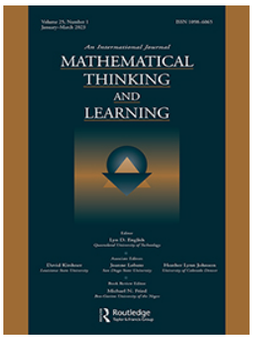

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Statistical modeling in teacher education

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ABSTRACT

The purpose of this research is to investigate prospective teachers' learning in the statistical modeling process. To reach this goal, we position the study in a socio-critical perspective of modeling that informed the design of a statistical investigation. Participants were 10 prospective teachers from a state university in northwestern Colombia who were taking a methods course in statistics. They worked out a statistical investigation inspired by a news article on global warming. The main source of data was fragments from prospective teachers' speech while working out the statistical investigation. The information was complemented with prospective teachers' narratives in which they reflected on their experience with statistics. The results reveal that the prospective teachers' discourse offers indications to suggest that the participants were in the process of developing statistical knowledge as well as developing awareness as critical citizens. Prospective teachers used statistics tools to (1) make sense of a crisis of society while progressing in the development of their statistical knowledge and (2) reflect on the critical issue while showing signs of social awareness development.

KEYWORDS

Statistical modeling; teacher education; critical citizenship; socio-critical perspective of modeling; teacher learning

Introduction

Research in statistics education in the last decade has been prolific in terms of promoting modeling in school statistics (Doerr et al., 2017; English, 2018; Pfannkuch et al., 2018); however, modeling has been modestly addressed in teacher education (Anhalt et al., 2018; Biembengut & Hein, 2013; Cetinkaya et al., 2016). Most research in teacher education in the field of statistics has focused on teacher knowledge of the subject (Groth, 2017), but very few studies have looked at the skills teachers could potentially develop when engaged in modeling activities. Research carried out in the last decade shows that statistics courses in teacher education programs still make emphasis on techniques with few applications related to real information from the students' own field of knowledge (Campos, 2016; Campos et al., 2013). In such a structure, knowledge and the mastery of the discipline by the teacher still play a central part (Campos, 2016) but do not support the development of other skills teachers need in their teaching activity. For example, the critical use of textbooks whose tasks are intended, for the most part, to use a given model and to perform certain operations but not to develop their own models (Anhalt et al., 2018).

Results from research have suggested that prospective teachers' difficulties with open-ended tasks that require modeling might be due to the limited exposure to modeling processes in their teacher education programs (Ng, 2013). In the same line of thought, Doerr (2007) has suggested that prospective teachers' participation in modeling activities assists them in understanding the nonlinear nature of the process and in forming the skills and thinking to develop models.

Prospective teachers should have opportunities to develop their modeling skills and to reflect on their teaching actions while they are students in statistical courses. Regardless of where those

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opportunities arise, it is essential that prospective teachers participate in experiences in which they can learn and develop their knowledge (Lovett & Lee, 2017). We believe that one of those opportunities could be to articulate statistical modeling in teacher education.

As Anhalt et al. (2018) have suggested, the implementation of modeling activities in teacher education could potentially promote conceptual understanding in prospective teachers as well as encourage appreciation of students' learning in their future practice. Furthermore, some research in the field of statistical education (Biehler et al., 2018a; Engel, 2017) has recognized that schools should educate students to be active citizens who can use and interpret statistics about society. According to Engel (2017) "statistics students need to develop the capacity to make sense of the staggering amount of information collected in our increasingly data-centered world" (p. 48). To achieve this, prospective teachers should be educated in the same way during their teacher education stage (Biehler et al., 2018a).

Statistical modeling contexts need to be motivating and meaningful. One option to provide meaningful contexts in teacher education is to combine the modeling approach with the study of statistics in society. Authors like Biehler et al. (2018b) and Engel (2019) call that approach *civic statistics*, which focuses on understanding statistical information on issues of social relevance for society that involve subjects such as economy, migration, health, wealth or environment, among others (Engel, 2017). Statistical modeling activities focused on the empirical study of these topics could be opportunities to promote both prospective teachers' learning of statistics and the development of social awareness.

