International Journal of Science and Mathematics Education (2023) 21:1407–1432
https://doi.org/10.1007/s10763-022-10301-6

A Macro Tool to Characterize and Develop Key Competencies for the Mathematics Teacher' Practice

Luis R. Pino-Fan[1] · **Walter F. Castro**[2] · **Vicenç Font Moll**[3]

Received: 29 January 2021 / Accepted: 28 June 2022 / Published online: 5 September 2022

Abstract

The initial and continuing training of mathematics teachers continues to be one of the most pressing challenges facing the research community in mathematics education. In this regard, various research agendas have been proposed and developed. Among these, two stand out: the characterization and development of (1) didactic and mathematical knowledge that allows the teacher to favor the management of his classes, and (2) skills necessary for professional practice. Although various models have been proposed to attend to each of these agendas separately, there are no models that allow an explicit integration between the notions of knowledge and teacher competence. On the other hand, various studies show the need to have theoretical-methodological tools that operationalize the categories of knowledge and skills proposed by scientific literature. In this article, the didactic-mathematical knowledge (DMK) model is presented as a theoretical-methodological alternative that allows for both the analysis and the development of essential knowledge and skills for the teacher's professional practice. In addition, it delves into a proposal of categories and subcategories of professional competencies necessary for teaching.

Keywords Didactic-mathematical knowledge · Knowledge of the teacher · Professional skills · Teacher training

✉ Luis R. Pino-Fan
luis.pino@ulagos.cl

Walter F. Castro
wfcastro82@gmail.com

Vicenç Font Moll
vfont@ub.edu

[1] Departamento de Ciencias Exactas, Universidad de Los Lagos, Osorno, Chile

[2] Universidad de Antioquia, Medellín, Colombia

[3] Universitat de Barcelona, Barcelona, Spain

Introduction

The study of both the knowledge and competencies, which a mathematics teacher should have to perform correctly in his professional practice, is a topic that has been investigated by the mathematics education community. The international research has led to conceptualizations (and models) on the components of knowledge that teachers use to teach mathematics (Hill et al.,2008; Neubrand, 2018; Petrou & Goulding, 2011; Rowland, 2014; Rowland et al., 2005; Schoenfeld & Kilpatrick, 2008;Shulman, 1987. Some studies have focused on the characterization and development of professional competencies necessary for the mathematics teacher (Kunter et al., 2013; Leuders et al., 2018; Zaslavsky & Sullivan, 2011). Other studies explore the assessment of teachers' knowledge as one of the most critical parameters of teaching quality (An & Wu, 2012; Bell et al., 2010); Blömeke & Delaney, 2012), although the research agenda for teacher competencies has given particular emphasis to the role of reflection on teaching practice, perceiving the reflective component as a critical competence for professional development and improvement of teaching (Davis, 2008; ; Fernández & Yoshida, 2004; Fortuny & Rodríguez, 2012; Gellert et al., 2013; Korthagen, 2010; Llinares, 2012).

Even though the previous proposals have signified advances in the approximation of responses to the problem of training and the type of knowledge and skills required by the mathematics teacher, Rowland and Ruthven (2011) already warned that there is no consensus on a theoretical framework that allows for a detailed description of the teacher's knowledge. Furthermore, as pointed out by Godino (2009), several of these proposals lack a systematic approach to the type of theoretical-methodological tools that allow observing, assessing, and improving the knowledge related to each of the categories that make up such models. Various studies (Blömeke & Delaney, 2012; Neubrand, 2018; Silverman & Thompson, 2008) show the need to have specific theoretical-methodological tools that help not only characterize but also develop knowledge and skills related to the primary tasks of the teaching work (i.e. design, implement, and reflect on the processes of study of mathematics). By a task we take the proposal by Chan and Leung (2013) that considers that a task is "a teacher designed 'thing-to-do' using the tool, either concrete or virtual manipulatives, for students to experience potential mathematical meanings carried by this tool" (p. 35). Empirical studies suggest that teachers can promote the evolution of mathematics knowledge through tasks and post-task mathematics discussions (Falcade et al., 2007; Jones, 2000; Mariotti, 2002).

The question that motivates this research is: What are the mathematical and didactic skills that a teacher exhibits when teaching? Then the article presents a theoretical-methodological proposal that combines the research agendas on the mathematics teacher's knowledge and skills with a comprehensive approach to the notion of competence and a pragmatist approach to the notion of knowledge. This article is a review/theoretical paper that presents a research-based proposal on the mathematics teacher's knowledge and skills with a comprehensive approach to the notion of competence. The aim is to present the notion of "teacher mathematical-didactical competence."

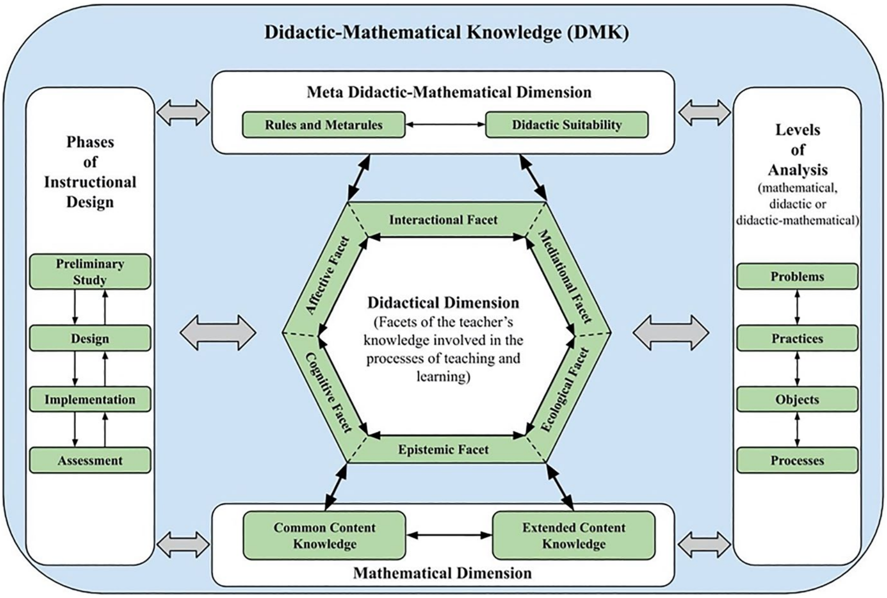

Fig. 1 Didactic-mathematical knowledge model

We start from the teacher's didactic-mathematical knowledge (DMK) model, whose research (e.g. Castro et al., 2018; Pino-Fan & Godino, 2015; Pino-Fan et al., 2015, 2018; Scheiner et al., 2019) concludes that this can be seen as a theoretical-methodological macro tool for the characterization and development of crucial competencies for the professional practice of the mathematics teacher. In addition, a proposal of essential competencies and sub-competencies for professional practice is presented, considered by the model, and different levels of grading for their development. To answer the research question, we present the teacher's knowledge model, and tables are presented that allow establishing a gradation of levels of achievement of professional skills, which are derived from teachers' observation and analysis.

The Didactic-Mathematical Knowledge Model

In the scientific literature, we can find various proposals and models that attempt to describe the components that make up the knowledge of the mathematics teacher (Neubrand, 2018; Scheiner et al., 2019). In this document, we will focus on the DMK model (Fig. 1) proposes that teachers' didactic-mathematical knowledge can be organized or developed according to three main dimensions: mathematical, didactical, and meta didactic-mathematical (Pino-Fan et al., 2015, 2018).

The mathematical dimension refers to the knowledge of school mathematics to be taught and its links with previous notions and later notions. The second dimension refers to knowledge about aspects involved in the processes of teaching and learning mathematics, that is, in-depth knowledge of school mathematics and its interaction with cognitive and affective aspects of students (how they think, how they learn, errors and difficulties in study-specific content), resources and media, classroom interactions, and ecological features (political, economic, social context, etc. that condition the teaching and learning processes). Finally, the third dimension, meta didactic-mathematical dimension, alludes to the knowledge that a professor must have to systematize the reflection on his practice and thus make judgments about his practice (the definitions and characteristics of each dimension are presented in (Pino-Fan et al., 2015, 2018).

One feature that should be highlighted is that the components of the didactical dimension are of didactic-mathematical nature since the teachers' knowledge of students' cognitive and affective aspects, interactions and resources, are closely linked to mathematics which is subject to teaching and learning. On the other hand, it is essential to note that, for each of the dimensions and sub-dimensions of the DMK model, it also proposes "analytical tools" that allow each of these dimensions to be operationalized (Pino-Fan et al., 2018). Such theoretical-methodological "tools" are based on the onto-semiotic approach (OSA) of mathematics knowledge and instruction (Godino et al., 2007, 2019).

The DMK Model and its Links with the Professional Competencies Study

One of the development perspectives of the DMK model was the entwinement of knowledge with competence. In various studies (e.g. Castro et al., 2018), it became clear that the three dimensions (with the six facets that make up the didactical dimension) can be used to analyze, describe, and develop the teachers' knowledge involved in the various phases of the teaching and learning of concrete topics: a preliminary study, planning or design, implementation, and assessment of their practice or that of others. In addition, as part of their didactic-mathematical knowledge, teachers must know and understand the features involved in each of these didactic design dimensions. Nevertheless, it is essential to ask: How are studies on teachers' knowledge linked to competencies?

The Notion of Competence in the Framework of the DMK Model

The DMK model considers the knowledge that teachers use in their professional practice. It was contrasted based on the didactic-mathematical reference knowledge, reconstructed based on the scientific literature, curricular analysis, and historical epistemological studies on the mathematical object under study. The question that naturally arose was, how are we considering and studying the teachers' didactical and mathematical competences? Mathematics teachers are expected to address fundamental didactic problems related to teaching this subject through theoretical and methodological tools, giving way to a set of specific competencies. Thus, the first two key questions that arise to develop the DMK model are: What is understood by the notion of competence? What are the critical competencies that mathematics teachers should have? (Font, 2011; Pino-Fan et al., 2017a).

