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Contradictions in mathematical modeling with digital technologies

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Abstract
This article presents an analysis of contradictions expressed by pre-service mathematics teachers when performing two modeling tasks using their own technological devices. The study is based on theoretical aspects of contradictions in the fields of expansive learning and modeling with digital technologies. Technology and modeling tasks were designed and tested with 14 participants in a mathematical modeling course offered to pre-service teachers in a Colombian university. Linguistic cues were used to carry out a thematic analysis for identifying contradictions in discursive manifestations. The results show how specific contradictions in modeling processes with technology are manifested by students and the kind of transformations they promote, as well as the changes that occur in how tasks are performed, and how students develop an idea of technology that goes beyond its traditional conception as a tool. The findings show the influence of other activity systems on the development of these processes and the need for future studies in learning research.

Keywords Teaching/learning strategies · Improving classroom teaching · Mathematical modeling; contradictions

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1 Introduction

Scientific literature offers evidence on uses of digital technologies in modeling processes in Mathematics Education (hereinafter *modeling*) and on how technology has been articulated to students' classroom tasks (Molina-Toro et al., 2019; Pereira et al., 2017). Molina-Toro et al. (2019) showed that in the international literature, there are two important roles of technology related to modeling processes: technology as a resource at the service of those who model, and technology as a means of reorganizing the modeling process. Molina-Toro et al. (2019) reported uses of technology related to visualization, simulation or recreation of phenomena, construction and validation of models, and reorganization of experimental processes; they also draw attention to the use of mobile devices and their constitutive role for the development of modeling processes.

Research to date on how mathematical modeling processes and technology are articulated is extensive; furthermore, the continuous growth and development of technological resources opens up new avenues for research on their integration into classrooms and their roles in student learning (Greefrath, 2011; Lei et al., 2016; Molina -Toro et al., 2019). Each technological development (for instance, new mobile devices or virtual and augmented reality) generates new possibilities for action in classrooms and for learning processes that can take full advantage of the potential of the "new" devices and all their related resources. Thus, research in this area need not be limited to accounting for cognitive processes or interactions between different subjects and tools but can promote new ways of interpreting realities beyond the classroom and alternative perspectives of understanding and promoting learning, so that the processes of teaching mathematics could evolve at the pace of society.

As a way of addressing the integration of technological resources with a different perspective on modeling processes and involving other agents — such as teachers, the educational community, and social and cultural school-related environments — this study takes Engeström's (2015) Expansive Learning Theory as a reference framework to identify theoretical and methodological elements that allow for the understanding of the evolution of human learning in settings where mathematics is a teaching purpose. In learning processes studied in this theory, there is a shift in the center of attention, from individuals to groups of persons, and then to activity systems. Expansive learning conceives of learning as a process in which contradictions emerge and are resolved, generating transformations of ideas that become established collectively and consolidate ways of acting socially. Engeström and Sannino (2010) return to the concept of contradiction to study the evolution of tensions in a group of people. Beyond analyzing isolated individuals, they sought to broaden how discursive manifestations of interacting subjects are understood; furthermore, they established criteria to identify the presence of contradictions in dialogues between people who share similar objectives in organizational settings.

Within this theoretical framework, to analyze the presence of contradictions in modeling processes with technologies in a group of students, it is necessary to observe the development of the process in environments where, as a reflection of the articulation of modeling and technology, the contradictions manifest themselves, and questions, arguments and new ideas originate and can evolve into expansive learning. Studying contradictions in modeling processes with technologies can also be a way to understand how different perspectives of mathematics, context, and culture appear in the language of participants, as well as how tensions among students are manifested and how they engage in various efforts to overcome difficulties in the development of the process. Several studies (Engeström & Sannino, 2010, 2011; Galleguillos & Borba, 2018) agree on the need to develop empirical studies that contribute to the identification of contradictions in activity systems.

Engeström (1999) recognized the importance of analyzing discourse and found evidence of how it alters the way in which subjects develop processes with a common purpose. Also, contradictions are the basis on which expansive learning is supported and are the starting point of the changing forces that promote the evolution of activity systems. Therefore, a clear understanding of contradictions in modeling processes with technologies and their role in transforming classroom mathematical activity is needed.

2 The role of contradictions in expansive learning theory

In the Expansive Learning Theory, contradictions play a fundamental role in the development of a given activity, since they are the driving force behind transformations of practice (Engeström, 1999, 2015). For Engeström and Sannino (2010), the object of an activity is always internally contradictory; however, this condition is what allows the object to be continuously moving, what motivates subjects, and what causes the emergence of new activity systems or transforms those already established, as part of the resolving tensions that promote change.

Tensions appear when contradictions influence activity and affect how the components of an activity system interact with the subject's ability to reach the object (Yamagata-Lynch, 2010). Consequently, tensions are strongly linked to contradictions and are a reference point for understanding deviations from previously consolidated practices. In the context of this study tensions are considered as opposing states can show the influence of digital technologies on the solution or construction of new questions when modeling and, at the same time, the transformation and development of these processes by their participants.

The contradiction is a foundational philosophical concept of expansive learning that is not theoretically defined and can only be identified through its manifestations (Engeström, 2000; Engeström & Sannino, 2011). This fact led Engeström and Sannino to formulate a systematic conceptual framework to identify contradictions in discursive manifestations. The authors maintain that previous work does not show the differences between the notion of contradictions and the notions of tension, paradox, opposition, or dichotomy and, therefore, the manifestations or discursive disturbances of the subjects could not be analyzed as internal contradictions in systems of activity; furthermore, they affirm that conflicts, dilemmas, and disturbances could be studied as manifestations of contradictions.

According to the investigative interests of this study, contradictions will be studied under the organizational system of the classroom, which implies a deep look at situations involving students' emotions and personal sensations, as well as a clear understanding of what represents an activity system like the one shown in Fig. 1.