In this research, we explore how statistical modeling in teacher education programs can act as a vehicle to promote prospective teachers' learning and knowledge development as well as social awareness. This study is justified by several aspects: (1) teachers do not only need to learn the statistical knowledge but need to explore different ways in which such knowledge could be accessible to the students they teach; (2) research has shown that modeling has been fruitful in school statistics to offer students possibilities to learn statistics in holistic ways (Campos, 2016; Doerr et al., 2017). This means offering possibilities to learn statistics holistically by experiencing the applicability of statistical content while developing the ability to investigate, discuss, reflect, criticize and communicate opinions.

We believe that statistical modeling in teacher education could also be successful in developing holistic statistical knowledge as well as promoting the development of critical citizenship. The goal of this study is to investigate prospective teachers' learning in the statistical modeling process. To be coherent with this goal, the research question we attempt to answer is: How do prospective teachers engage with the statistical modeling process as they work on tasks designed from a socio-critical perspective?

Theoretical framework

Statistical modeling

Research in teacher education largely points to limited opportunities to experience the activity of modeling (Anhalt et al., 2018; Cetinkaya et al., 2016; Ng, 2013). Despite the growing interest of including modeling in teacher education (Biembengut & Hein, 2013), there are still few programs that integrate it as a tool to strengthen teachers' knowledge (Anhalt et al., 2018; Biehler et al., 2018a, 2018b; Huberty et al., 2018). Some researchers (Campos et al., 2013) recognize that providing opportunities to experience the modeling activity in teacher education programs can motivate prospective statistics teachers to immerse themselves in research environments of different social issues of interest. As Campos et al. (2015) propose, through modeling it is possible to:

> create motivation, facilitate learning, give meaning to the content worked, value the applicability of concepts and develop students' critical spirit. This way, we stimulate students to transform their reality whilst we promote the understanding of the socio-political role of statistics. (p. 504)

By considering these limitations in previous research, we position this study in a socio-critical perspective of modeling (in the sense understood by Kaiser & Sriraman, 2006). Such a perspective takes into account the construction and use of statistical models to solve problems that have their origin in situations of reality. However, the modeling process goes beyond the instrumental role of the model of purpose, process and prediction (Makar & Allmond, 2018). Statistical models are seen as a set of tools ranging from time series to simpler representations such as statistical graphs and tables, which allow reasoning and thinking about big statistical ideas like variability, uncertainty, centrality and inference, when solving situations of reality (Campos, 2016).

That is to say, the principles of randomness and uncertainty, which lead statistics to move away from deterministic aspects, are in correspondence with the critique of a false-true ideology (certainty ideology). Getting away from such an ideology can be facilitated by the development of a critical awareness about the role of statistics in the political and social contexts in which the subjects are immersed. In this respect, the modeling activity promotes the critical participation of students in society by discussing political, economic and environmental issues in which the model functions as a technological tool (Araújo, 2009, 2012). Through the statistical modeling activity, students identify other ways of seeing the world in which they live, while expanding their spectrum of possibilities for action and interaction in society (Silva & Kato, 2012).

The development of critical awareness is the result of the individuals' formation as critical citizens. Critical citizenship is related to the active participation of people in their communities and governments, where they interrogate the structures that produce conditions of injustice, and work to change them (Weiland, 2019). Authors like Skovsmose (1999) have referred to those conditions of injustice —racism, sexism, elitism— and exploitation of nature as the crises of society. Subjects as critical citizens can use statistics to influence, shape and transform socially constructed structures and discourses around them (Weiland, 2017). Contributions such as those of Zapata-Cardona and Marrugo-Escobar (2019) have emphasized that statistics should be a tool to help citizens understand and transform their world beyond the domain of concepts and procedures. With this in mind, the development of critical citizenship is the ultimate goal of statistical modeling from a socio-critical perspective.

In a socio-critical perspective of modeling, the learners carry out all the actions related to the construction of the model (production and analysis of data, use of statistical tools and statistical reasoning to make conclusions based on data) and use the findings provided by the model to transform their surroundings (Zapata-Cardona, 2018). It means that the model becomes a tool to deepen the statistical knowledge of the learner but at the same time allows the development of awareness as a critical citizen (Zapata-Cardona & Marrugo-Escobar, 2019). This perspective requires the contexts in which the modeling process is carried out to be real (Stillman et al., 2013). Artificial or fictitious contexts would detract from the purpose of modeling (Barbosa, 2006) for the development of social consciousness. Therefore, the contexts have the role of generating a connection with the "world outside" (Zapata-Cardona, 2018).