According to Weinert (2001), competency-based approaches can be classified into three broad groups: (a) cognitive approach, (b) motivational approach, and (c) integral approach or action competence. However, other studies show the relevance of considering the notion of competence from a more comprehensive perspective, taking a certain distance from a cognitivist stance, which predominates in research in mathematics education (see, for example, Niss & Højgaard, 2019; Kunter et al., 2013). Thus, the competence is understood from the perspective of the competent action, considering it as the set of knowledge and dispositions, which allows the effective performance within typical professional contexts (Font, 2011; Pino-Fan et al., 2017)

This formulation of the term "competence" has to be developed to be operational and measurable. For that purpose, it is necessary to characterize competence (definition, levels of development, and descriptors) that allows for its development and assessment. The teacher must mobilize skills, didactic-mathematical knowledge, and attitudes to perform a practice that intends to solve a professional problem. Furthermore, the teachers' practice can be assessed by some descriptors of competence associated with a certain level of competence. Thus, the DMKC model considered the mathematics teacher's two key competencies: mathematical competence and didactic intervention and analysis competence. The fundamental nucleus of the second refers to the "Designing, applying and assessing learning sequences, through didactic analysis techniques and status criteria, to establish planning, implementation, assessment, and proposals for improvement cycles" (Breda et al., 2017, p. 1897). In the next section, we will develop the two competencies contemplated in the DMKC model. There is consensus in the mathematics education research community about didactic analysis as a teacher's competence. Although there are different definitions (Barquero & Bosch, 2015; Klafki, 1995; Prince & Felder., 2006), in this work it is considered that the teacher manifests it and uses it differently depending on various factors. Thus, it is not assumed either as an exclusive competence of researchers, nor that it is full fledge applied by teachers—the grading made of it in the sub-competencies accounts for the different levels it can be used.

Methodology

The methodology is qualitative and exploratory (Cohen et al., 2011; Creswell, 2009). We developed the research over 9 years in various countries: Chile, Peru, Panama, Colombia, Spain, Mexico, Ecuador, and Brazil. The participants are both preservice teachers and in-service teachers. The instruments used to identify and validate the levels of mathematical competence of the teacher are guides on objects, meanings, processes, and levels of suitability (Breda & Lima, 2016; Castro, 2011; Godino, 2009; Godino et al., 2013; Parra-Urrea, 2021; Pino-Fan, 2013; Rivas, 2012). The researchers used the tools during class sessions, analyzed them, and discussed the findings (Arsal, 2014; Fernández, 2011) with the preservice and in-service teachers. The participating teachers sought to improve teaching performance and achieve more effective classroom and content management (Donnelly & Fitzmaurice, 2011). The proposed levels and sublevels are deduced from data taken in at least eight countries' research over 9 years.

Due to space limitations, it is impossible to show the instruments used; however, they can be found in the above references.[1] The proposed levels derived from the teachers' teaching practice observed and recorded by researchers, additionally from discussions with the participating teachers. In general, the levels and their gradation, proposed in Tables 1, 2, 3, 4, 5, 6, and 7, are the result of systematic observation and analysis of in-service teachers' and preservice teachers' practices at two different moments: (1) before the instruction of the DMKC model and its analysis tools; and (2) during the instruction of the DMKC model and its analysis tools. For example, in Table 7, levels 0 and 1 correspond to the moment before teaching the DMKC model, while levels 2 and 3 correspond to the moment after teaching the DMKC model. Now, to assign one of these four levels to the practice of a teacher, the following process was followed: (1) during the moment before the instruction of the model and its tools, content analyses of the teacher's narratives were made, which allowed observing the implicit use of DMKC tools (Breda et al., 2017), and then level 0 or level 1 was assigned as a result of a triangulation process of experts. (2) Afterwards, an instruction process (training cycles for teachers) was carried out on the DMKC model and its tools; moment two began after teaching the DMKC. Teachers' practices and narratives were analyzed, depending on the explicit and correct tools use. Finally, it was assigned level 2 or 3 as a result of experts' triangulation.

For 9 years, this process carried out in two moments allowed us to observe "regularities" in the teachers' practices, which allowed us to refine the development descriptors of each level. Hummes (2022) study is the most recent research that presents the route described above.

Characterization of Professional Competences Within the Framework of the DMKC Model

The DMKC model suggests two competencies for the mathematics teacher's professional activity: (1) mathematical competence; (2) competence of analysis and didactic intervention. Below we will discuss and characterize each of these two key competencies. In addition, each one has sub-competencies based on several proper research conducted during the last 15 years and on international literature. Figure 2 schematically shows the proposal developed in this article. The activation of each of the three dimensions and the six teacher knowledge facets of DMK are tested when designing, teaching mathematics tasks, and assessing students learning.

Teacher's Mathematical Competence

The research community has widely discussed the notion of mathematical competence in mathematics education, which has made various formulations. For example,

[1] See Author (Pino-Fan et al., 2015) and the web http://enfoqueontosemiotico.ugr.es/pages/fprofesores.html.

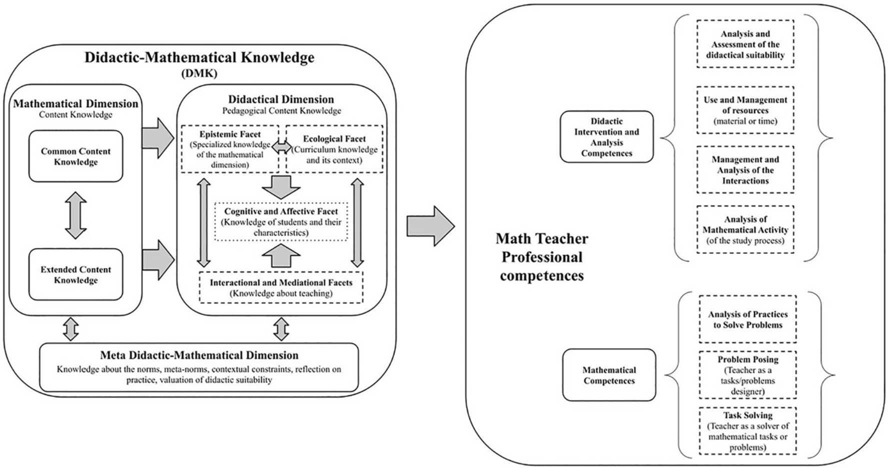

Fig. 2 Math teacher knowledge and competences

within the framework of the Competencies and the Learning of Mathematics Project (KOM), it is defined as the insightful, detailed disposition of someone to act appropriately in response to all kinds of mathematical (intra- or extra-mathematical) challenges related to given situations (Niss & Højgaard, 2019). The KOM project proposal includes eight competencies, four related to "poising and answering questions and with mathematics" and four related to "dealing with mathematical language and tools" (Niss, 2015, p. 41). The four associated with this last grouping are representation, symbols and formalism, communication, aids, and instruments; the first two are linked to the epistemic facets of DMK; communicative competence is linked to the interactional facet, and the latter is linked to the mediational facet, but in this case distributed both in those expressed by the teacher and in those that the teacher wishes to promote by the students. The richness of KOM proposal is maintained in ours, but it is redistributed according to the DMK model.

Another proposal, adopted in various educational curricula, is the one prepared by the Organization for Economic Co-operation and Development (OECD) in the context of the Program for International Student Assessment (PISA), which defines mathematical competence as:

> The individual ability to identify and understand the role that mathematics plays in the world, to make well-founded judgments, to use and engage with mathematics, and to meet the needs of personal life as a constructive, engaged, and reflective citizen. (Organization for Economic Co-operation and Development [OECD], 2004, p. 3)

The mathematical competence of the subject who learns mathematics (students) should not be confused with the mathematical competence of the person who teaches mathematics (teacher or future teacher). While, in the first case, the challenges and given situations are of a mathematical nature—intra or extra (Niss & Højgaard, 2019)—in the second case, the challenges are related to the critical tasks for the teacher's professional activity (design, implement, and assess mathematics teaching processes). Thus, the teacher' mathematical competence requires both the knowledge involved in the mathematical dimension of the DMK model—to be able to solve mathematical problems that he could propose to the students—and the knowledge involved in the epistemic facet of the didactical dimension of the DMK model, to plan the tasks that will be proposed for the student learning. For this, the teacher will have to consider curricular adaptation, the wealth of mathematical meanings of the notion studied, context, and resources. Three sub-competencies are proposed for this competence, each one with different levels of achievement: (1) task solving, (2) proposition of problems, and (3) analysis of the responses to a problem posed.

Mathematical Sub-competency 1: Task Solving

This sub-competency arises around problem-solving. Many projects have been developed using various approaches ((Barmby et al., 2014;Cai et al., 2020). It is necessary to highlight that problem-solving is an important teaching tool to incorporate mathematical knowledge into the real world (Villalobos, 2008). Furthermore, it is considered one of the mathematical competencies, both by the Organization for Economic Co-operation and Development [OECD] (2004) and the KOM project (2002). The National Council of Teachers of Mathematics (2000) highlights the importance of considering problem-solving as the central axis of school mathematics and promotes studies and research related to mathematics teaching and learning.