This triangular model is frequently applied as a lens to analyze educational processes and constitutes a unit of analysis for the researcher (Engeström, 1999). This frame of reference includes four levels of contradictions in human activity. At level 1, contradictions are related to exchange value or use value; at level 2, contradictions appear among the constituent elements of the activity system. At level 3, more advanced objects or motives are introduced into the system; at level 4, contradictions are related to central and 'neighbor activities' (Engeström, 2015). Additionally, Engeström and Sannino (2011) present four types of discursive manifestations of contradictions that, in their view, allow a better understanding of them, namely:

- Dilemmas as aspects of social beliefs shared by individuals, giving rise to ways of thinking.
- Conflicts as a form of resistance or disagreement.
- Critical conflicts as situations that involve feelings or guilts and affect personal sense.
- Double bonds as processes that lead individuals to ask rhetorical questions that show the impossibility of continuing their work.

The four types of discursive manifestations are accompanied by cues or linguistic signs that serve as a tool to identify a contradiction. Table 1 presents the proposal built by Engreström and Sannino (2011), which contributes to the analysis of dialogues between individuals in activity systems from educational contexts.

According to the authors, linguistic cues need to be simple and unambiguous since, sometimes, an expression does not necessarily imply the formation of a

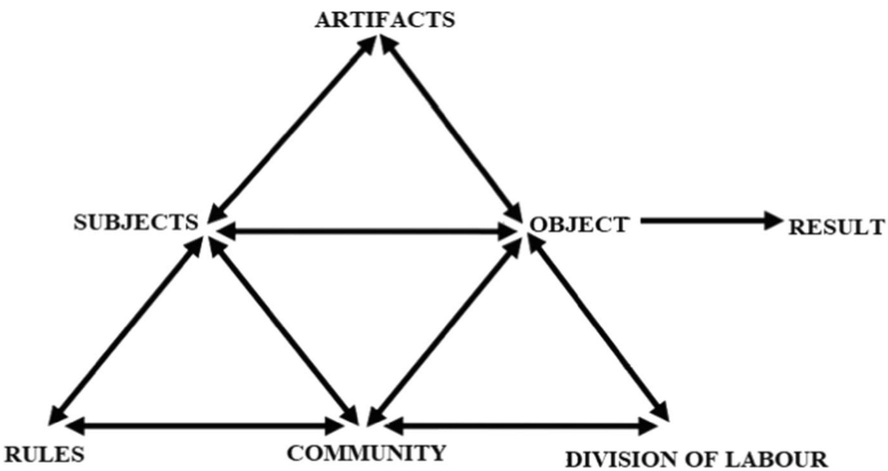

Fig. 1 Structure of the activity system, adapted from Engeström (1987, 2015)

Table 1 Linguistic signs in discursive manifestations presented by Engeström and Sannino (2011)

Manifestations	Examples of linguistic signs
Dilemmas	"On the one hand […] On the other hand"; "yes, but"; "I did not mean that"; "I really meant to say."
Conflicts	"Not"; "I disagree"; "this is not true"; "yes"; "I can accept this"
Critical conflict	Consider personal, emotional, moral narrative structures, vivid metaphors, "Now I realize that […]"
Double bond	"We", "we must", "we have" rhetorical questions, expressions of helplessness "Let's do it", "we will do it"

contradiction and could instead divert the attention of the subject, hence the need to analyze the activity as a network where multiple voices interact and expansive transformations occur.

This theory suggests that contradictions must be creatively solved and must generate new models of mediation and new systems of activity in which disagreements or oppositions are overcome. In this sense, under the lens of this study, some cultural traditions for the development of modeling processes in the classroom could also be transformed in such a way that digital technologies could provide new insights that contribute to the solution of tensions generated when addressing these processes in school activities.

In educational contexts, some discursive manifestations could be used to generate collective work and transform future activities. That is to say, students' expressions and points of view could be considered as input that allows for expanding the modeling process, experimenting with alternative solutions, or building other objects of activity. Likewise, the need to promote spaces where creativity and scientific research can appear naturally presents interesting and challenging scenarios to analyze the manifestation of contradictions. Such analysis can serve as a starting point to transform school activity systems and their contexts. Through the lens of this study, there are a number of other considerations that may arise as a result of the unlimited introduction of digital technologies in modeling processes for classroom work; therefore, it is necessary to have a better understanding of discursive manifestations that may develop from alternative ways of working in the classroom and which could become opportunities to develop new activity systems.

3 Mathematical modeling with digital technologies

Using the theory of Expansive Learning, the inclusion of cultural artifacts in human activity allows subjects' actions to be analyzed not solely individually, but as part of a collective system of actions that can have a strong influence on neighboring activity systems. This characteristic, which the literature presents as an evolution of activity theory (Engeström, 2015), provides, in the context of this study, the ability to understand the power of transformation of technologies in school activities related to modeling processes.

Technology can fit into various roles relating to modeling. It can serve as a virtual learning environment for modeling (Rosa & Orey, 2019), it can be an instrument that can reorganize the nature of modeling processes, and, beyond being a temporary part of a school activity, or even being present throughout the development of activity, it can be considered as an instrument allowing students to unleash a series of reasonings and actions to address modeling situations (Molina-Toro et al., 2019). In the present study, this serves as a point of reference to analyze characteristics of the discursive manifestations that students express when they link digital technologies to these processes and produce expansive transformations.

Mathematical modeling has various perspectives and interpretations in international literature. An aspect common to many is the possibility of promoting ways of studying "reality" through mathematics. For some researchers, modeling creates bridges between mathematics and reality when new concepts are introduced (Possani et al., 2010). It can also be seen as a pedagogical approach (Borba & Villarreal, 2005), as an instrument of research (Bassanezi, 2002), as a scientific and pedagogical activity that favors interdisciplinary practices (Cifuentes & Negrelli, 2012) or as a training object for professionals and teachers (Romo-Vázquez et al., 2019). Despite the diversity of approaches to the study of modeling, technology has come to occupy a privileged place in the development of modeling processes, and it is less and less common to find studies that do not include it.