With the advent of data science and new forms of data and visualizations, it became clear that new opportunities arose to rethink and study the nature and role of statistical modeling. According to Huberty et al. (2018), "statistical modeling processes involve connecting data, chance and context" (p. 1113) and such a process is closely related to a research cycle followed in the empirical enquiry (Wild & Pfannkuch, 1999). "The cycle incorporates the posing of investigative questions about real-world situations, and transforming those questions into relevant data that are collected and analyzed using a myriad of representations in order to guide inferences about the real-world situation" (Pfannkuch et al., 2018, p. 1116).

Since data are inherently variable and they are collected from a random sample, the researcher structures and represents the data to show the present variability. Therefore, dealing with variation is considered a critical goal of the modeling process. In other words, variation, modeling and context are ubiquitous components throughout the statistical research cycle (Pfannkuch et al., 2018). Whether students receive data or collect data themselves, as soon as they begin to structure and represent data,

they are modeling, dealing with variation and thinking in context. Complementary, authors such as Konold et al. (2017) recognize the modeling process as a key aspect not only for the production of statistical summaries or for adjusting curves to data, but as a key element of each component of the "research cycle" (in the terms proposed by Wild & Pfannkuch, 1999).

Teacher learning and knowledge

There is no doubt that a strong preparation of the statistics teacher leads to strong instruction (Batanero, 2009; Frischemeier & Biehler, 2018), however, teacher education is not comparable to teaching students. Statistics teachers, in their education process, have a twofold challenge; they need to learn statistics and learn how to help others to make learning happen (Groth, 2017). For this reason, teacher education programs need to provide tools for teachers to develop strong foundations for the subject matter as well as opportunities to reflect on skills needed later on for teaching (Leavy & Hourigan, 2016). We hypothesize that these two requirements are satisfied by introducing statistical modeling in teacher education by means of *statistical investigations*.

Teacher learning and knowledge developed in a statistical modeling process is an investigative type of knowledge considering that it has been inspired by the framework for statistical problem solving given in the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report* (Franklin et al., 2007) for preK-12 grade levels. That framework —which shares considerable similarities with the "research cycle" used in the empirical enquiry (Wild & Pfannkuch, 1999)— contemplates four components in statistical investigations (a) formulating questions, (b) collecting data, (c) analyzing data, and (d) interpreting results. In that respect, such an investigative type of knowledge involves certain abilities similar to those essential for statistics problem solving.

According to Burgess (2011), teachers need to develop a deep understanding of several statistical concepts as well as cultivate ways of encouraging greater conceptual understanding of statistics in their students. To fulfil this goal, they need multiple experiences with understanding the investigative process itself, experiences that will later help them develop understanding of students' statistical thinking and understanding. Part of the statistics teacher activity is to bring knowledge and conceptual understanding into play in order to judge, from a statistical point of view, whether students' responses in relation to variation in data are reasonable. Anhalt et al. (2018) stating that the modeling activity in teacher education contributes to the development of knowledge required to teach offered a complementary argument. A similar argument is offered by Biehler et al. (2018b) who recognize that statistical modeling can offer activities for teachers to develop an understanding of key statistical concepts, to improve their ability to explore and learn from data, and to learn to build evidence-based statistics arguments.

In the field of teacher education (Kelly, 2006), as well as in statistics education (Utsumi et al., 2016), the development of teacher knowledge can take place through reflective, discursive and collaborative experiences. In this research, we combine these theoretical frameworks to make sense of the development of teacher knowledge and teacher learning in the statistical modeling activity.

Conception of learning

Teachers' learning of statistics and in general teachers' learning discussed in this work are inspired by a social theory of learning. Such a perspective is based on the sociocultural theory of learning proposed by Vygotsky (Kozulin, 2010; Vygotsky, 1930/1978) which emphasizes that learners —in our case prospective teachers— come to know their world as a result of social interaction that offers opportunities to share knowledge and to activate reflection (Kelly, 2006; Vygotsky, 1930/1978).

