Z. Phys. C 69, 683–686 (1996)

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Is $\mathrm{U}(1)_H$ a good family symmetry?

William A. Ponce[1,2], Arnulfo Zepeda[2], Jesús M. Mira[1]

[1] Departamento de Física, Universidad de Antioquia, A.A. 1226, Medellín, Colombia
[2] Departamento de Física, Centro de Investigación y de Estudios Avanzados del IPN, Apartado Postal 14-740, 07000 México D.F., México

Received: 29 May 1995

Abstract. We analyze $\mathrm{U}(1)_H$ as a horizontal symmetry and its possibilities to explain the known elementary-fermion masses. We find that only two candidates, in the context of $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \otimes \mathrm{U}(1)_H$ nonsupersymmetric, are able to fit the experimental result $m_b \ll m_t$.

I Introduction

The pattern of fermion masses, their mixing, and the family replication, remain as the most outstanding problems of nowadays particle physics. The successful standard model (SM) based on the local gauge group $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ can tolerate, but not explain the experimental results. Two main features that a consistent family theory should provide are:

(i) Within each charge sector, the masses increase with family by large factors:

$m_u \ll m_c \ll m_t$; $m_d \ll m_s \ll m_b$; $m_e \ll m_\mu \ll m_\tau$.

(ii) Even if one restricts to the heaviest family, the masses are still quite different:

$m_\tau \sim m_b \ll m_t$.

The horizontal survival hypothesis [1] was invented in order to accommodate (i), under the (wrong) assumption that $m_\tau \sim m_b \sim m_t$. The idea of radiative symmetry breaking in a supersymmetric extension of the SM [2] depends crucially on the existence of one quark with a mass comparable to the SM breaking scale, but it can not explain why this was the top quark instead of the bottom quark. The modified horizontal survival hypothesis [3] was introduced in order to explain the full extent of (i) and (ii), but a dynamically realization of this hypothesis is still lacking. Of course, these hypothesis and ideas rest on the assumption that all the dimension four Yukawa couplings in a well behaved theory should be of order one.

Related to (i) and (ii) is the fact that the Cabibbo-Kobayashi-Maskawa quark mixing matrix is near to the identity, but it is a common prejudice to assume that the appropriate family symmetry may explain this fact as a consequence of (i) and (ii). In what follows we will enlarge the SM gauge group with an extra $\mathrm{U}(1)_H$ horizontal local gauge symmetry (the simplest multi-family continuous symmetry we can think of). We then show that the structure $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \otimes \mathrm{U}(1)_H$ by itself is able to explain (ii), and that the simplest supersymmetric (SUSY) extension of this model without a μ-term can not cope with (ii).

2 $\mathbf{SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_H}$ as an anomaly-free model

Our attempt is to keep the number of assumptions and parameters down to the minimum possible, and try to construct a model which explains both features (i) and (ii) at the lowest possible energy scale. We therefore demand cancellation of the triangular (chiral) anomalies [4] by the power counting method, including the mixed gravitational (grav) anomaly [5]. The alternative of cancelling the anomalies by a Green-Schwarz mechanism [6] has been already considered in Refs. [7], and corresponds to the construction of a model string-motivated which demands the inclusion of physics near the Plank scale.

$\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \otimes \mathrm{U}(1)_H$ as a continuous gauge group, with $\mathrm{U}(1)_H$ as a family symmetry, was introduced long ago in [8], (revived recently in the context of SUSY string-motivated models in [7, 9, 10]). There are two different versions of the model, corresponding to two different ways of cancelling the chiral anomalies. One is the demanding cancellation of the anomalies for each family and the other one is cancelling the anomalies between families.