Research in this field presents different perspectives on the link between technologies and modeling processes. In one perspective, modeling is developed through cycles, focused on observing how students go through each of its phases (Greefrath & Vorhölter, 2016; Perrenet & Zwaneveld, 2012; Rodríguez Gallegos & Quiroz Rivera, 2016; Siller & Greefrath, 2010). In another, modeling is developed in environments incorporating technology, in which the production of knowledge by students is analyzed (Molina-Toro et al., 2018; Soares & Borba, 2014). While the focus of these two perspectives is different, the presence of technology is vital, both to promote the transition from one phase to another in modeling cycles, as well as to reorganize threads and ways of acting while studying various phenomena.

Broadening students' understanding of certain situations by linking together observations, measurements, and other quantifiable or describable characteristics is, as shown by Confrey (2007), another way of providing mathematical sophistication to students through modeling. Analyzing modeling with technology from the perspective of expansive learning involves paying attention to key aspects of the way mathematics is done in the classroom. In this work, then, modeling is assumed to be a learning environment in which students engage with the study of mathematics through the development of tasks that combine mathematical models, cultural and social contexts, processes of experimentation with technology, and processes of inquiry and expansion of mathematical knowledge. In this sense, a given technology cannot be seen as a substitute for what other technologies do, or as a mere juxtaposition of tools and modeling situations. On the contrary, it should promote a mathematical activity that brings together new devices, contexts, norms, experiences, and students' beliefs, among others.

The conception of technology in this study includes theoretical aspects of the philosophical perspectives of Mitcham (1994) and Vries (2016), which address social and educational dimensions of technology in human beings. Vries uses the term 'technology' in a broad sense, meaning a human activity that transforms the natural environment according to his needs. For this researcher, technology is not only a matter of our hands, but also a matter of our minds. Mitcham addresses theoretical questions about technology and states that many historians use the word technology to refer to artifacts of primitive or modern manufacture and use. According to this last author, reflections on technology have conceptualized it in four main ways: as objects (tools made by man), as knowledge (ethical principles and uses of objects), as activity (events that connect knowledge and the will to create new tools) and as volition (motivations, impulses, and choices). These four categories, according to Mitcham, provide a new perspective from which technology can be approached; however, in results shown by Vries (2016), people typically only conceptualize technologies as objects or artifacts, and important aspects of the formation of subjects are ignored.

In these philosophical perspectives on technology, the authors make it clear that fundamental issues are still under discussion; however, their frameworks allow an approach to technology that extends theoretical horizons so that its articulation to modeling processes can be following current technological developments. For instance, they propose new ways of establishing objects' measurements depending on the measuring tool and new ways of obtaining and treating data; and they propose working with mobile applications and other new technologies to expand spaces of experimentation emerging from modeling processes.

4 Research question

The new roles that technology is taking on within modeling processes could define new forms of activity in classrooms and in the activity systems that may influence other activity systems outside the school. As a consequence, this article reports the results of a study that was aimed to address the following question: What contradictions are identified in students who participated in modeling processes in mathematics education using digital technologies?

5 Methodology

The empirical phase of this research was carried out in a mathematical modeling course led by three teacher educators with investigative experience in modeling, with 15 students who were part of a mathematics teacher training program. This section presents details of the context and participants, the empirical data resulting from the seminar, and, finally, the procedures for data analysis.

5.1 Context and participants

This research was carried out during the second semester of 2019. The mathematical modeling course was part of a mathematics teacher bachelor's program offered by a school of education at a public university in Medellín, Colombia. In Colombia, it is the universities that are responsible for training mathematics teachers for primary and secondary education (Guacaneme-Suárez et al., 2017). These programs have a duration of five years, and are based on mathematical, didactic, and pedagogical knowledge subjects, as well as a practicum. For details on policies and the mathematics teacher preparation system in Colombia, see Guacaneme-Suárez et al. (2017).

The findings reported in this study are related to a subgroup of four volunteer students who expressed their willingness to participate in the research. They were fourth year students in the bachelor's program. Their ages ranged from 19 to 23 years. The four participants completed mathematics education courses (e.g., Didactics of algebra, geometry, statistics), mathematics courses (e.g., geometry, arithmetic, mathematical analyses), and a part of pedagogical courses (e.g., curriculum, educational politics, educational psychology). The students are referred to using pseudonyms: Cristina, Diana, Estela, and Sandra. The choice of the members for this subgroup was motivated by the variety of information found in the data, the constant participation of its members during the development of the course, and the spontaneous and continuous use they made of their mobile devices throughout the seminar.

5.2 The data and thematic analysis

The course included 16 sessions of four hours each. In several sessions, the participants discussed theoretical papers on modeling and modeling tasks (Villa-Ochoa et al., 2017). The modeling tasks that participants were asked to complete were designed to draw links between the reference literature and modeling processes with students in school practices. The tasks were formulated so that participants had to analyze different measurement processes (lengths, three-dimensional objects in two-dimensional spaces, and time, among others) and discuss their proposed solutions and the particularities of their results, diverting their attention from focusing only on mathematical contents. In addition, an activity systems analysis methodology was used as proposed by Yamagata-Lynch (2010), where the unit of analysis was defined as an object-oriented activity. In the case of this study, those activities represented tasks in modeling environments with technologies.

With the ethics protocol approved and the informed consent of the students, data was collected in the form of video recordings, audio recordings, and written productions of the participants. Consistent with the analysis of activity systems, these records were systematized in the *Atlas.ti* software and were coded (as shown in Table 1) according to the linguistic cues proposed by Engeström and Sannino (2010) which seek to identify contradictions in discursive manifestations. A thematic analysis was carried out, which consisted of (i) analyzing and transcribing student dialogues in which they expressed their points of view regarding the ways to solve the task and their reasons when carrying out the proposed actions (object-oriented actions); (ii) identifying linguistic signs in correspondence with the criteria established in Table 1 and patterns of manifestations where they appeared; (iii) evaluating the signals that were identified and the type of contradictions that they represent for the participants and the activity system, and finally (iv) studying the evolution of these contradictions during the modeling processes according to the theoretical framework.