Learning is a permanent social process of the constitution of consciousness, which is "a subjective reflection and proper positioning on the external world. Consciousness is the subjective emotional, affective process, through which each of us as an individual reflects on the world and is oriented in it"[1] (Radford, 2017, p. 21). As depicted by Vygotsky, learners move beyond a certain level of

understanding through the help of more knowledgeable others (Engeström & Sannino, 2010; McLaughlin et al., 2005; Vygotsky, 1930/1978) because learning is an intersubjective act (Radford, 2014). This means that in learning from a social perspective, the interaction among peers is an essential element of the process. As described by Elhammoumi (2010), Vygotsky saw human individuals as "the result of historical, cultural and social development rather than the starting point of our explanation" (p. 665). In this perspective, learning is defined as a process that enables every individual to reach his/her rightful potential.

> Vygotsky's approach to learning disabilities started from the potential of the disabled child (or individual) rather than from his/her disability. Human abilities can reach its potential only in a society based on *each according to his/her needs*. (Elhammoumi, 2010, p. 670)

Moving to the field of teacher education, teacher learning takes place within social, cultural and historical settings. Teacher learning is a situated activity that involves teachers engaging in the process of knowing-in-practice (Kelly, 2006). Thus, there is a crucial role assigned to the meaningful, authentic context in which the learning takes place. Learning is "the social processes through which learners face historical and culturally constituted forms of thought and action and gradually become familiar with them, in a critical way"[2] (Radford, 2018, p. 67).

This particular perspective of learning conceives prospective teachers as active participants in the learning process, assigns great importance to giving learners the opportunity to share and discuss ideas for solutions, and confers high value to meaningful contexts where statistical knowledge can emerge. This combination of interaction and meaningful context serves as a basis for reaching more general and formal levels of understanding. Using statistical modeling to support prospective teachers in the learning of statistics and its teaching fits quite well within the teacher education field. However, until now, the use of statistical modeling in teacher education is limited in the field of statistics education (Biehler et al., 2018a, 2018b; Huberty et al., 2018). The present study is an attempt to start closing such a gap.

Methodology

The goal of the present study is to investigate prospective teachers' learning in the statistical modeling process. Both processes, statistical modeling and teacher education, are objects of a qualitative nature that could be properly addressed under a qualitative research paradigm. Such a paradigm includes a flexible and a holistic process where people's ideas, feelings and experiences prevail. The emphasis of this research paradigm is on comprehending the personal ways of understanding the world and the particular realities of the participants according to their subjectivity (Denzin & Lincoln, 2012). The rich and complex process of modeling is used to generate local theories of how prospective teachers learn and acquire statistical knowledge and develop critical citizenship when engaging in modeling activities. We explore this field and generate new insights by applying qualitative analysis methods.

Setting

The study was conducted during the spring semester of 2018 at a large (45,000 students) comprehensive university in the Northwest of Colombia. The Department of Art and Science offers one section of a four hours a week methods course in statistics each semester. Enrollments per semester are approximately 15 students and a faculty member usually teaches the course. One of the goals of the course is for the prospective teachers to become fluent in planning instruction. We provided them with different ways of approaching the teaching of statistics that could turn into helpful tools in their future teaching activity.

Participants

The participants in this research were 10 prospective teachers (4 female and 6 male) pursuing a mathematics education degree in a five-year program. They participated voluntarily and did not

Table 1. Participant teachers.

Name	Gender
Alberto[3]	male
Francy	female
Gabriel	male
Jacobo	male
James	male
Laura	female
Mary	female
Melany	female
Patricio	male
Samuel	male

receive any academic stimulus or economical compensation for taking part in the study. Participants were taking a methods course in statistics, which is usually offered in their third year after a descriptive statistics course. The participants had no experience teaching statistics, as student teaching practice —practicum— starts in the fourth year of the program. Table 1 presents the list of participants of the study.

Data collection process

Prospective teachers were presented with a *statistical investigation* and asked to discuss and explore possible solutions. A *statistical investigation* is a holistic way to organize statistical instruction (Zapata-Cardona, 2016), which is inspired by the philosophical roots of mathematical critical education (D'Ambrosio, 1985; Skovsmose, 1999). A *statistical investigation* starts with a crisis of society and uses statistical tools to study it empirically, understand it and to react to it. The purpose of a statistical investigation is the development of statistical knowledge and the construction of social consciousness. Hereof, it promotes both strong statistics knowledge and the development of critical citizenship (Skovsmose, 1999). In our conception of modeling, statistical investigations exemplify the idea of statistical modeling since they involve the construction of a model to study, understand and solve problems of reality related to crises of society.