In this approach, the teacher is seen mainly as the provider of mathematical problems, which must be associated with solution techniques that consider the students' aptitudes, his previous knowledge, curricular guidelines, and the wealth of mathematical objects, heuristics, or resolution methods, among other aspects (Felmer et al., 2016, 2019; Pochulu et al., 2016). It is considered that the mathematics teacher "must know and be able to implement mathematical practices necessary to solve the mathematical problems usually approachable by students of the corresponding level, and must know how to articulate them with the subsequent thematic blocks" (Godino et al., 2017, p. 91). This sub-competence considers the teacher's activity, fundamentally, as a solver of mathematical tasks or problems associated with a specific mathematical notion studied at a particular moment. Four levels (L) of development are proposed in this subcategory, which is based on the notions of common knowledge, expanded knowledge, and knowledge about the epistemic facet, offered by the DMK model. (Table 1)

Mathematical Sub-competency 2: Problem Posing

Task design is considered a key element to achieve quality teaching (Mason & Johnston-Wilder, 2004; Tzur et al., 2008; Watson & Ohtani, 2015; Zaslavsky & Sullivan, 2011). Pochulu et al., (2016) informs that the tasks are the starting point of the students' activity, which, in turn, promotes their learning. Proposing the

Table 1 Achievement levels of the task solving sub-competence

L0	The teacher reproduces both the formulation of mathematics tasks and their procedures without considering: •Mathematical object relations •Processes •Enhanced knowledge •The representations of the mathematical object •The variety of procedures that solve the mathematical task He uses textbooks or programs of the grade he teaches to establish mathematical tasks and their solutions, which will be discussed during the instructional process
L1	The teacher solves problems at the educational level he teaches, identifies the changes in the variables of the problems, but generally, these are linked to "types of problems," which leads to the implementation of a type of procedure or the generation of justifications, or similar arguments, when solving mathematical tasks or problems
L2	The teacher solves problems corresponding to the educational level and the courses he teaches using different representations of the mathematical object. On occasions, he also uses various procedures and elaborates other arguments. For example, he can solve problems in which several meanings of the studied notion must be used, but he does not distinguish between the relationships that can be established among the various meanings of the notion studied. Likewise, he can link the mathematical object being studied with other mathematical objects of the educational grade he teaches (or earlier). Still, he cannot connect them with mathematical objects of the later stage or require it as prior knowledge
L3	The teacher solves problems at the educational grade and the next (associated with the mathematical notion studied at the time). He can solve mathematical tasks using various procedures and representations of the mathematical object. Likewise, he proposes multiple justifications or arguments when solving mathematical tasks and can mobilize more than one meaning of the studied notion, relating and articulating the meanings among themselves (Biehler, 2005; Pino-Fan et al., 2011)

tasks that will be part of the study process is the responsibility of the teacher. There are tasks (problems, exercises, etc.) that he must select, analyze or design for his students based on teaching material, and determine the extent to which the problems, or the way to organize the content, are adequate to achieve the objectives (Llinares, 2011). The design or selection and the analysis of the tasks (problems, exercises, and activities) proposed in the curriculum (study plans, textbook) is a characteristic and critical activity of the mathematics teacher's practice (Fernández, 2011; Ponte, 2005).

Consequently, this second sub-competence arises, associated with task posing. The teacher uses his specialized mathematical knowledge (DMK epistemic facet) to propose tasks related to studying a mathematical object. Likewise, the teacher considers the possible conflicts or errors that students face during the development of mathematical practices (DMK cognitive facet) and the interests and contexts of students (DMK affective facet), which would allow to design or propose tasks to study mathematical notions. He also considers various meanings of mathematical notions to create or select tasks and considers multiple elements related to the didactic dimension of the DMK model (Malaspina, 2017; Pino-Fan et al., 2015).

The achievement levels for this sub-competence are described in Table 2. However, it is pertinent to point out that level zero (L0) has similar characteristics in the sub-competencies in task solving and proposing since it refers to "replicating/repeating" tasks from other media (books, internet, curriculum).

Table 2 Achievement levels of the problem posing sub-competence

L0	The teacher reproduces both the statement of tasks and their solutions, and he makes use of the internet, textbooks, or programs of the subject of the grade in which he teaches classes. The latter is done to establish the mathematical tasks that will be proposed in the study process
L1	The teacher proposes tasks suited to the educational level he teaches. To do this, he considers features related to the curriculum (procedures, arguments or justifications, representations) and students' prior knowledge and redesigns the mathematical object's tasks but does not foresee misconceptions, errors, difficulties, or different answer possibilities. These are primarily task adaptations taken from other sources regarding the proposed tasks
L2	The teacher proposes tasks corresponding to the educational level, considering features related to curriculum (procedures, arguments or justifications, representations) and the student's previous knowledge. In addition, he considers various plausible responses, misconceptions, conflicts, or errors concerning mathematical practice for his proposal, the most appropriate contexts according to the students' characteristics, interests, and needs. Tasks can be adaptations or modifications of tasks taken from other sources or of his design. The teacher does not consider that different types of tasks situations might require different meanings on the notion under study
L3	Additionally, the teacher considers new tasks associated with the mathematical topic and proposes challenges for the students. The tasks link the mathematical object studied with other mathematical objects (in the school grade, earlier, or later grades). The mathematical notions' meanings that he proposes to study broadens his vision of types of situations/problems in terms of the contexts in which such notions can be used (the intra- and extra-mathematical connections of the notions under study are considered). At this level, the teacher anticipates student conflicts or mistakes and sometimes induces them to create learning opportunities

Mathematical Sub-competence 3: Analysis of Practices to Solve Problems

This sub-competence is associated with the study process's didactic analysis (a priori, in situ, a posteriori). Consequently, it connects with the second key competence suggested by the DMKC model, the didactic analysis, and intervention competence. The term didactic analysis has different meanings. Puig (1997) defines it as "the analysis of the contents of mathematics that is performed at the service of the organization of its teaching in educational systems" (p. 61). In a broader view, Font (2011) and Giménez et al. (2013) define it as designing, applying, and evaluating learning sequences, using didactic analysis techniques and quality criteria, in establishing cycles of planning, implementation, assessment and to present improvement proposals. Some authors suggest that one of the activities associated with didactic analysis is the analysis of mathematical activity, and they consider that the identification of the mathematical objects and meanings implemented during the mathematical activity allows the teacher to understand the progression of learning and evaluate the mathematical competencies of students (Giacomone, 2019).

It should also be noted that within the framework of the didactic analysis, a challenging task for the teacher is to analyze the students' mathematical activity, which would allow him to adequately evaluate the mathematical competencies of his students (Stahnke et al., 2016). Rubio (2012) states that when teachers are not competent in the analysis of mathematical practices, processes, and objects, they exhibit limitations in evaluating the mathematical competencies of their students.

This sub-competence considers the analysis of planned mathematical practice developed by a student regarding the solution of a task. For this, the teacher will require knowledge related to the epistemic facet of the DMK model (Pino-Fan et al., 2015) and tools proposed for the operationalization of this facet. Table 3 presents the four development levels.

Analysis Competence and Didactic Intervention

This general competence involves putting knowledge related to the didactic and meta didactic-mathematical dimensions of the DMK model into action (Pino-Fan et al.,2015). Furthermore, it is formed by different sub-competencies (Breda et al., 2017; Pino-Fan et al., 2017): (1) sub-competence in the analysis of the mathematical activity; (2) sub-competence in analysis and management of the interactions; (3) sub-competence in the use and management of resources; and (4) sub-competence in the analysis and assessment of the didactical suitability.

Sub-competence in the Analysis of Mathematical Activity

This sub-competence involves what is described in mathematical sub-competence 3. It does not focus solely on analyzing the mathematical practice underlying the plausible responses that the teacher foresees (a priori) or the responses that students give to the situations/proposed problems (a posteriori). Here, the teacher puts his didactic-mathematical knowledge (DMK) into play to analyze the mathematical activity that can emerge in any of the four stages of didactic design—a preliminary

Table 3 Levels of achievement of the sub-competence analysis of practices to solve problems

L0	The teacher analyzes the students' mathematical practices, identifying evident mathematical elements: procedures or definitions of some concepts used. Teacher values such mathematical aspects in terms of correct or incorrect uses. At this level, the teacher does not perform an a priori analysis of the mathematical practices he expects to implement with the tasks he proposes; thus, he does not anticipate errors or conflicts on students' solutions
L1	The teacher performs what is indicated in L0. Still, the a posteriori analysis of his students' mathematical practices is done by taking, as a reference, the a priori analysis developed based on the response it hopes to obtain. The teacher plans the answer that he hopes to get with a specific proposed task. In any case, the analyses are performed based on his experience without considering any analysis methodology
L2	The teacher uses theoretical-methodological tools to analyze mathematical practices, both those expected (a priori) and those implemented by the students (a posteriori). With the theoretical-methodological tool onto-semiotic configuration proposed to operationalize the epistemic facet of the DMK model (Pino-Fan et al., 2015, 2018), the teacher can identify his students' expected practices, representations and their adequateness, concepts/definitions, properties/propositions, procedures, and arguments; and can identify features that were "generators" of errors or conflicts in the students' practices
L3	At this level, the teacher has appropriated some theoretical-methodological tools. For example, the teacher knows the onto-semiotic configuration and uses it as a tool to analyze mathematical practices. Additionally, it considers various expected practices in which different meanings context-related are implemented and identifies and analyzes configuration elements in students' real-time practices, allowing the teacher to make decisions and take measures to overcome errors or conflicts

study, planning or design, implementation, and evaluation (Pino-Fan & Godino, 2015). The mathematical activity can arise in an operative or discursive way in the teacher's practices or by the students.

At this point, it is necessary to distinguish "mathematical activity" from "mathematical practice." In this paper, it is considered that "The subject's activity, external and internal, is mediated and regulated by a psychic reflection of reality. What the subject sees in the object world are motives and goals, and he must receive conditions of the activity in one way or another, presented, understood, retained, and reproduced in his memory…" (Leontiev, 2009, p. 114). On the other hand, we interpret the practice as a coherent system of internal mental processes that are externalized and recognized in ostensive acts directed by a motive to achieve conscious goals. In our case, the communication of a mathematical task solution, explanations of concepts or properties, when solving students' doubts, when using technological or manipulative resources, etc. The teacher can recognize them in forms of representation of external acts.