The object of analysis for this study was student activity while solving modeling tasks using their mobile digital devices. In analyzing the data, the activity system in which the records were produced was taken into account, as well as the specific roles and varying expressions of teachers and students while solving the modeling tasks. The classroom activity system as a whole, as suggested by Yamagata-Lynch (2010), was the setting in which observations were made, dialogues with participants were carried out and all records were collected. It should be noted that the dialogues between teachers and students did not include any particular interest in the use of technologies by the students, nor the discussion of theoretical references used in this research. The following section presents the main results of the analysis.

6 Results

The results are structured based on two modeling tasks carried out by students: (i) measuring a beltway and (ii) choosing a gas station. These tasks were developed by students in the second and seventh sessions of the seminar, respectively, and are presented in order to analyze the influence that students' own devices have on the initial modeling processes proposed during the seminar. Modeling in this seminar was not a rigid process, that is, a single homogeneous process was not developed with all the students during all sessions. Instead, some periods were established in which students played the role of modelers, and in others, various theoretical, methodological, and evaluative elements of modeling in Mathematical Education were discussed.

6.1 Measurement of a beltway

The first modeling task the students addressed was finding the length of a beltway that surrounded the university. Based on the nature of the task, the students needed to find ways to measure a large distance (object of the activity); they came to realize that the strategies and instruments that they usually used to measure small distances were insufficient. The group of students left the classroom and set a reference point on the beltway to begin their journey and take measurements. In principle, students' first discursive manifestations pointed to ways of establishing the measurement by counting steps; however, other considerations began to arise in the group proposing the use of their mobile phones. The decision to use these devices arose spontaneously when considering the length of the beltway, as shown in the following dialogue:

> Diana: let's see what applications are there [referring to cell phone applications, aka "apps"]
> Estela: a pedometer?
> Cristina: stopwatch no! whatever!

> Diana: we can try recording the stopwatch, recording a video or how is it called? Let's look at what applications there are!
> *(Dialogue among students. 7 / June / 2019)*

Due to Diana's proposal, the group of students began to consider the use of mobile phones for the task. They realized that their previous proposals did not appear to be appropriate for this measuring task (e.g. lack of precision, the need to have several ways to find the measurement and to have a simple way to measure). Following Diana's proposal, each one of the students began to have a specific role in the development of the process. The students wanted to find a way to avoid going all the way around the beltway; however, there was no consensus on the strategy to follow. This fact revealed the presence of tensions between them. Some example expressions of these tensions include: "But the idea is to count without going all the way around [without doing the whole journey]", "but what is half of this?", "No, no, no, it's easier to do it from here to there, the entire circumference and then multiply it by two. It is more symmetrical this way than that [pointing both possible directions in which the route can be made]", "it is not symmetrical". According to Engeström and Sannino (2011), these manifestations, in the form of dilemmas and conflicts, may be evidence of the presence of contradictions related to the selection of a way to carry out the task.

After this exchange of expressions, Cristina focused her attention on "counting steps", Diana on "making a video to determine the time needed to make the tour" and Estela on "observing the measurement on the pedometer of her cell phone". The diversity of strategies offers an example of how the tasks were distributed in the activity system of the subgroup. Each of these strategies subsequently allowed a particular solution to the task and a way of getting several sources of measurement to confront their results, which became a form of validation within the group.

At the end of the tour, the students met again in the classroom to write a report about the solution to the task. At this moment, new tensions between participants appeared regarding the effectiveness of the various strategies. For example, Cristina, who used the step counting strategy, lost count of the number of steps she was taking and had to make adjustments to finish the measurement. Estela, who was using the pedometer of her cell phone, found an inconsistency in the measure due to improper handling of the device and could not obtain the total number of steps. Tensions regarding the choice of a strategy vs. reliability of results appeared. The difficulties in writing the report led the group to select Diana's mobile strategy, as is evident in the following dialogue, taken from the video recording of the seminar session.

> Look at the health application of my cell phone, it reports the exact time. No, it's like a range of hours. It reports from today, June 7, from 4:00 to 5:00 [in the afternoon]. I gave 2987 steps, these steps ... ehhh [pauses briefly], we have to discount some steps to get an approximation because I arrived at 4:08, I was not here at 4 o'clock. I entered from Barranquilla [an avenue near the university] to here at Block 9 and we went down and up to here to get there [the starting point of the measure]. Here we arrive at 5:00 o'clock. So, we can do

an approximation and subtract a little, a few steps, how many do we subtract? I don't know. (Diana, June 7, 2019)

The students decided to take into account Diana's calculations since her cell phone application showed no inconsistencies compared to the measure that the others had taken; Although Cristina and Estela had problems with their results, their adjustments allowed for an approximate measurement of the length of the beltway. Diana made an internet search about what her application was doing, that is, how it counted the steps and how a pedometer was used. This information was shared with her colleagues and provided another element taken into account in writing the report. Cristina adjusted the number of steps in relation to the number delivered by the health application on Diana's cell phone and, although in Estela's application the average measurement of one step was 66 cm, when observing the length of Diana's steps they decided to also use 70 cm as a reference.

Based on this information, the students chose to multiply 2,500 steps by 66 cm, which resulted in a length of the beltway of 1.65 km (using Estela's strategy); however, with Diana's strategy, the result changed to 1.75 km. Both results were in the range of measurements that they established with the video strategy (using some paint marks on the street) and with the measurement given by one of the university security guards.

6.2 Choosing a gas station

The second modeling task proposed to the students had two purposes. The first was to offer another look at academic literature, since the task was designed based on an example from Blum (2015), a document that students had recently read. The second was to create an environment in which various perspectives on modeling could be discussed with students and proposed as possible tools in their future mathematics teaching practices. For this task, the teachers presented the students with three different locations in the city of Medellín; a house (as a reference point) and two gas stations (Texaco 80 and Texaco Palacé gas stations), which differed in price by 800 COP [0.25 USD]. The students' task was analyzed in which of the two gas stations it was cheaper to fill the tank, when starting at the reference point.