A news article published in a local newspaper about global warming (Velásquez Gómez, 2017), inspired the statistical investigation designed for this research. The news article described the increment in temperatures in the continental platform as well as in the oceans, the reduction of ice on both poles and the corals threatened. The news article also made some predictions about the meteorological phenomena of "El Niño" and "La Niña" for the second semester of 2017. Prospective teachers invested two 120-minute class sessions working out this statistical investigation. They read the news article and then the following questions were posed to orient the discussion:

(1) What opportunities does this problem offer you to teach statistics?
(2) To the question "is our city (planet) warming up?" a student gave this answer: Yes, because last year climate change was evident in the city. How would you help this student to go further in their reasoning?
(3) To the question "is our city (planet) warming up?" a student gave this answer: No, because global warming is a myth. How would you help this student to go further in their reasoning?
(4) Use statistical tools to answer the statistical question "Is our city (planet) warming up?"

To answer the last question, prospective teachers were encouraged to work in small teams (two or three students) and report their work back to the whole class in the second session of the lesson. They discussed and offered solutions to the statistical question by carrying out the statistical investigation using different paths; some used basic statistical knowledge and others refined statistical knowledge to

model the situation. Some teams used software such as Excel to facilitate the exploration and analysis of the data. The instructor was present in all the sessions to moderate small-group and whole-class discussions and to ask challenging questions. She never offered direct instruction with a twofold intention (1) to explore different ways prospective teachers modeled the situation, and (2) to avoid any possible influence on prospective teachers' decisions.

Each class session was video recorded and transcribed to capture fidelity in prospective teachers' discussions in the whole-class (data from small-group discussion is not available) to facilitate the analysis. The main source of data was fragments from prospective teachers' speech. The rationale behind this decision is based on the fact that speech constitutes consciousness (Lerman, 2001) and such speech plays an essential role in the organization of higher psychological functions (Vygotsky, 1930/1978). Those fragments of speech were integrated with prospective teachers' narratives where they described their experience and background with statistics and offered some reflections. The narrative assignment was a reflective writing exercise done outside the class time at the end of the lesson and oriented by questions and guidelines like the following: How has my experience been with statistics as a student? How has my experience been as a user of statistics? How prepared do I feel to undertake the challenges of teaching statistics? Describe a memory I remember the most as a statistics student. Narratives in teacher education function as a catalyst for teacher knowledge that links reflection, socially and historically situated contexts and scientific knowledge (Villarreal & Esteley, 2014).

Analysis process

We, the first and the second author, carefully watched the videos from the class sessions to create a descriptive sequence of the discussions that took place during the lessons and to identify episodes for further analysis that appeared to uncover prospective teachers' engagement in the modeling process. The identified episodes were transcribed verbatim —originally in Spanish that was the language of instruction— and then translated into English to facilitate the analysis. With the transcriptions, we annotated and coded the episodes independently following an inductive strategy and then discussed in light of the theoretical framework to reach a consensus. Although we did not specifically calculate an inter-rater reliability measure, we agreed on most ratings. The analysis focused on different processes: (1) evidence of the investigative type of knowledge in the modeling process, and (2) indications that suggest the development of consciousness.

Prospective teachers' discussions, together with their narratives, were studied in a constant comparative routine (Fram, 2013) to make sense of teachers' speech and track the statistical modeling process and indications of critical citizenship they developed through the statistical investigation. First, we selected some episodes that are representative of the prospective teachers' discussions that might be considered as evidence of the investigative type of knowledge that emerged during the statistical modeling process. Second, we selected some episodes that are representative of prospective teachers' discussions and that suggest indications of development of critical citizenship. Although we have transcriptions and prospective teachers' narratives from all participants, we only selected those fragments that helped us the most to illustrate the answer to our research question. To account for the engagement in the statistical modeling process by prospective teachers, the analysis of the information required to delve into the data in an inductive way following a cyclical process, since there was no attempt to verify hypotheses raised before the study (Bogdan & Biklen, 1994). An inductive form is a logical reasoning strategy based on the systematic observation of facts and phenomena from the empirical data to propose theories (Hernández Carrera, 2014).