Rubio (2012) describes the design and implementation of a training period in the Secondary School Teachers Training Master Program of Universitat de Barcelona. Teachers are first taught the technique for the analysis of practices, objects, and processes proposed by OSA (onto-semiotic configuration) and then a method for the evaluation of mathematical competencies of the students. This study's objective was to test the following hypothesis: teachers' professional competence in analyzing mathematical practices and mathematical objects and processes activated in such practices is "in-depth knowledge" that evaluates and develops the students' mathematical competence. Rubio (2012) concludes that after all the experiments conducted, it can confirm such a hypothesis. Furthermore, it is stated that if teachers are not competent in the analysis of mathematical activity, processes, and objects, they will not be skilled in the evaluation of mathematical competencies. Thus, Rubio's thesis (2012) results indicate a sub-competence of the competence in analysis and didactic intervention that a mathematics teacher has to develop to foster and evaluate his students' competences: competence in analysis of the mathematical activity. In other words, the study of the mathematical practices, objects, and mathematical processes activated in them.

This first sub-competence enables teachers to analyze mathematical activity. This type of analysis is essential in the teacher's education, and it is a type of analysis that is somehow difficult for teachers and prospective teachers (Stahnke et al., 2016).

As mentioned before, the lack of consensus over a paradigm that defines how to analyze the mathematical activity in mathematical education is problematic. The DMK model assumes that the theoretical tools of OSA (practice, primary and secondary objects emerging from the practices, the meaning of a mathematical object in terms of practices, partial meanings, mathematical processes) allow such analysis in terms of practices, mathematical objects, and processes Godino et al., 2007, 2019). With these theoretical notions, when the meanings are understood pragmatically in terms of practices (Font et al., 2013), one can, firstly, answer questions such as: What are the partial meanings of the mathematical objects intended to be taught? How are they expressed together?

Identifying the objects and processes involved in mathematical activity by the teacher allows for comprehension of the learning process's progression, management of the necessary institutionalization processes, and evaluation of the students' mathematical competences. Thus, it is possible to answer the question: What are the configurations of primary mathematical objects and processes involved in the practices that constitute the intended contents' diverse meanings (epistemic configuration)? Mathematics teachers must know and comprehend tools such as the configuration of objects and processes activated in a particular mathematical practice and skillfully teach and learn mathematics (Lugo-Armenta & Pino-Fan, 2021; Pino-Fan et al., 2017b, 2018). Table 4 presents development levels for this sub-competency.

Table 4 Analysis of mathematical activity of the study process

L0	At this level, no features of the didactic analysis component are observed. In the didactic intervention component, as part of the teacher's mathematical activity, practices (operational or discursive) are observed that are considered incorrect from the mathematical point of view. There are ambiguities (in the wording of definitions, procedures, or properties) that can confuse students. Metaphors are ill-used, which can cause learning conflicts
L1	Features of the didactic analysis are still not observed; however, regarding the didactic intervention, the teacher no longer experiences what is indicated in L0. No mathematical practices are considered incorrect, nor ambiguities (in definitions, explanations, procedures, properties) that may confuse students. Metaphors are used in a more controlled way, although explanations, verifications, or demonstrations are not adapted to the educational level taught
L2	For the interventionist component, in addition to L1, the teacher promotes various ways to represent the mathematical object under study and promotes discussion by and among students. The definitions, procedures, properties, and explanations are clearly and correctly stated and, like the checks or demonstrations, appropriate to the educational level they address. The teacher is concerned with ensuring that students have prior knowledge to study the subject (either verifying that they have learned previously or are planning their study). However, in the mathematical activity performed, the representativeness of the richness of the mathematical object's meanings under investigation is not yet observed (Pino-Fan et al., 2011, 2018) Features of the didactic analysis components are observed when faced with moments when the teacher can reflect on his activity (or colleague's activity). He recognizes errors and mathematical ambiguities in the explanations, definitions, propositions, or metaphors used (this with the knowledge of some theoretical-methodological tool that has not yet been mastered, for example, the onto-semiotic configuration, or with the experience acquired from years of teaching service). The latter allows proposing alternatives to overcome the conflicts detected
L3	The teacher knows and has systematized the use of some theoretical-methodological tools (e.g. onto-semiotic configuration) to implement his intervention and its didactic analysis. In addition to what is described in L2, he foresees and uses various procedures and arguments regarding the same problem situation. He suggests tasks and explanations that promote different mathematical meanings of the object under study and uses intra- and extra-mathematical contexts that promote senses richness. When applying the didactic analysis, the teacher identifies both the key elements present in mathematical activity—representations, concepts/definitions, properties/propositions, procedures, and arguments—and the meanings used by the students, identifying conflicts for the students. Nonetheless, he helps propose alternatives to overcome such conflicts. Furthermore, he promotes and identifies mathematical and cognitive processes relevant to mathematical activity (e.g. generalization, modeling, argumentation, problem-solving, intra- and extra-mathematical connections, representation changes, and conjectures). Thus, curricular adaptation to student differences is guaranteed

Sub-competence in Management and Analysis of the Interactions

The notion of didactic configuration has been introduced in OSA to analyze the interactions in instruction processes (Godino et al., 2006). It is about a theoretical-methodological construct to model the articulation of teachers' and students' performance regarding a specific task and content (a configuration of primary objects and processes) of teaching and learning, where knowledge arises from the interaction. Mathematics teachers have to be competent in the design and management of didactic configurations. It intends to answer the question: What type of interactions between people and resources will be implemented in instructional processes, and what are the consequences in the learning process? How can interactions and conflicts be managed to optimize learning? Therefore, the teacher should know the many types of didactic configurations (dialogic) that can be implemented, their effect on students' learning, and how to design and manage them in specific instruction processes. To do this, the teacher must use his knowledge of the interactional facet and its links with other DMK model facets.

Both for the management and the analysis of interactions, it is necessary to remember that a normative system regulates such interactions. The different stages of the design process and implementation are supported by and depend on a complex set of norms and meta-norms of various origins and nature (Assis et al., 2012; ; Godino et al., 2009; Molina, 2019, Partanen & Kaasıla, 2015; Sánchez & García, 2014) that need to be explicitly recognized in order to comprehend the development of instruction processes and direct them towards optimal suitability levels. For example, there are rules regarding how it should be written or how it should be solved when studying equations.

Also, there are non-mathematical norms, such as whether calculators are used, the evaluation method, participating in class, etc. Therefore, mathematics teachers must become competent in the normative analysis of mathematical instruction processes to answer questions: What norms determine instructional procedures? Who, how, and when are the criteria established? What and how can these be changed to optimize mathematical learning? The studies cited present categories and subcategories to analyze the normative dimension of the study processes. The categories have been adopted as one of the methodological tools that operationalize the meta didactic-mathematical dimension of the DMK model. This dimension refers to the knowledge that allows us to reflect on the rules that regulate and impact interaction and student learning.

In short, this sub-competence refers to the management and analysis of interactions and rules that regulate the exchanges, and Table 5 shows the proposed gradation.

Sub-competence in the Use and Management of Resources

Various resources, such as materials, adaptations, manipulatives, mathematical software, applications, and calculators, constitute opportunities and challenges for teaching mathematics. There are proposals to coordinate the teacher's didactic knowledge with technological resources (Archambault & Barnett, 2010; Chai et al., 2013; Koh

Table 5 Sub-competence achievement level management and analysis of the interactions

L0	The intervention uses a "traditional interactional dynamic," in which the students mainly act as receptors. The topic's presentation is not entirely adequate (i.e. clear and well organized); the key concepts about the mathematical notion studied are not emphasized. Interaction between students is not promoted, nor is their autonomy. There is no empathy for the students' difficulties; it is not meaningful or motivating to solve the tasks for the students. The teachers identify some fundamental roles (or norms) of the agents involved in the study process in the analysis aspect. For example, "the student must pay attention to the teacher and solve the proposed problems" and "the teacher must explain and define the concepts."
L1	The teacher adequately presents the topic (i.e. a clear and well-organized presentation, does not speak too fast, emphasizes the fundamental concepts of the notion of function), and favors dialog and communication among students. Students' participation in class dynamics is promoted, doubts are answered, and, when necessary, clarifications are made about their misconceptions. The teacher also fosters students' positive attitudes, such as participation in the proposed activities, perseverance, and responsibility. Likewise, he understands students' conflicts and encourages them to overcome them. He also encourages students to value the usefulness of mathematics in everyday life. Based on the preceding information, the teacher can identify which interactional dynamics were best suited to the students' characteristics and which ones were not adjusted to what they wanted to achieve with their learning
L2	In addition to what was indicated in L1, the teacher, based on experience or knowledge of some theoretical-methodological tool, contemplates moments in which the students: pose questions and present solutions; explore examples and counterexamples to investigate and conjecture; use a variety of tools to reason, make intra- or extra-mathematical connections, solve tasks, and communicate their solutions, in other words, promotes student autonomy. Furthermore, the teacher uses various rhetorical and argumentative resources to capture the students' attention, understand the students' conflicts, encourage them, and propose questions to contrast their answers. Concerning the aspect of didactic analysis, the teacher analyzes the incomplete interventions of his students; he correctly interprets the silences of the students, their facial expressions, and their questions; examines the practices of his students (procedures, arguments, definitions), seeking to argue or explain based on consensus; identifies some norms that promote certain expected practices (Molina, 2019); and identifies situations, problems, or contexts that are of interest to students. The latter helps him predict how students solve specific tasks and estimate those they will find exciting and challenging. Additionally, he recognizes and uses actions that could help evaluate student progress
L3	Theoretical-methodological "tools" are used systematically (e.g. Assis et al., 2012; Godino, 2009; Molina, 2019), which allow for the identification and use of norms that regulate the interactions of the teaching process and enhance student learning. Besides what was raised in L2, qualities of precision and rigor of mathematics are highlighted, and self-esteem is promoted, avoiding negative predisposition to mathematics study. The teacher solves the students' issues of apathy and disinterest, fosters self-esteem, prevents rejection or fear of mathematics, and encourages learning. Students' cognitive progress (achievement of learning objectives) is systematically observed and analyzed. The latter makes it possible to assess the usefulness of mathematics in daily and professional life is used and studied, and argumentation is favored in situations of equality. That is, the argument is valued in itself and not by who formulates it

et al., 2015). The National Council of Teachers of Mathematics (2000) suggested that the use of technology is essential in mathematics classes since its proper implementation and management can positively influence teaching and, in turn, increase the learning possibilities of students. For Mishra and Koehler (2006), technological pedagogical content knowledge is the knowledge that is situated, multifaceted, and required for the reflective use and integration of technologies in the classroom. Teachers have to become familiar with the technologies (algebraic computer system, dynamic geometry software, spreadsheets), learn about available resources, and integrate them into classes.