The students discussed the necessity of finding a reference point to locate themselves on the map and use some type of tool to solve the problem. They also discussed the importance of determining various information, such as the address of each gas station, the distance from the house to each gas station, the number of liters consumed on each trip and the amount of money that each trip represents. Some of the proposals for developing a solution focused on determining a mathematical function to solve the task. The students questioned the gallon/liter conversion expression, and made comments such as: "But I see, the more distance you travel to the station, the more gas you will spend". They also intensified the opposite positions as disagreements regarding the solution of the task. Afterward, Diana used Google Maps to show her classmates the distance from the chosen reference point to each gas station. This data allowed the participants not only to know the distance from the reference point to each one of the stations, but also to estimate the time that it would take for a vehicle to make the journey. The students carried out a series of conversions and operations to find out the expected consumption of gasoline and the money it represented. At this point, Diana mentioned aspects such as the routes between the chosen reference point and the gas station, based on her knowledge as a driver. This knowledge, together with the interpretation of the map information, generated tensions that determined the way to carry out the task.

Diana: But what did you type? [asking Estela about the way she searches for an address on Google Maps]
Sandra: We typed the exact address.
Estela: I typed the exact address, look, I typed 'Carrera 84 and Calle 52' [the station is located at the intersection of avenue 84 and street 52].
Diana: Texaco? Yes, street 51, avenue 84.
Cristina: But does that report the number 84? Is not it necessary to type 'avenue 84 number…'? [she asks for the conventional way of completely writing an address; her statement takes the form of a dilemma]
Estela: I looked it up on the map [talking about locating a point on the map], I didn't type it like that.
Diana: No, no, give me the number ... but that is very subjective because it depends on where you type it, and it is different if the map application autocompletes the address [conflict manifestation]. It is like when you use Uber and you type the address to show on the map, you type it exactly where you want and consider it to be. But when you click on the map and receive the address, it gives you the exact address, it has to give the address number. Because, if you put it where you think it is [approximately], it may move [the location].
Cristina: Well, look, yes, it gives me 1.4 km to the Texaco at 80th [referring to the gas station, looking at the measurement on Google Maps], and the Texaco Palacé is ...
(Dialogue of a group of students 20 / June / 2019)

This episode shows, on the one hand, tensions in the form of dilemmas or conflicts manifested by Cristina and Diana; on the other hand, it shows the influence of other activity systems (neighboring activity systems) on the development of the task. In this case, Diana's knowledge of Google Maps generates a non-acceptance of the group dynamics and shows how some considerations regarding the task can be better determined. The interest in constructing a more functional model faced the students with the need to consider and select various variables.

Cristina: [When searching for Palacé Avenue and using Google Maps] So there, the distance would be 5.8 km.
Diana: I think there is something wrong because it shows that it is from that address [indicating the starting point] to the gas station, and it is not the same as going from the gas station to there, because there are different streets [the route for a car may vary depending on the direction between two points of the city]. You have to put this first, then the other and then the other.
Cristina: Oh yes, ok. Erase all that and type ... from here [Palacé Avenue]

Diana: And look, you know what? That depends on which path you take, because there are some paths that are 6.6 km long and others that are 6.2 and 6. So there are many variables.
Sandra: Oh yes! There is 6, too.
(Dialogue of a group of students 20 / June / 2019)

In these episodes, there is evidence of a collective activity in which communication is constant between the participants and individual actions influence the activity system.

In line with Lektorsky (2009), the fact that Diana has thought about the fact that the route to the gas station is different from the route back to the starting point, an analysis that she makes from her experience as a driver, can be considered as an extraction of interaction to self-reflect the process, an interaction between the individual activity system and the collective activity system. This influence of neighboring activity systems continued to promote other types of tensions. The expression "I think there is something wrong …", is a point that determines a break in the process that the group had already been carrying out. Consistent with this interpretation, the development of these contradictions leads to a transformation in the way of proposing a solution. To complete the task, students take Diana's considerations into account and once again use Google Maps to obtain data on round-trip routes, times, distances, and the shortest path. In the written report, students proposed two algebraic functions with which they determined the cost of gasoline at both gas stations and the money that can be saved by filling the car at the Palacé Avenue station.

6.3 Analysis of contradictions in this study

The most relevant contradictions in the two analyzed tasks were directly related to the types of tasks, and are linked to the processes that the students carried out to establish standardized measures in modeling tasks. The presence of dilemmas and conflicts was present in students' manifestations, contexts, and arguments regarding the actions to be carried out to solve the tasks; These manifestations became an input to analyze the activity systems.

Figure 2 shows the Activity System that was analyzed in the first task. The contradictions that were identified in this activity system are, based on Engeström (2015), Level 2 contradictions, that is, contradictions between constituent elements of the activity system. These contradictions appeared together in two ways: subject-artifacts and subject-rules. In the subject-artifact binomial, although the students had some knowledge about the scope of the devices they used, the contradictions were evident when they had to expand this domain to solve the task and find a result from a new perspective.