Results and discussion

We organized the data including some characteristics from the statistical modeling process pictured in the investigative type of knowledge and some indications that suggest that prospective teachers were on the path to develop critical citizenship. The development of critical citizenship is a lifetime process, this research only shows some indications that such a process has started taking place.

Investigative type of knowledge

Collecting data

In the whole-class discussion, six out of the ten prospective teachers suggested data production from reliable sources to study the situation empirically. Data production is an essential part of the modeling process. Data collection combines accurate measurements, decisions about what data to gather, where to look to better answer the statistical question, how to gather reliable data and other related tasks. Data collection process is an important component in empirical research, since, in it the modeler has to think critically about the data quality (Wild & Pfannkuch, 1999). In this component, questions arise regarding the way in which the data is collected, if they help answer the question under study and if they contribute to the solution of the problem. In other words, properly collected data is considered a primary requirement to make reliable judgments about real situations (Pfannkuch & Wild, 2004). The following excerpts illustrate fragments from prospective teachers' speech that focus on collecting data:

Samuel-We need previous [temperature] records. (April 9, 2018, lines 211–213)

Samuel-We could investigate records from similar seasons, records from previous years, temperature records. [. . .] We need to set the atmosphere to investigate, to inquire, and based on the records make a conclusion. (April 9, 2018, lines 291– 295)

Laura-There is a need to gather data and to explore how the data could be contrasted. [. . .] (April 9, 2018, lines 361–366)

An important aspect of the data production process is the quality of the data. Some prospective teachers suggested paying attention to this characteristic in their discussion. Mary provides an example in the following excerpt:

Mary-Data need to be gathered from reliable sources like the SIATA,[4] from official organizations dedicated to the study of these kinds of issues. (April 9, 2018, lines 357–360)

Prospective teachers mentioned the importance of reliable data to run exploratory data analysis before any formal contrast. The following quote from Samuel illustrates this claim:

Samuel-Data collection should be worked out together with data exploration tools. We need to gather data, investigate records from reliable sources to explore through graphics, bar diagrams and others. (April 9, 2018, lines 398–403)

Fragments from prospective teachers' speech reveal that participants combined several skills in the data collection process. Prospective teachers recognized that the quality of the data is needed to derive reliable conclusions and inferences. A preference for reliable data and recognition of the inadequacies of personal experience and anecdotal evidence are a foundational part of statistical thinking (Wild & Pfannkuch, 1999). This has to do with the recognition that data is needed to judge a situation, and therefore, opinions may need to be reviewed in the light of the empirical evidence (Pfannkuch & Wild, 2004).

Analyzing data

Analyzing data requires a combination of statistical knowledge and contextual knowledge. It involves going back and forth between the contextual and the statistical world (Wild & Pfannkuch, 1999). Prospective teachers described different ways to analyze the data. All prospective teachers in the small teams made references to approaches they took to analyze the data. They described a variety of strategies, some focused on descriptive analysis, others suggested multiple regression tools and others suggested using probability distributions to find a p-value to carry out a hypothesis contrast. The

following two quotes from two teams illustrate some of the strategies which are representative of the discussions that took place in the sessions:

Samuel and Laura-We took a table of records of the average temperatures from past decades. In those records, we saw the increment of temperature in each decade. (April 18, 2018, lines 8–9)

Gabriel, Alberto and Francy-We gathered information on average temperatures of Medellín from 1958 to 2018. Just by looking at the tendency, intuitively we could say that the city is warming up. [. . .] In the 70′ and 80′ the temperature never went up to 24° [Celsius] but in recent years it has reached 24°. We thought about doing a mean difference test. (April 18, 2018, lines 91–158)