However, technologies are not the only resources. To adapt to the various contexts in which mathematics' teaching and learning processes are performed, teachers must competently use material resources (textbooks, blackboard, manipulatives) and adequately allocate time during the study process. This sub-competency supposes an amalgam between the teacher's mathematical competence and the other sub-competencies of the analysis and didactic intervention to know, use, and reflect on the possibilities, challenges, and complex overlaps that the teaching resources entail. Table 6 presents the proposed levels of development for this sub-competency.

This sub-competency will allow the teacher to practice and answer questions such as: Is the time allocated to studying the mathematical object appropriate? What resources should be used to promote the learning of a mathematical object under

Table 6 Sub-competence achievement level management and analysis of the interactions

L0	Material resources (textbooks, manipulatives, computers, slide presentations) are items to be used rather than elements that can help construct student learning. Time is an element considered as part of the management of resources and procedures
L1	The relevance of material resources (manipulatives, calculators, and computers) is analyzed and used to introduce or reinforce mathematical notions or solve mathematical tasks. However, the procedures and arguments that emerge along the use of such materials are not adapted to the intended meaning (i.e. they could promote other meanings for the notion understudy). Or, the objective of using the material is lost, and it ends up teaching its use rather than using it to promote learning. At this level, the teacher does not know that using some resources involves using metaphors or analogies, which can cause mathematical confusion in students. Regarding teaching time, the teacher analyzes whether it is sufficient for the proper development of activity within the class (e.g. solving a task, group work, interaction with the software)
L2	Various materials are used to promote learning, and analyses are performed based on their relevance or effectiveness (according to the students' objectives, context, and characteristics) The procedures and arguments that emerge in the use of such materials are adapted to the intended meaning. The teacher analyzes and uses the resources with the proposed objective (i.e. it does not divert the aim of teaching the use of the help). Faced with materials such as texts, the teacher analyzes, questions, and complements the relevant tasks. Time, within the classroom, is another element that he studies and uses in his classes (teacher considers time to understand the problem, group work, individual work, and institutionalize the built-in knowledge). However, the teacher still needs to consider the non-contact times and other times not derived directly from the classroom interactions (if the math class is on Friday in the last block of hours before the students finish their day, then no new content is introduced). In addition, the metaphors and analogies derived from the use of resources are still an aspect to be improved
L3	Relevance is analyzed (in terms of expected learning objectives, context, students' characteristics, the educational center); manipulatives and computer materials are used to introduce mathematics ideas, languages (representations, visualization), procedures, and arguments. Each of these aspects is adapted to the intended meaning. The resources used allow identifying the diversity of mathematical object meanings. The metaphors and analogies derived from using resources are controlled and explained to avoid confusing the students. The teacher considers the classroom size, the class schedule, and the classroom conditions to design and conduct teaching. Other features are analyzed and included: time allocation for teaching or tutoring, for individual learning, for understanding a task, for the institutionalization of a notion, for ensuring that students have learned the necessary prior knowledge, for the most important or core contents of the subject, and for the contents that present more difficulty

consideration? What are the advantages and disadvantages of the resources I intend to use? How do I propose efficient collaborative workspaces through the use of material or technological resources? What obstacles to teaching and learning does the use of the resources under consideration pose?

Sub-competence in the Analysis and Assessment of the Didactical Suitability

This sub-competency could be seen as the most complex to develop due to the diversity of aspects that constitute it, which allow understanding, development, and analysis of didactic interventions. It refers to the teacher's competence to reflect on his practice or that of others at various moments in the process: a priori reflection, in situ reflections, and a posteriori review. In the scientific literature, we can find multiple proposals for a posteriori review (Brown & Coles, 2012) and in situ reflection, or "teacher noticing" (Castro et al., 2018; Schack et al., 2017). However, it has been informed that these approaches are not exhaustive (Kunter et al., 2013) and reveal the need for theoretical-methodological tools that guide and systematize reflection. The teacher must develop their reflective skills to foresee, act, and seek potential improvements for the study processes, guaranteeing and improving learning opportunities.

This comprehensive sub-competence requires all the previous sub-competencies—both mathematical competence and the competence of didactic analysis and intervention; consequently, criteria and descriptors are required to systematize the didactic analysis of the teacher and thereby improve the teaching. The DMK model proposes didactic suitability as an essential tool (Breda et al., 2017; Pino-Fan et al., 2017a). Once a specific topic has been selected in a particular educational context, the notion of didactic suitability helps answer questions such as: What is the degree of didactical suitability of the teaching and learning processes implemented? What changes should be made in the design and implementation of the instruction process to increase its didactic suitability in the future?

Didactical suitability of an instruction process refers to the degree to which such method includes specific characteristics considered as suitable (optimal or ideal) to attain adaptation between students' meanings (learning) and the intended or implemented institutional purposes (teaching), considering the circumstances and available resources (environment).

The notion of didactical suitability is separated into six specifics types:

- Epistemic suitability refers to the mathematics taught ideally as "good mathematics," which considers what is prescribed in curriculum and mathematics textbooks
- Cognitive suitability expresses the degree to which the intended or implemented learning is within the students' zone of potential development and the proximity of the attained understanding to the intended or implemented learning
- Interactional suitability refers to the degree to which the modes of interaction allow for identifying and solving conflicts of meaning and favor autonomy in learning

- Mediational suitability refers to the degree of availability and adaptation of the material and time resources necessary to develop the teaching and learning processes
- Affective suitability refers to students' degree of implication (interest, motivation) in the study process
- Ecologic suitability refers to the degree of adaptation to the school's comprehensive education plan, including the school's study process, the curricular guidelines, the environment, etc.

Various studies (Beltrán-Pellicer et al., 2018; Breda et al., 2018; Godino et al., 2005; Ramos & Font, 2008) have proposed criteria and descriptors for each suitability type. The studies' results help the operationalization, development, and systematization of the meta didactic-mathematical dimension of the DMK model. In general terms, it is a rubric system that allows for a complete or balanced reflection on the elements that make up a "quality" teaching process in mathematics. The proposed didactic suitability criteria and descriptors can be used for the teacher's didactic intervention (design, implementation, and evaluation) and for analyzing and evaluating such intervention; nonetheless, such suitability criteria and descriptors can be used by teachers in different degrees of depth, from an intuitive use based on experience (i.e. they do not know the "tool" but intuitively recognize some features of the six suitability types), to "expert" use of the tool. In this sense, various studies developed within the framework of the DMK model (Breda et al., 2017; Rubio, 2012; Seckel, 2016) have shown that even when the teachers do not know the didactical suitability criteria with all their components and indicators, they implicitly use them to assess their teaching (Burgos & Godino, 2022; Burgos et al., 2020). In any case, for teachers to reflect and value their teaching, it is necessary to create opportunities (Castro et al., 2018; Pino-Fan & Parra, 2021). They can systematize and habitually analyze and assess the teaching process. Schoenfeld and Kilpatrick (2008, p. 348) explain it as follows, "Once it is habitual, reflection can become the main mechanism for improving one's teaching practice." Based on the above, it is possible to propose a gradation for this sub-competence (Table 7).

Final Reflections

This article presents a "macro tool" proposal that identifies and characterizes critical competencies for the mathematics teacher's practice. The tool considers two competencies that include sub-competencies that broaden and describe teachers' knowledge and subordinate competencies that a teacher must exhibit to solve professional problems. Likewise, it provides achievement levels. The proposal is based on research performed within the DMK model framework, which is based on the theoretical notions of the OSA (Godino et al., 2007,2019; Presmeg, 2014), which in turn provides both an academic and pragmatic foundation to the proposal.

The proposal on the knowledge and skills of the teacher has advantages and limitations. The benefits refer to the advance made from a recommendation of mathematical competencies, referred to as "mathematical solving problems" (Niss, 2015),

Table 7 Development levels of the analysis and assessment of didactic suitability

	Categories		
	Study process phase	Type of analysis	Depth of analysis
Level 0	A posteriori reflection does not identify plausible improvements or have references to perform or contrast its reflection elements	The superficial and ambiguous analysis does not describe, explain, or assess the study process or episode	Superficial narrative (written or discursive). The narrative does not account for what happened in the episode
Level 1	A priori or a posteriori reflection of the episode. An attempt is made to contrast the results of the analysis of these two moments	*Descriptive* It favors knowing what happened in the episode or study process	Narrative that captures the essential elements of the episode under analysis. Whoever reads or listens to the narrative gets an idea of what happened in the episode
Level 2	A priori, in situ, and a posteriori episode reflection. The analysis of the three moments is not yet coordinated, or the in situ reflection is still scarce	*Explicative* It seeks to answer: Why did what happened in the episode (concerning a phenomenon, conflict, error, etc.)?	Complete and understandable narrative, which involves a detailed analysis, trying to follow a model (e.g. if a description of the mathematical activity is made, components of the onto-semiotic configuration are used to identify some practices, primary objects, and processes; or the suitability criteria are used explicitly or implicitly)
Level 3	Coordinated reflection of the study process moments: a priori, in situ, and a posteriori	*Appraisal* It includes the elements of the two previous levels and answers the question: What can or should be improved in the episode and why?	Expert analysis of the narrative according to the systematic use of a model (e.g. a detailed description of the mathematical activity is made, the practices, primary objects, and processes, meanings of the notions used, are exhaustively identified; systems of norms that conditioned the interactions and learnings in the episode; or the eligibility criteria explicitly)

to one where teacher' competencies are identified to design, recognize, and promote student mathematical activity. The proposed levels and sublevels are validated by research over 9 years. The competency and sub-competency levels have been identified in various investigations and validated in others. Therefore, it is expected that the proposal materialized in Tables 1, 2, 3, 4, 5, 6, and 7 can be helpful to guide the initial teacher training and professional development process, promoting knowledge and skills in mathematics teachers.