2.1 Cancellation of anomalies in each generation

Assuming there are no right-handed neutrinos, using the $\mathrm{U}(1)_Y$ and $\mathrm{U}(1)_H$ charges displayed in Table 1, and demanding freedom from chiral anomalies for $\mathrm{SU}(3)_c$

Table 1. $U(1)_Y$ and $U(1)_H$ charges for the known fermions. $i = 1, 2, 3$ is a flavor index related to the first, second and third families. The Y_{SM} values stated are family independent

	$\psi_{i,L} = (N_i, E_i)_L$	$E^c_{i,L}$	$\chi_{i,L} = (U_i, D_i)_L$	$U^c_{i,L}$	$D^c_{i,L}$	$N^c_{i,L}$
Y_{SM}	-1	2	1/3	$-4/3$	2/3	0
$Y^i_{H_\eta}$	Y_{ψ_i}	Y_{E_i}	Y_{χ_i}	Y_{U_i}	Y_{D_i}	Y_{N_i}

$\otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_H$, we get:

$$[SU(2)_L]^2 U(1)_H : Y_{\psi_i} + 3Y_{\chi_i} = 0 \quad (1)$$

$$[SU(3)_c]^2 U(1)_H : 2Y_{\chi_i} + Y_{U_i} + Y_{D_i} = 0 \quad (2)$$

$$[U(1)_Y]^2 U(1)_H : 2Y_{\psi_i} + 4Y_{E_i} + \frac{2}{3} Y_{\chi_i} + \frac{16}{3} Y_{U_i} + \frac{4}{3} Y_{D_i} = 0 \quad (3)$$

$$U(1)_Y [U(1)_H]^2 : -Y^2_{\psi_i} + Y^2_{E_i} + Y^2_{\chi_i} - 2Y^2_{U_i} + Y^2_{D_i} = 0 \quad (4)$$

$$[\text{grav}]^2 U(1)_H : 2Y_{\psi_i} + Y_{E_i} = 0 \quad (5)$$

$$[U(1)_H]^3 : 2Y^3_{\psi_i} + Y^3_{E_i} + 6Y^3_{\chi_i} + 3Y^3_{U_i} + 3Y^3_{D_i} = 0. \quad (6)$$

The solution to (1)–(6) is [8, 11]

$$Y^i_{H_\eta} = \alpha_i Y_{SM_\eta},$$

where α_i is an arbitrary number different for each family, and Y_{SM_η} is the $U(1)_Y$ charge for the η multiplet.

These $U(1)_H$ charges cannot explain the feature (ii) which demands that at tree level only the top quark acquires a mass, and therefore that the Higgs field with $U(1)_H$ charge $Y_{H\phi}$ satisfies:

$$Y_{\chi_3} + Y_{U_3} = Y_{H\phi}$$

$$Y_{\chi_3} + Y_{D_3} \neq -Y_{H\phi} = Y_{H\phi^*}.$$

But once the first of these equations is satisfied, (2) above implies $Y_{\chi_3} + Y_{D_3} = -Y_{H\phi}$. Therefore, if a top quark mass arises at tree level ($Y_{H\phi} = \alpha_3$), a bottom mass arises as well at the same level.

Adding right-handed neutrinos $N^c_{i,L}$ to our set of fundamental fields does not change this conclusion since (2) stays valid [the only changes are in (5) and (6) which are now replaced by

$$[\text{grav}]^2 U(1)_H : 2Y_{\psi_i} + Y_{E_i} + Y_{N_i} = 0$$

$$[U(1)_H]^3 : 2Y^3_{\psi_i} + Y^3_{E_i} + 6Y^3_{\chi_i} + 3Y^3_{U_i} + 3Y^3_{D_i} + Y^3_{N_i} = 0]. \quad (7)$$

2.2. Cancellation of anomalies between families

If the $U(1)_H$ anomalies are cancelled by an interplay among families, (1)–(6) should be understood with a sum over $i = 1, 2, 3$. (4) then reads

$$\sum_i (-Y^2_{\psi_i} + Y^2_{E_i} + Y^2_{\chi_i} - 2Y^2_{U_i} + Y^2_{D_i}) = 0. \quad (8)$$