The subject-rule binomial shows that the adoption of certain methods and strategies for solving the task did not fit into a mathematical tradition in which obtaining certain measurements is associated with the use of certain devices linked to recurring practices established as norms inside the classrooms. The development of this contradiction required students to know in greater detail the functionalities of the mobile device applications (tools) with which they were making measurements, a

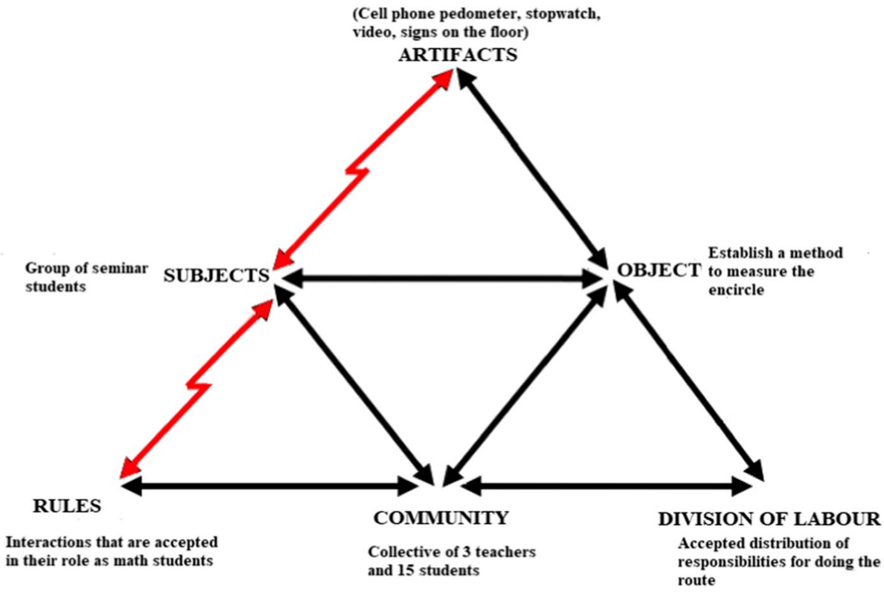

Fig. 2 Contradictions (indicated by red jagged arrows) in the activity system of task 1

fact that in light of the theory of Expansive Learning may be an indicator of a transformation of the object.

In the second task, in addition to the previously identified contradictions, the influence of neighboring activity systems was observed. In this case, Diana's knowledge as a driver led her to question the dynamics of the group while determining the length of the round trips to and from each gas station. Those contradictions can be considered Level 4: the influence of neighboring activity systems transform the initial activity system in which the task was conceived. As a consequence, participants were able to determine the amount of money a driver can save by loading fuel at a gas station using mathematical functions. The presence of level 2 and level 4 contradictions are illustrated in Fig. 3.

Although in both tasks the students used their own mobile devices, the apps they used allowed them to determine a means to structure and discuss some ideas related to the problem's solution. The tensions between the subjects and the instruments that they used to establish measurements between locations on the map developed until it was possible to find a way to represent the links between variables algebraically. However, the characteristics of the route, and the way it should be traveled, allowed them to structure a new solution that considers what traditionally happens when driving a vehicle and making a round trip between one place and another. For Lektorsky (2009), the need to transform collective activity is the result of an internal conflict that exists in a system. In this case, internal conflicts produced transformations in how the students solved their tasks. As a consequence, they adopted other ways of establishing measures in modeling processes where they had the ability to decide which tools to incorporate.

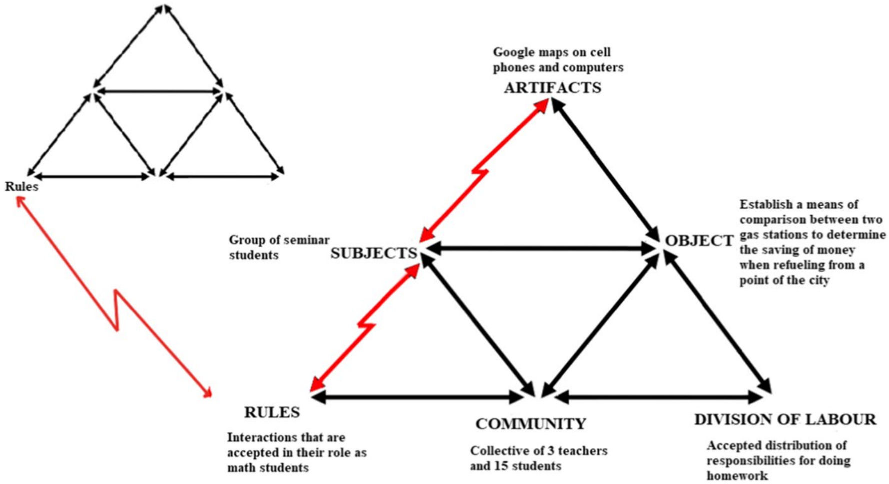

Fig. 3 Contradictions (indicated by red jagged arrows) in the activity system of task 2

The student dialogues are evidence that technology has a leading role not only in its conceptualization as an artifact (Mitcham, 1994), but also in its transformation to an object of study by the participants and the volitional aspects that this entails; that is, there existed a reason for students to expand their knowledge about the tool and the decisions they can make with it. These characteristics of the technology are conceptualized by Mitcham (1994) as knowledge when they imply understanding of the functional nature of the artifact, and as volition when they influence the will, intention and choice that may be associated with it. For the first task, the students needed to expand their knowledge of the pedometer to make use of the applications they had on their cell phones; for the second task, the students needed to focus their attention on the application of maps available on their cell phones and computers and understand how to better determine a location and establish different routes.

The object of the activities in this study was to solve modeling tasks in contexts that are not traditional in the classroom. This led the students to need to make links between their technological devices and the conventional uses they have in neighboring activity systems. These links extended the utility of the students' devices and their knowledge, and represents possible evidence of expansive transformations in school activity. This is important for this research in two ways. First, it leads to transformations of the object of the activity, that is, it allows students to spontaneously and temporarily focus their attention on expanding the horizon of knowledge of their own tools and consequently, their actions over objects. Second, solving these tasks shows an understanding of how technologies can be used in the classroom without underestimating their social use.

In line with Engeström and Sannino (2010), these transformations by technology in modeling tasks are one way in which expansive learning can be represented. Our results provide evidence that the contradictions presented here evolved to expansive transformations in the activity systems that were analyzed. In these two tasks, the students incorporated new ways in which the measurements could be obtained and, in addition, incorporated other artifacts that generated changes in the development of the modeling processes that they developed. Similar findings were presented by Souto (2014), who reports a transformation of the object of activity when introducing Geogebra to study conics in an online teacher training course; using the software transformed the problem space and shaped the way in which participants acted.