An advantage of the proposal of levels of mathematics competence is that the research that led to their identification has been carried out in various countries, and the teaching practices that helped identify them coincide, and attends to various cultural and curricular contexts.

The disadvantage of the proposal is that it looks pretty complex, which makes it challenging to implement and operationalize with preservice and in-service teachers. The instruments to be used are varied, and the interpretation of information requires knowledge of the onto-semiotic approach to cognition and mathematical instruction (OSA). The latter can assume the levels and sublevels as a gradation of competencies that they must exhibit instead of seeing them as evidence of the complexity of their teaching work. In addition, the complexity of the proposal can alienate both researchers and teachers. However, an interesting aspect is that what is presented in Tables 1, 2, 3, 4, 5, 6, and 7 is compatible with theoretical and methodological proposals for developing didactic or content analysis; therefore, it is not required to use the OSA and the DMKC model exclusively.

Although an example that favors the proposal's understanding would be desirable, for space reasons, it has been impossible to introduce it in this article since its inclusion would be detrimental to the in-depth explanation of the competencies, sub-competencies, and the levels of gradation proposed here. However, the DMK model's systematic use to observe each sub-competency and their gradation levels was progressive over time. Thus, the proposal presented in this article is the result of the systematic observation of teacher training research implemented with the use of the DMK model in the last 9 years (from 2011 to date); some of the studies can be found and downloaded from the following website:

http://enfoqueontosemiotico.ugr.es/pages/fprofesores.html

The conditions that promote such competencies and mobility between the sublevels for the Mathematical Competence and the Didactic Analysis and Intervention Competence are open research topics. However, it is reasonable to assume that both are developed jointly and are interdependent. For example, suppose we accept that the teacher's mathematical knowledge is an indicator of student achievement (Schoenfeld, 2010); in that case, it is necessary to propose tools and strategies to improve the teacher's didactic-mathematical knowledge. Thus, the opportunities for quality mathematics education can be increased. The proposal presented in this document is an alternative to study and develop the mathematics teacher's competencies in response to the demands imposed on the performance of teachers and students. However, much research is required to adjust the competencies and determine criteria to favor the mobility between sublevels of competence.

Acknowledgements This work has been developed within the framework of the project Fondecyt 1200005, funded by Agencia Nacional de Investigación y Desarrollo (ANID) of Chile.

Funding This article has been developed within the framework of the research project Fondecyt 1200005, funded by Agencia Nacional de Investigación y Desarrollo (ANID) of Chile.

Declarations

Ethics Approval The authors have complied in the writing of the article with all the ethical guidelines established by the journal.

Informed Consent The authors of this article are informed and accept the ethics policies, standards, and guidelines of the journal.

Conflict of Interest The authors declare no competing interests.

References

An, S., & Wu, Z. (2012). Enhancing mathematics teachers' knowledge of students' thinking from assessing and analyzing misconceptions. *International Journal of Science and Mathematics Education, 10*(3), 717–753. https://doi.org/10.1007/s10763-011-9324-x

Archambault, L. M., & Barnett, J. H. (2010). Revisiting technological pedagogical content knowledge: Exploring the TPACK framework. *Computers & Education, 55*(4), 1656–1662. https://doi.org/10.1016/j.compedu.2010.07.009

Arsal, Z. (2014). Microteaching and pre-service teachers' sense of self-efficacy in teaching. *European Journal of Teacher Education, 37*(4), 453–464. https://doi.org/10.1080/02619768.2014.912627

Assis, A., Godino, J. D., & Frade, C. (2012). As dimensões normativa e metanormativa em um contexto de aulas exploratório-investigativas [The normative and meta-normative dimensions in a context of exploratory-investigative classes]. *RELIME: Revista Latinoamericana de Investigación en Matemática Educativa, 15*(2), 171–198.

Barmby, P., Bolden, D., & Thompson, L. (2014). *Understanding and enriching problem solving in primary mathematics*. Critical Publishing.

Barquero, B., & Bosch, M. (2015). Didactic engineering as a research methodology: From fundamental situations to study and research paths. In A. Watson & M. Ohtani (Eds.), *Task design in mathematics education - ICMI study 22* (pp. 249–273). https://doi.org/10.1007/978-3-319-09629-2_8

Bell, C. A., Wilson, S. M., Higgins, T., & McCoach, D. B. (2010). Measuring the effects of professional development on teacher knowledge: The case of developing mathematical ideas. *Journal for Research in Mathematics Education, 41*(5), 479–512. https://doi.org/10.2307/41110411

Beltrán-Pellicer, P., Medina, A., & Quero, M. (2018). Movies and TV series fragments in mathematics: Epistemic suitability of instructional designs. *International Journal of Innovation in Science and Mathematics Education, 26*(1), 16–26, 2018.

Biehler, R. (2005). Reconstruction of meaning as a didactical task: The concept of function as an example. In J. Kilpatrick, C. Hoyles, & O. Skovsmose (Eds.), *Meaning in mathematics education* (pp. 61–81). New York, NY: Springer. https://doi.org/10.1007/0-387-24040-3_5

Blömeke, S., & Delaney, S. (2012). Assessment of teacher knowledge across countries: A review of the state of research. *ZDM, 44*(3), 223–247. https://doi.org/10.1007/s11858-012-0429-7

Breda, A., & Lima, V. M. (2016). Case study on the didactic assessment over a final work of a master for mathematics teachers in service. *REDIMAT, 5* (1), 74-103. https://doi.org/10.4471/redimat.2016.1955

Breda, A., Pino-Fan, L., & Font, V. (2017). Meta didactic-mathematical knowledge of teachers: Criteria for the reflection and assessment on teaching practice. *EURASIA*: *Journal of Mathematics, Science and Technology Education, 13*(6), 1893–1918. https://doi.org/10.12973/eurasia.2017.01207a

Breda, A., Font, V., & Pino-Fan, L. (2018). Criterios valorativos y normativos en la Didáctica de las Matemáticas: el caso del constructo idoneidad didáctica [Evaluative and normative criteria in

Didactics of Mathematics: The case of didactic suitability construct]. BOLEMA: Boletim de Educação Matemática, 32(60), 255–278. https://doi.org/10.1590/1980-4415v32n60a13
Brown, L., & Coles, A. (2012). Developing “deliberate analysis” for learning mathematics and for mathematics teacher education: How the enactive approach to cognition frames reflection. *Educational Studies in Mathematics, 80*(1–2), 217–231. https://doi.org/10.1007/s10649-012-9389-7
Burgos, M., Castillo, J., Beltrán, P., Giacomone, B., & Godino, J.D. (2020). *Didactical analysis of a lesson on proportionality of a primary school textbook using tools of the onto-semiotic approach. Bolema, 34*(66), 40–68. https://doi.org/10.1590/1980-4415v34n66a03.
Burgos, M., & Godino, J. D. (2022). Assessing the epistemic analysis competence of prospective primary school teachers on proportionality tasks. *International Journal of Science and Mathematics Education. Online First*. https://doi.org/10.1007/s10763-020-10143-0
Cai, J., Chen, T., Li, X., Xu, R., Zhang, S., Hu, Y., & Song, N. (2020). Exploring the impact of a problem-posing workshop on elementary school mathematics teachers’ conceptions on problem posing and lesson design. *International Journal of Educational Research, 102*, 101404. https://doi.org/10.1016/j.ijer.2019.02.004
Castro, W. F. (2011). Evaluación y desarrollo de competencias de análisis didáctico de tareas sobre razonamiento algebraico elemental en futuros profesores [doctoral thesis, Universidad de Granada]. http://enfoqueontosemiotico.ugr.es/pages/tesisdoctorales.html
Castro, W. F., Pino-Fan, L., & Velásquez-Echavarría, H. (2018). A proposal to enhance preservice teacher’s noticing. EURASIA: *Journal of Mathematics, Science and Technology Education, 14*(11), em1569. https://doi.org/10.29333/ejmste/92017
Chai, C. S., Ng, E. M. W., Li, W., Hong, H.-Y., & Koh, J. H. L. (2013). Validating and modelling technological pedagogical content knowledge framework among Asian preservice teachers. *Australasian Journal of Educational Technology, 29*(1), 41–53. https://doi.org/10.14742/ajet.174
Chan Y. C. & Leung, A. (2013). Rotational symmetry: Semiotic potential of a transparency toolkit. In C. Margolinas (Ed.), *Rotational symmetry: Semiotic potential of a transparency toolkit, task design in mathematics education. Proceedings of ICMI study 22* (pp. 35–44). Oxford, UK.
Cohen, L., Manion, L., & Morrison, K. (2011). *Research methods in education*. Routledge.
Creswell, J.W. (2009). Research design: Qualitative, quantitative, and mixed methods approaches. Sage Publications.
Davis, B. (2008). Is 1 a prime number? Developing teacher knowledge through concept study. *Mathematics Teaching in the Middle School, 14*(2), 86–91. https://doi.org/10.2307/41182638
Donnelly, R., & Fitzmaurice, M. (2011). Towards productive reflective practice in microteaching. *Innovations in Education and Teaching International, 48*(3), 335–346. https://doi.org/10.1080/14703297.2011.593709
Falcade, R., Laborde, C., & Mariotti, M. A. (2007). Approaching functions: Cabri tool as instruments of semiotic mediation. *Educational Studies in Mathematics, 66*, 317–333.
Felmer, P., Pehkonen, E., & Kilpatrick, J. (Eds.). (2016). Posing and solving mathematical problems: Advances and new perspectives. Cham, Switzerland: Springer. https://doi.org/10.1007/978-3-319-28023-3
Felmer, P., Liljedhal, P., & Koichu, N. (Eds.). (2019). Problem solving in mathematics instruction and teacher professional development. Cham, Switzerland: Springer. https://doi.org/10.1007/978-3-030-29215-7
Fernández, C., & Yoshida, M. (2004). *Lesson study: A Japanese approach to improving mathematics teaching and learning*. Lawrence Erlbaum. https://doi.org/10.4324/9781410610867
Fernández, C. (2011). Análisis de temas en los libros de texto de matemáticas [Analysis of topics in mathematics textbooks]. UNO: *Revista de Didáctica de las Matemáticas, 56*, 77–85.
Font, V. (2011). Competencias profesionales en la formación inicial de profesores de matemáticas de secundaria [Professional competences in initial secondary education mathematics teachers’ training]. *UNIÓN: Revista Iberoamericana de Educación Matemática, 26*, 9–25.
Font, V., Godino, J. D., & Gallardo, J. (2013). T*he emergence of objects from mathematical practices. Educational Studies in Mathematics, 82*(1), 97–124. https://doi.org/10.1007/s10649-012-9411-0
Fortuny, J. M., & Rodríguez, R. (2012). Aprender a mirar con sentido: facilitar la interpretación de las interacciones en el aula [Learn to notice and interpret classroom interactions]. *Avances de Investigación en Educación Matemática, 1*, 23–37. https://doi.org/10.35763/aiem.v1i1.3
Gellert, U., Becerra, R., & Chapman, O. (2013). Research methods in mathematics teacher education. In A. Bishop, M. A. K. Clements, C. Keitel-Kreidt, J. Kilpatrick, & F. K.-S. Leung (Eds.), Third