Samuel and Laura gathered data of city temperatures from previous years and organized them in a way that allowed them to see a trend or pattern. They focused on exploratory data analysis to make their claims but they did not go beyond this point. In contrast, Gabriel, Alberto and Francy suggested a different strategy that included a sophisticated statistical tool to compare two groups. Both teams combined contextual knowledge with statistical knowledge to make sense of the data. However, what is noteworthy from sharing aloud various approaches to deal with the statistical investigation is that prospective teachers were able to see different routes. This opens possibilities for prospective teachers to increase the complexity of their statistical knowledge and to share different directions of the modeling process. Such an "engagement in particular instances of social activities with others" (Kelly, 2006, p. 513) leads to the transformation of the individual and to teacher learning. Particularly, the potential for prospective teachers' knowledge development is increased when they have the opportunity to share and discuss ideas, and engage in collaborative problem solving, inquiries and dialogs. In any event, learning —and teachers' learning in particular— is an intersubjective process (Radford, 2014). In the same line of thought, Vygotsky stated that "the learning process can never be reduced simply to the formation of skills but embodies an intellectual order that makes it possible to transfer general principles discovered in solving one task to a variety of other tasks" (1930/1978, p. 75–76). By this, we do not imply that prospective teachers' learning and knowledge developed during the statistical modeling process will suffer a straightforward transfer to other scenarios. Rather, we insinuate that by promoting the out loud discussion of diverse strategies to carry out the same modeling activity could contribute to the development of understanding of key statistical concepts and procedures, to the improvement of their ability to explore and learn from data, and to the building of evidence-based statistics arguments. This is consistent with Anhalt et al. (2018) arguments who suggest that the understanding of the strategies and procedures that prospective teachers use in the modeling process will provide valuable insight in their future practice as teachers.

Investigative cycle

Although collecting and analyzing data are both fundamental components of the investigative cycle, in this part we highlight prospective teachers' fragments of the discussion where they considered a complete and articulated investigative cycle. While prospective teachers talked about collecting and analyzing data, those topics did not constitute an isolated discussion but were considered within a complete investigative cycle as particular characteristics of the statistical modeling process.

Some prospective teachers approached the proposed statistical investigation considering a complete investigative cycle. This approach reveals that prospective teachers thought holistically about the statistical investigation, which might suggest an investigative type of knowledge. We illustrate this statement with the following excerpts from prospective teachers' discussions:

Albert-It is basically this, gather data, compare the years and carry out an analysis of the data. (April 9, 2018, lines 297–298)

Laura -We need to establish the level of significance. We could establish a hypothesis and compare with the level of significance whether the hypothesis of global warming is rejected or not. We could determine whether there was a [temperature] increment or not, or whether it was due to randomness or not. (April 9, 2018, lines 207– 210)

By having the opportunity to get involved in an investigative cycle to empirically study an issue related to global warming, participants were able to propose ways to collect, analyze, interpret and

criticize data and arguments (Zapata-Cardona, 2016). Those components did not appear in isolation, but in a holistic way. All these elements are part of the investigative type of knowledge required in the modeling process. Such knowledge involves essential elements of the empirical inquiry process suggested by Wild and Pfannkuch (1999) —recognize the need to work with real data, reason with statistical models and integrate the statistics and context throughout the research process— that contribute to answering the statistical question under study. Although investigative type of knowledge implies a complete investigative cycle, it does not end there, and it might become a tool (or starting point) to cultivate a critical consciousness in people through the empirical study of social phenomena.

Social awareness

Prospective teachers valued the connection of the statistical investigation proposed with crises of society. Eight out of ten participants highlighted crises as links to students' experience and life and as opportunities to develop awareness. We illustrate this claim with the following excerpts from prospective teachers which represent the discussion that took place during the sessions and personal reflections:

Laura-What is important here is that this is an issue that is affecting the world. It is an issue we are living with that interests everybody. It is a real context, it is not fictitious and students could embrace it. (April 9, 2018, lines 426–435)

Alberto-We take advantage of teaching statistics but at the same time acknowledge that there is a social problem. This could generate a lasting impact on students' learning. (April 9, 2018, lines 470–482)

Samuel -These kinds of situations seek to develop awareness. (April 9, 2018, lines 501–502)

These excerpts from prospective teachers suggest that the statistical investigation allowed them to gaze upon themselves as participants in the world and not as mere observers. They saw the critical issue as a topic that could interest their future students, a potential tool for statistics teaching and a powerful way to develop students' awareness. These results are in coherence with the principles of a socio-critical perspective of modeling that considers involving socio-political aspects, constructing knowledge in dialogue with others —democratizing knowledge, valuing ethics and social justice, stimulating participants in the process of reflection and promoting social responsibility (Campos et al., 2013).