The modeling environment in which the students and their technological devices were involved allowed for an increase in the number of physical possibilities to solve the tasks, as well as in the number of ways in which ideas or questions in their minds could combine. In this sense, this study also offers a look at how students expand their knowledge about technology, and thereby overcome some of the problems mentioned by Williams and Goos (2012), who describe how technology is often seen as a black box in which mathematics is hidden.

7 Conclusion

The idea of studying contradictions and their transformative role in academic training processes is not new in the scientific literature. Similar perspectives on this activity system in school environments have already been provided by Mwalongo (2016) and Gedera (2016), with results that promote discussion in the field of education. However, combining the study of contradictions in modeling processes with the study of technology can be used in future work to investigate how the development of modeling processes can be transformed when students use their own technologies, expanding their utility, and offering new ways of knowledge and action in the study of mathematics.

The students' discursive manifestations were the starting point to identify contradictions, but students' expressions also showed other linguistic forms that may appear, generating new possible research that could extend this frame of reference. In this sense, this work examines and tests the empirical and methodological usability offered by the theory, as pointed out by Engeström et al. (2013).

The students showed limited use of the tools, but the individual and collective actions that appeared in the activity system allowed them to transform the modeling process. In the conception of technology of this study, what initially appeared as a tool became knowledge (as a result of the collective interest in knowing more about it), activity (it allowed the solution of the modeling task), and volition (regarding the decisions made by students in order to complete the task). These elements show how there can be an evolution of the conception of technology (Mitcham, 1994; Vries, 2016).

Aspects related to volition, consistent with this conception of technology, are changing the way people make decisions and apparently, as the findings of this study show, are also promoting challenges in modeling processes in mathematics education. Devices such as smart-watches can currently indicate if a person has a pulse outside the normal range and, thereby, indicate to the user the need to visit a doctor; Other applications on mobile devices can record physical activity and show high or low intensity peaks in an exercise routine, so a person could decide whether to increase or decrease the training frequency. It would be interesting to develop experiences of interdisciplinary modeling work with these characteristics among health sciences students in the future.

Finally, it is necessary to expand the field of knowledge of modeling processes with technologies with studies similar to this one and, as mentioned by Molina-Toro et al. (2019), observe how student model, how they elaborate their conjectures and how learning is consolidated in those processes. Using students' devices in the classroom may be a way to allow students to learn mathematics independent of state resources and free of technological dependencies that transcend the school context. A study of the activity systems that emerge in modeling processes with digital technologies, and the consequent emergence of expansive learning, suggests a need to expand both theoretical and empirical research in this field.

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Conflicts of interest/competing interests We have no conflicts of interest to declare.

References

Bassanezi, R. C. (2002). *Ensino-aprendizagem com modelagem matemática*. Contexto.

Blum, W. (2015). Quality teaching ofmathematical modelling: What do we know, what can we do? In S. J. Cho (Ed.), *The Proceedings ofthe 12th International Congress on Mathematical Education* (pp. 73–96). Springer International Publishing. https://doi.org/10.1007/978-3-319-12688-3_9

de Borba, M. C., & Villarreal, M. E. (2005). *Humans-with-media and the reorganization of mathematical thinking*. Springer. https://doi.org/10.1007/b105001

Cifuentes, J. C., & Negrelli, L. (2012). Uma Interpretação Epistemológica do Processo de Modelagem Matemática: implicações para a matemática. *Bolema: Boletim de Educação Matemática*, *26*(43), 791–815. https://doi.org/10.1590/S0103-636X2012000300003

Confrey, J. (2007). Epistemology and modelling—overview. In W. Blum, P. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education* (pp. 125–128). Springer. https://doi.org/10.1007/978-0-387-29822-1_10.pdf

Engeström, Y. (1987). *Learning by expanding: An activity-theoretical approach to developmental research*. Orienta-Konsultit.

Engestrom, Y. (2015). *Learning by expanding. Learning by expanding: An activity-theoretical approach to developmental research* (Second Edition). Cambridge University Press. https://doi.org/10.1017/CBO9781139814744

Engeström, Y. (1999). Communication, discourse and activity. *The Communication Review, 3*(1–2), 165–185. https://doi.org/10.1080/10714429909368577

Engeström, Y. (2000). From individual action to collective activity and back: developmental work research as an interventionist methodology. In P. Luff, J. Hindmarsh, & C. Heath (Eds.), *Workplace studies* (pp. 150–166). Cambridge University Press. https://doi.org/10.1017/cbo9780511628122.008

Engeström, Y., Rantavuori, J., & Kerosuo, H. (2013). Expansive learning in a library: Actions, cycles and deviations from instructional intentions. *Vocations and Learning, 6*(1), 81–106. https://doi.org/10.1007/s12186-012-9089-6

Engeström, Y., & Sannino, A. (2010). Studies of expansive learning: Foundations, findings and future challenges. *Educational Research Review, 5*(1), 1–24. https://doi.org/10.1016/j.edurev.2009.12.002

Engeström, Y., & Sannino, A. (2011). Discursive manifestations of contradictions in organizational change efforts: A methodological framework. *Journal of Organizational Change Management, 24*(3), 368–387. https://doi.org/10.1108/09534811111132758

Galleguillos, J., & Borba, M. C. (2018). Expansive movements in the development of mathematical modeling: Analysis from an activity theory perspective. *ZDM Mathematics Education, 50*(1–2), 129–142. https://doi.org/10.1007/s11858-017-0903-3

Gedera, D. S. P. (2016). The application of activity theory in identifying contradictions in a university blended learning course. In D. S. P. Gedera & P. J. Williams (Eds.), *Activity theory in education* (pp. 53–69). SensePublishers. https://doi.org/10.1007/978-94-6300-387-2_4