international handbook of mathematics education (pp. 327–360). New York, NY: Springer. https://doi.org/10.1007/978-1-4614-4684-2_11

Giacomone, B. (2019). Análisis a priori de tareas matemáticas. Un componente del análisis didáctico. *Uno Revista De Didáctica De Las Matemáticas, 86*, 25–31.

Giménez, J., Font, V., & Vanegas, Y. (2013). Designing professional tasks for didactical analysis as a research process. In C. Margolinas (Ed.), Task Design in Mathematics Education: Proceedings of ICMI Study 22 (pp. 579–587). Oxford, England: International Commission on Mathematical Instruction

Godino, J. D. (2009). Categorías de análisis de los conocimientos del profesor de matemáticas [Categories of analysis of the mathematics teacher's knowledge]. *UNIÓN: Revista Iberoamericana de Educación Matemática, 20*, 13–31.

Godino, J. D., Contreras, A., & Font, V. (2006). Análisis de procesos de instrucción basado en el enfoque ontológico-semiótico de la cognición matemática [Analysis of instruction processes based on the ontologicalsemiotic approach to mathematical cognition]. Recherches en Didactiques des Mathematiques, 26(1), 39-88.

Godino, J. D., Batanero, C., & Font, V. (2007). *The onto-semiotic approach to research in mathematics education. ZDM – Mathematics Education, 39*(1), 127–135. https://doi.org/10.1007/s11858-006-0004-1

Godino, J. D., Wilhelmi, M. R., & Bencomo, D. (2005). Suitability criteria of a mathematical instruction process. A teaching experience of the function notion. *Mediterranean Journal for Research in Mathematics Education, 4*(2), 1–26.

Godino, J. D., Batanero, C., Rivas, H., & Arteaga, P. (2013). Componentes e indicadores de idoneidade de programas de formação de professores em educação matemática [Suitability components and indicators of teachers' education programs in mathematics education]. REVEMAT: Revista Eletrônica de Educação Matemática, 8(1), 46–74. https://doi.org/10.5007/1981-1322.2013v8n1p46

Godino. J.D., Giakamone, B., Batanero, C., & Font, V. (2017). Enfoque Ontosemiótico de los Conocimentos y Competencias del Profesor de Matemáticas [Onto-Semiotic Approach to Mathematics Teacher's Knowledge and Competences]. *Bolema, 31*(57), 90–113. https://doi.org/10.1590/1980-4415v31n57a05

Godino, J. D., Batanero, C., & Font, V. (2019). The onto-semiotic approach: implications for the prescriptive character of didactics. *For the Learning of Mathematics, 39*(1), 38–43.

Hill, H. C., Ball, D. L., & Schlling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39*, 372–400.

Hummes, V. (2022). Uso combinado del lesson study y de los criterios de idoneidad didáctica para el desarrollo de la reflexión sobre la práctica en la formación de profesores de matemáticas. (Unpublished doctoral dissertation). Universitat de Barcelona, España.

Jones, K. (2000). Providing a foundation for deductive reasoning: Students' interpretations with using dynamic geometry software and their evolving mathematical explanations. *Educational Studies in Mathematics, 44*(1–3), 55–85.

Klafki, W. (1995). Didactic analysis as the core of preparation of instruction (Didaktische Analyse als Kern der Unterrichtsvorbereitung). *Journal of Curriculum Studies, 27*(1), 13–30. https://doi.org/10.1080/0022027950270103

Koh, J. H. L., Chai, C. S., Wong, B., & Hong, H.-Y. (2015). Technological Pedagogical Content Knowledge (TPACK) and design thinking: A framework to support ICT lesson design for 21st century learning. *The Asia-Pacific Education Researcher, 24*(3), 535–543. https://doi.org/10.1007/s40299-015-0237-2

Korthagen, F. (2010). La práctica, la teoría y la persona en la formación del profesorado [Practice, theory, and person in teachers' training]. *Revista Interuniversitaria De Formación Del Profesorado, 68*(24), 83–101.

Kunter, M., Baumert, J., Blum, W., Klusmann, U., Krauss, S., & Neubrand, M. (Eds.). (2013). Cognitive activation in the mathematics classroom and professional competence of teachers. Results from the COACTIVE Project. New York, NY: Springer. https://doi.org/10.1007/978-1-4614-5149-5

Leuders, T., Dörfler, T., Leuders, J., & Philipp, K. (2018) Diagnostic competence of mathematics teachers: Unpacking a complex construct. In T. Leuders, K. Philipp, & J. Leuders (Eds.), Diagnostic competence of mathematics teachers: Unpacking a complex construct in teacher education and teacher practice (pp. 3–31). Cham, Switzerland: Springer. https://doi.org/10.1007/978-3-319-66327-2_1

Llinares, S. (2011). Tareas matemáticas en la formación de maestros: caracterizando perspectivas [Mathematical tasks in teachers' training: Characterising perspectives]. *NÚMEROS: Revista de Didáctica de las Matemáticas, 78*, 5–16.

Llinares, S. (2012). Construcción de conocimiento y desarrollo de una mirada profesional para la práctica de enseñar matemáticas en entornos en línea [Knowledge building and development of professional noticing for the mathematics teaching. What are we learning?]. *Avances de Investigación en Educación Matemática, 2*, 53–70. https://doi.org/10.35763/aiem.v1i2.18

Lugo-Armenta, J.G., & Pino-Fan, L. (2021). *Inferential statistical reasoning of math teachers: experiences in virtual contexts generated by the Covid-19 pandemic. Education Sciences, 11*(7), 363. https://doi.org/10.3390/educsci11070363

Malaspina, U. (2017). La creación de problemas como medio para potenciar la articulación de competencias y conocimientos del profesor de matemáticas [Problem posing as a means to foster the articulation of mathematics teacher's competences and knowledge]. In J. M. Contreras et al. (Eds.), *Actas del Segundo Congreso International Virtual sobre el Enfoque Ontosemiótico del Conocimiento y la Instrucción Matemáticos*. Available at: http://enfoqueontosemiotico.ugr.es/civeos/malaspina.pdf

Mariotti, M. A. (2002). Justifying and proving in the Cabri environment. *International Journal of Computers for Mathematical Learning, 6*(3), 257–281.

Mason, J., & Johnston-Wilder, S. (2004). Designing and using mathematical tasks. London, England: Tarquin.

Mergler, A. G., & Tangen, D. (2010). Using microteaching to enhance teacher efficacy in pre-service teachers. *Teaching Education, 21*(2), 199–210. https://doi.org/10.1080/10476210902998466

Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record, 108*(6), 1017–1054.

Molina, O.J. (2019). Sistema de normas que influyen en procesos de argumentación: un curso de geometría del espacio como escenario de investigación [System of norms that influence argumentation processes: A space geometry course as a research setting] (Doctoral thesis). Retrieved from http://enfoqueontosemiotico.ugr.es/documentos/Tesis_OMolina.pdf

National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Richmond, VA: Author.

Neubrand, M. (2018). Conceptualizations of professional knowledge for teachers of mathematics. *ZDM, 50*(4), 601–612. https://doi.org/10.1007/s11858-017-0906-0

Niss, M. (2015). Mathematical competencies and PISA. In K. Stacey & R. Turner (Eds.), *Assessing mathematical literacy: The PISA experience* (pp. 35–55). Springer International Publishing.

Niss, M., & Højgaard, T. (2019). Mathematical competencies revisited. *Educational Studies in Mathematics, 102*(1), 9–28. https://doi.org/10.1007/s10649-019-09903-9

Organization Economic Co-operation and Development. (2004). Learning for tomorrow's world: First results from PISA 2003. Paris, France: Author.