Obviously a solution to the new anomaly constraint equations which are linear or cubic in the Y_{η_i} is

$$\sum_{i=1}^{3} Y^i_\eta = 0$$

for each η. We will limit ourselves to this type of solutions and within this set we will consider only those for which the ψ_i and U_i H-hypercharges are fixed to satisfy either

$$Y_{\psi_1} = \delta_1 \equiv \delta,\ Y_{\psi_2} = \delta_2 = -\delta,\ Y_{\psi_3} = \delta_3 = 0,$$

$$Y_{U_1} = \delta'_1 \equiv \delta',\ Y_{U_2} = \delta'_2 = -\delta',\ Y_{U_3} = \delta'_3 = 0,$$

or any set of relations obtained from the former equations by a permutation of the indices $i = 1, 2, 3$. The solutions can then be divided onto four classes according to the way the cancellations occur in (8).

Class A. $Y_{E_i} = Y_{\psi_i} = \delta_i$ and $Y_{D_i} = Y_{\chi_i} = Y_{U_i} = \delta'_i$; $i = 1, 2, 3$. A model with a tree-level top quark mass arises if $Y_{H_\phi} = Y_{\chi_i} + Y_{U_j}$ for some i and j. There are five different models in this class characterized by $Y_{H_\phi} = \pm 2\delta', \pm\delta'$ and 0 respectively. Any of this five models becomes nonviable if it gives rise to a tree-level bottom mass. That is if there exists a k and a l for which $Y_{\chi_k} + Y_{D_l} = -Y_{H_\phi}$. For example, if $Y_{H_\phi} = 2\delta'$ then $i = j = 1$ and $k = l = 2$ satisfy the previous equations; this is signaled in Table 2 by the entry $(1, 1)_U$; $(2, 2)_D$ in the Class A column and the $2\delta'$ row. The fact that in Table 2 there is at least one D-type entry for every U-type one for all the five models of Class A, means that none of them is viable. This fact can be easily understood by noticing that $Y_{\chi_i} + Y_{U_j}$ changes sign under

Table 2. Summary of three-level mass term for all the possible models for the local gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_H$. A Higgs field with a hypercharge Y_{H_ϕ} different to the ones in the first column does not produce a mass term in the quark sector

Y_{H_ϕ}	CLASS A	CLASS B	CLASS C
$2\delta'$	$(1, 1)_U$; $(2, 2)_D$		$(1, 1)_U$
$-2\delta'$	$(2, 2)_U$; $(1, 1)_D$		$(2, 2)_U$
0	$(1, 2)_U$; $(2, 1)_U$; $(3, 3)_U$; $(1, 2)_D$; $(2, 1)_D$; $(3, 3)_D$	$(3, 3)_U$; $(3, 3)_D$	$(1, 2)_U$; $(2, 1)_U$; $(3, 3)_U$; $(3, 3)_D$
δ'	$(1, 3)_U$; $(3, 1)_U$; $(2, 3)_D$; $(3, 2)_D$	$(3, 1)_U$; $(3.2)_D$	$(1, 3)_U$; $(3.1)_U$; $(2, 3)_D$
$-\delta'$	$(2, 3)_U$; $(3, 2)_U$; $(1, 3)_D$; $(3, 1)_D$	$(3, 2)_U$; $(3, 1)_D$	$(2, 3)_U$; $(3, 2)_U$; $(1, 3)_D$
$\delta + \delta'$		$(1, 1)_U$; $(2, 2)_D$	$(2, 2)_D$
$-\delta + \delta'$		$(2, 1)_U$; $(1, 2)_D$	$(2, 1)_D$
$\delta - \delta'$		$(1, 2)_U$; $(2, 1)_D$	$(1, 2)_D$
$-\delta - \delta'$		$(2, 2)_U$; $(1, 1)_D$	$(1, 1)_D$
δ		$(1, 3)_U$; $(2, 3)_D$	$(3, 1)_D$
$-\delta$		$(2, 3)_U$; $(1, 3)_D$	$(3, 2)_D$