerability: A case study on the transalpine transport corridor” Transp. Policy, vol. 20, pp. 13–21, 2012. DOI: 10.1016/j.tranpol.2011.10.002.

[73] L. G. Hernández and O. G. Gómez, “Identification of critical segments by vulnerability for freight transport on the paved road network of Mexico” Investig. Geográficas, Boletín del Inst. Geogr. UNAM, no. 74, pp. 48–57, 2011.

[74] B. Y. Chen, W. H. K. Lam, A. Sumalee, Q. Li, and Z.-C. Li, “Vulnerability analysis for large-scale and congested road networks with demand uncertainty” Transp. Res. Part A Policy Pract., vol. 46, no. 3, pp. 501–516, 2012. DOI: 10.1016/j.tra.2011.11.018.

[75] J. Yang, H. Sun, L. Wang, L. Li, and B. Wu, “Vulnerability Evaluation of the Highway Transportation System against Meteorological Disasters” Procedia - Soc. Behav. Sci., vol. 96, no. 6, pp. 280–293, 2013. DOI: 10.1016/j.sbspro.2013.08.035.

[76] R. A. El-Rashidy and S. M. Grant-Muller, “An assessment method for highway network vulnerability” J. Transp. Geogr., vol. 34, pp. 34–43, 2014. DOI:10.1016/j.jtrangeo.2013.10.017.

[77] M. Sanchez-Silva, M. Daniels, G. Lleras, and D. Patiño, “A transport network reliability model for the efficient assignment of resources” Transp. Res. part B, vol. 39, no. 1, pp. 47–63, 2005. DOI:10.1016/j.trb.2004.03.002.

[78] H. Al-Deek and E. Emam, “New Methodology for Estimating Reliability in Transportation Networks with Degraded Link Capacities” J. Intell. Transp. Syst. Technol. Planning, Oper., vol. 10, no. 3, pp. 117–129, 2006. DOI:10.1080/15472450600793586.

[79] N. Khademi, B. Balaei, M. Shahri, M. Mirzaei, B. Sarrafi, M. Zahabiun, and A. S. Mohaymany, “Transportation network vulnerability analysis for the case of a catastrophic earthquake” Int. J. Disaster Risk Reduct., vol. 12, pp. 234–254, 2015. DOI:10.1016/j.ijdrr.2015.01.009.

[80] M. A. P. Taylor, “Critical transport infrastructure in urban areas: Impacts of traffic incidents assessed using accessibility-based network vulnerability analysis” Growth and Change, vol. 39, no. 4, pp. 593–616, 2008. DOI: 10.1111/j.1468-2257.2008.00448.x.

[81] T. Nagae, T. Fujihara and Y. Asakura “Anti-seismic reinforcement strategy for an urban road network” Transp. Res. Part A Policy Pract., vol. 46, no. 5, pp. 813–827, 2012. DOI:10.1016/j.tra.2012.02.005.

[82] D. Watling and N.C. Balijepalli “A method to assess demand growth vulnerability of travel times on road network links” Transp. Res. Part A Policy Pract., vol. 46, no. 5, pp. 772–789, 2012. DOI:10.1016/j.tra.2012.02.009.

[83] M. S. Ridout, C. G. B. Demetrio, and J. P. Hinde, “Models for count data with many zeros” in Proceedings of the XIXth International Biometric Conference, 1998.

[84] D. Lambert, “Zero-Inflated Poisson Regression, with an Application to Defects in Manufacturing” Technometrics, vol. 34, no. 1, pp. 1–14, 1992. DOI: 10.2307/1269547.

[85] M. Flynn and L. Francis, “Zero-Inflated Models and Hybrid Models” Casualty Actuar. Soc., pp. 148–224, 2009.

[86] S. H. Ahmad, M. Asqc, and K. Abdul, “A Simple Technique for Computing Network Reliability” IEEE Transactions on Reliability, vol. 31, no. 1, pp. 41–44, 1982. DOI: 10.1109/TR.1982.5221220.

[87] A. Moreno, V. Velez, J. Montoya, and L. Rhenals, “La lluvia y los deslizamientos de tierra en Antioquia : Análisis de su ocurrencia en las escalas interanual, intraunal y diaria” Rev. EIA, vol. 5, pp. 59–69, 2006.

[88] Colfecar, “Informe Vial” Working Report, 2010.

[89] DANE, “Informe de coyuntura económica regional” Working Report, 2011.

[90] E. Dalziell, “Risk Assesment methods in road network evaluation: A study of the impact of natural hazards in the Desert Road, New Zealand” Ph. D. Dissertation, University of Canterbury, 1998.

[91] A.J. Richardson and M.A.P. Taylor, “Travel Time Variability on Commuter Journeys” High Speed Ground Transportation Journal, vol. 12, no. 1, pp. 77-99, 1978.