Parra- Urrea, Y. (2021). Conocimiento didáctico-matemático de futuros profesores chilenos de enseñanza media sobre la noción de función: Una experiencia en contextos de microenseñanza [doctoral thesis, Universidad de LosLagos]. http://edumat.ulagos.cl/tesis/

Partanen, A. M., & Kaasıla, R. (2015). Sociomathematical norms negotiated in the discussions of two small groups investigating calculus. *International Journal of Science and Mathematics Education, 13*(4), 927–946. https://doi.org/10.1007/s10763-014-9521-5

Petrou, M., & Goulding, M. (2011). Conceptualising teachers' mathematical knowledge in teaching. In T. Rowland & K. Ruthven (Eds.), Mathematical knowledge in teaching (pp. 9–25). Dordrecht, The Netherlands: Springer. https://doi.org/10.1007/978-90-481-9766-8_2

Pino-Fan, L., Godino, J. D., & Font, V. (2011). Faceta epistémica del conocimiento didáctico-matemático sobre la derivada [Epistemic Facet of the Didactic-Mathematics Knowledge About The Derivative]. *Educação Matemática Pesquisa, 13*(1), 141–178. https://revistas.pucsp.br/index.php/emp/article/view/4423

Pino-Fan, L. (2013). Evaluación de la faceta epistémica del conocimiento didáctico-matemático de futuros profesores de bachillerato sobre la derivada [Evaluation of the epistemic aspect of the didactic-mathematical knowledge of future secondaryteachers on the derivative] (Doctoral thesis). Universidad de Granada. http://enfoqueontosemiotico.ugr.es/pages/tesisdoctorales.html

Pino-Fan, L., & Godino, J. D. (2015). Perspectiva ampliada del conocimiento didáctico-matemático del profesor [Extendedperspective of didactic-mathematical knowledge of the teacher]. *PARADIGMA, 36*(1), 87–109.

Pino-Fan, L., Assis, A., & Castro, W. F. (2015). Towards a methodology for the characterization of teachers' didacticmathematical knowledge. *EURASIA: Journal of Mathematics, Science and Technology Education, 11*(6), 1429–1456. https://doi.org/10.12973/eurasia.2015.1403a

Pino-Fan, L., Font, V., & Breda, A. (2017a). Mathematics teachers' knowledge and competences model based on the ontosemiotic approach. In B. Kaur, W. K. Ho, T. L. Toh & B. H. Choy (Eds.), Proceedings of the 41st Conference of the International Group for the Psychology of Mathematics Education (Vol. 4, pp. 33-40). Singapore: PME.

Pino-Fan, L., Font, V., Gordillo, W., Larios, V., & Breda, A. (2017b). Analysis of the meanings of the antiderivative used by students of the first engineering courses. *International Journal of Science and Mathematics Education, 16,* 1091–113. https://doi.org/10.1007/s10763-017-9826-2

Pino-Fan, L., Godino, J. D., & Font, V. (2018). Assessing key epistemic features of didactic-mathematical knowledge of prospective teachers: the case of the derivative. *Journal of Mathematics Teacher Education, 21*(1), 63–94. https://doi.org/10.1007/s10857-016-9349-8

Pino-Fan, L. & Parra-Urrea, Y. (2021). Criterios para orientar el diseño y la reflexión de clases sobre funciones ¿Qué nos dice la literatura científica? [Criteria to guide the design and reflection of classes on functions. What does the scientific literature tell us?]. UNO. *Revista de Didáctica de las Matemáticas, 91*, 45–54.

Pochulu, M., Font, V., & Rodríguez, M. (2016). Desarrollo de la competencia en análisis didáctico de formadores de futuros profesores de matemática a través del diseño de tareas [Development of the competence in didactic analysis of trainers of future mathematics teachers through task design]. RELIME: Revista Latinoamericana deInvestigación en Matemática Educativa, 19(1), 71–98. https://doi.org/10.12802/relime.13.1913

Ponte, J. P. (2005). Gestão curricular em Matemática. In GTI (Ed.), O professor e o desenvolvimento curricular (pp. 11–34). Lisboa: APM

Presmeg, N. (2014). Contemplating visualization as an epistemological learning tool in mathematics. *ZDM, 46*(1), 151–157. https://doi.org/10.1007/s11858-013-0561-z

Prince, M. J., & Felder, R. M. (2006). Inductive teaching and learning methods: Definitions, comparisons, and research bases. *Journal of Engineering Education, 95*(2), 123–138. https://doi.org/10.1002/j.2168-9830.2006.tb00884.x

Puig, L. (1997). Análisis fenomenológico. En L. Rico (Coord.) La educación matemática en la enseñanza secundaria (págs. 61–94). Barcelona: Horsori / ICE.

Ramos, A. B., & Font, V. (2008). Criterios de idoneidad y valoración de cambios en el proceso de instrucción matemática [Suitability and assessment criteria of changes in the mathematics instruction process]. *RELIME: Revista Latinoamericana de Investigación en Matemática Educativa, 11*(2), 233–265.

Rivas, M. (2012). Análisis epistémico y cognitivo de tareas de proporcionalidad en la formación de profesores de educación primaria. [doctoral thesis, Universidad de Granada]. http://enfoqueontosemiotico.ugr.es/pages/tesisdoctorales.html

Rowland, T., & Ruthven, K. (Eds.). (2011). *Mathematical knowledge in teaching*. Springer. https://doi.org/10.1007/978-90-481-9766-8_1

Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education, 8*(3), 255–281.

Rowland, T. (2014). Frameworks for conceptualizing mathematics teacher knowledge. In S. Lerman (Ed.), Encyclopedia of mathematics education (pp. 235–238). Dordrecht, The Netherlands: Springer. https://doi.org/10.1007/978-94-007-4978-8_63

Rubio Goycochea, N. (2012). Competencia del profesorado en el análisis didáctico de prácticas, objetos y procesos matemáticos [Teachers competence in didactic analysis of mathematical practices, objects, and processes] (Doctoral thesis). Retrieved from Tesis Doctorals in Xarxa. (http://www.tdx.cat/handle/10803/294031)

Sánchez, V., & García, M. (2014). Socio-mathematical and mathematical norms related to definition in pre-service primary teachers' discourse. *Educational Studies in Mathematics, 85*(2), 305–320. https://doi.org/10.1007/s10649-013-9516-0

Schack, E. O., Fisher, M. H., & Wilhelm, J. (Eds.). (2017). Teacher noticing: Bridging and broadening perspectives, contexts, and frameworks. New York, NY: Springer. https://doi.org/10.1007/978-3-319-46753-5

Scheiner, T., Montes, M. A., Godino, J. D., Carrillo, J., & Pino-Fan, L. (2019). What makes mathematics teacher knowledge specialized? Offering alternative views. *International Journal of Science and Mathematics Education, 17*(1), 153–172. https://doi.org/10.1007/s10763-017-9859-6

Schoenfeld, A. H. (2010). *How we think: A theory of goal-oriented decision making and its educational applications*. Routledge.

Schoenfeld, A., & Kilpatrick, J. (2008). Towards a theory of profiency in teaching mathematics. In D. Tirosh & T. L. Wood (Eds.), Tools and processes in mathematics teacher education (pp. 321–354) Rotterdam, The Netherlands: Sense Publishers. https://doi.org/10.1163/9789087905460_016

Seckel, J. M. S. (2016). Competencia en análisis didáctico en la formación inicial de profesores de educación general básica con mención en matemática. (Doctoral dissertation). Universitat de Barcelona, Barcelona. Retrieved from http://hdl.handle.net/2445/99644

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1–22. https://doi.org/10.17763/haer.57.1.j463w79r56455411

Silverman, J., & Thompson, P. W. (2008). Toward a framework for the development of mathematical knowledge for teaching. *Journal of Mathematics Teacher Education, 11*(6), 499–577. https://doi.org/10.1007/s10857-008-9089-5

Stahnke, R., Schueler, S., & Roesken-Winter, B. (2016). Teachers' perception, interpretation, and decision-making: A systematic review of empirical mathematics education research. *ZDM, 48*(1–2), 1–27. https://doi.org/10.1007/s11858-016-0775-y

Tzur, R., Sullivan, P., & Zaslavsky, O. (2008). Examining teachers' use of (non-routine) mathematical tasks in classrooms from three complementary perspectives: Teacher, teacher educator, researcher. In O. Figueras & A. Sepúlveda (Eds.), Proceedings of the joint meeting of PME 32 and PME-NA XXX , Vol. 1, pp. 133–137). Mexico City: PME.

Vásquez, C., & Alsina, A. (2015). Conocimiento didáctico-matemático del profesorado de educación primaria sobre probabilidad: diseño, construcción y validación de un instrumento de evaluación [Primary school teachers' didactic-mathematical knowledge when teaching probability: Development and validation of an evaluation instrument]. *BOLEMA: Boletim de Educação Matemática, 29*(52), 681–703. https://doi.org/10.1590/1980-4415v29n52a13

Villalobos, F. X. (2008). Resolución de problemas matemáticos: Un cambio epistemológico con resultados metodológicos [Mathematical problem-solving: An epistemological change with methodological results]. *Revista Iberoamericana Sobre Calidad, Eficacia y Cambio En Educación, 6*(3), 36–58.

Watson, A., & Ohtani, M. (Eds.). (2015). *Task design in mathematics education*. Springer. https://doi.org/10.1007/978-3-319-09629-2

Weinert, F. E. (2001). Concept of competence: A conceptual clarification. In D. S. Rychen & L. H. Salganik (Eds.), *Defining and selecting key competencies* (pp. 45–65). Hogrefe & Huber Publishers.

Zaslavsky, O., & Sullivan, P. (Eds.). (2011). *Constructing knowledge for teaching secondary mathematics*. Boston, MA: Springer. https://doi.org/10.1007/978-0-387-09812-8