Greefrath, G. (2011). Using technologies: New possibilities of teaching and learning modelling – overview. In G. Kaiser, W. Blum, R. Borromeo Ferri, G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling, ICTMA 14* (pp. 301–304). Springer. https://doi.org/10.1007/978-94-007-0910-2_30

Greefrath, G., & Vorhölter, K. (2016). *Teaching and learning mathematical modelling: Approaches and developments from German speaking countries* (pp. 1–42). Springer International Publishing. https://doi.org/10.1007/978-3-319-45004-9_1

Guacaneme-Suárez, E. A., Obando-Zapata, G., Garzón, D., & Villa-Ochoa, J. A. (2017). Colombia: Mathematics education and the preparation of teachers. Consolidating a professional and scientific field. In A. Ruiz (Ed.), *Mathematics teacher preparation in central America and the Caribbean* (pp. 19–37). Springer International Publishing. https://doi.org/10.1007/978-3-319-44177-1_2

Souto, D.L.P. (2014). *Transformações expansivas na produção matemática online*. Cultura Acadêmica. Retrieved from https://repositorio.unesp.br/handle/11449/126237. Accessed 27 Sept 2018

Lei, J., Luo, P. H., Wang, Q., Shen, J., Lee, S., & Chen, Y. (2016). Using technology to facilitate modeling-based science education: Lessons learned from a meta-analysis of empirical research. *Journal of Educational Technology Development and Exchange (JETDE)*, *9*(2), art.4. https://doi.org/10.18785/jetde.0902.04

Lektorsky, V. A. (2009). Mediation as a Means of Collective Activity. In A. Sannino, H. Daniels, & K. D. Gutierrez (Eds.), *Learning and Expanding with Activity Theory* (pp. 75–87). Cambridge University Press. https://doi.org/10.1017/CBO9780511809989.006

Mitcham, C. (1994). *Thinking through technology: The path between engineering and philosophy* (1st ed.). The University of Chicago Press.

Molina-Toro, J. F., Rendón-Mesa, P. A., & Villa-Ochoa, J. A. (2019). Research trends in digital technologies and modeling in mathematics education. *Eurasia Journal of Mathematics, Science and Technology Education*, *15*(8), em1736.

Molina-Toro, J. F., Villa-Ochoa, J. A., & Suárez-Téllez, L. (2018). La modelación en el aula como un ambiente de experimentación-con-graficación-y-tecnología. Un estudio con funciones trigonométricas. *Revista Latinoamericana de Etnomatemática*, *11*(1), 87–115. Retrieved from http://revista.etnomatematica.org/index.php/RevLatEm/article/view/506. Accessed 1 Oct 2018

Mwalongo, A. I. (2016). Using activity theory to understand student teacher perceptions of effective ways for promoting critical thinking through asynchronous discussion forums. In D. S. P. Gedera & P. J. Williams (Eds.), *Activity theory in education* (pp. 19–34). SensePublishers. https://doi.org/10.1007/978-94-6300-387-2_2

Pereira, R., Seki, J., Palharini, B., Neto, J., Silva, A., Damin, W., & Martins, B. (2017). Modelagem matemática e tecnologias digitais educacionais: Possibilidades e aproximações por meio de uma revisão sistemática de literatura. *REnCiMa, 8*(2), 80–94.

Perrenet, J., & Zwaneveld, B. (2012). The many faces of the mathematical modeling cycle. *Journal of Mathematical Modelling and Application, 1*(6), 3–21.

Possani, E., Trigueros, M., Preciado, J. G., & Lozano, M. D. (2010). Use of models in the teaching of linear algebra. *Linear Algebra and Its Applications, 432*(8), 2125–2140. https://doi.org/10.1016/j.laa.2009.05.004

Rodríguez Gallegos, R., & Quiroz Rivera, S. (2016). El papel de la tecnología en el proceso de modelación matemática para la enseñanza de las ecuaciones diferenciales. *Revista Latinoamericana de Investigación En Matemática Educativa, 19*(1), 99–124. https://doi.org/10.12802/relime.13.1914

Romo-Vázquez, A., Barquero, B., & Bosch, M. (2019). El desarrollo profesional online de profesores de matemáticas en activo: una unidad de aprendizaje sobre la enseñanza de la modelización matemática. *Uni-pluriversidad, 19*(2), 161–183. https://doi.org/10.17533/udea.unipluri.19.2.09

Rosa, M., & Orey, D. C. (2019). Mathematical modelling as a virtual learning environment for teacher education programs. *Uni-pluriversidad, 19*(2), 80–102. https://doi.org/10.17533/udea.unipluri.19.2.04

Siller, H.-St., & Greefrath, G. (2010). Mathematical modelling in class regarding to technology. In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello (Eds.), *Proceedings of the 6th Congress of the European Society for Research in Mathematics Education (CERME 6)*, January 28–February 1, 2009, Lyon (France)

Soares, D. S., & Borba, M. C. (2014). The role of software Modellus in a teaching approach based on model analysis. *ZDM - International Journal on Mathematics Education, 46*(4), 575–587. https://doi.org/10.1007/s11858-013-0568-5

de Vries, M. J. (2016). Teaching about Technology. An Introduction to the Philosophy of Technology for Non-philosophers. Springer International Publishing. https://doi.org/10.1007/978-3-319-32945-1

Villa-Ochoa, J. A., Castrillón-Yepes, A., & Sánchez-Cardona, J. (2017). Tipos de tareas de modelación para la clase de matemáticas. *Espaço Plural, 18*(36), 219–251.

Williams, J., & Goos, M. (2012). Modelling with mathematics and technologies. In M. Clements, A. Bishop, C. Keitel, J. Kilpatrick, & F. Leung (Eds.), *Third international handbook of mathematics education* (pp. 549–569). Springer New York. https://doi.org/10.1007/978-1-4614-4684-2_18

Yamagata-Lynch, L. C. (2010). *Activity systems analysis methods*. Springer US. https://doi.org/10.1007/978-1-4419-6321-5

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