Alberto, one of the prospective teachers pointed out in his narrative a reflection that was inspired by a statistical investigation worked out at the beginning of the semester —within the same method course and the same setting of the present research— related to gender discrimination. Although the discussion from that statistical investigation was not part of the data gathered for this report, it is worth noting how that experience with the statistical investigation affected Alberto. In his narrative, he expressed how developing the statistical investigation had become a positive experience, and from there he began to become aware of the differentiated ways in which women are treated in multiple areas of society. The following excerpt illustrates this:

Alberto -A memory I have from statistics is the class where a hypothesis test with a hypergeometric distribution was carried out to determine if there was discrimination against women candidates who showed up for a job in a bank. I never imagined that something like this could be analyzed through statistics. What struck me the most is that while I was learning statistics, I simultaneously learned about the conflicts that women have to face. (Alberto's narrative, April 9, 2018, lines 37–41)

Prospective teachers' reflections revealed the potential of statistical modeling not only for teaching and learning statistical concepts, but also for the promotion of critical citizenship by raising awareness about their role in society (Silva & Kato, 2012). Scientific education of teachers should contribute to the critical understanding of the world and promote reflection on the role of science in society (Barbosa, 2006; Skovsmose, 1999).

The statistical investigation for the statistical modeling activity reported in this study was a crucial, meaningful, and authentic context where learning took place. Prospective teachers' learning was closely related to the statistical investigation, to the discussion they have in the class and to the experiences they had with the crisis under study. In that respect, we coincide with Elhammoumi's (2010) statement and anticipate that teachers' learning was a function of the activity, context, and culture. Within the theoretical foundations of a socio-critical perspective of learning, "human beings cannot be conceived of as set apart from the world and their cultures" (Radford, 2016, p. 188).

Carrying out a statistical modeling activity by means of a statistical investigation supported prospective teachers to connect scientific knowledge to a real situation. This connection among subject knowledge and real-world knowledge is a crucial element in situated learning, which plays a fundamental part in the development of critical citizenship. Such a relationship is not a trivial one since "human mental functions are formed and transformed by the real material life" (Elhammoumi, 2010, p. 664). In a Vygotskian point of view, consciousness is considered a human mental function, which is an ensemble of social relations mediated by social relations, tools, and concrete social-cultural -historical environments. Statistical modeling using statistical investigations carries with it a conception of learning anchored in culture (Zapata-Cardona, 2016) that does not focus exclusively on the acquisition of knowledge, but also takes into account the social dimension of learners as beings (Radford, 2014).

Conclusions and implications

Our special interest in this study was to investigate prospective teachers' learning and critical citizenship in the process of statistical modeling by means of statistical investigations. The results described in the previous section revealed that statistical modeling has the power to elicit prospective teachers' learning as well as social awareness.

The results of this study show that statistical modeling in teacher education seems to contribute to the development of prospective teachers' learning as well as the development of their social awareness, which is at the heart of critical citizenship. One of the practical implications of this result for teacher education is to consider statistical modeling in methods courses in statistics for prospective teachers as a possibility for strengthening teachers' learning and improving the comprehension of their socio-political conditions. Although the brief experience presented here showed some indications of teachers' learning and some hints of social awareness development, it suggests the need to explore multiple experiences of this kind in teacher education.

A point that needs attention in this kind of research is the statistical knowledge concerning the knowledge about the world promoted by the statistical investigations. Scholars in the field of mathematics education (e.g., Mistele & Jacobsen, 2016) have shown the tensions teachers have balancing these two components: subject knowledge and knowledge about social issues. Sometimes the real-world situation is not able to illuminate the statistical knowledge and sometimes the statistical knowledge is just ignored to reflect on the situation. This requires skills from the teacher educator to create and develop the proper balance; however, it constitutes a real professional challenge in teacher education.

Without losing the focus on teachers' learning, adopting a socio-critical perspective of statistical modeling could enrich teacher education research and practice. Statistical modeling might allow prospective teachers to get involved in learning environments to understand the social and political reality in which they live and help them to connect scientific knowledge to the world "outside". In essence, the intention is to build learning environments based on the active participation of learners in a democratic society. For teachers to be educated as critical citizens, it is not enough to master statistical skills and knowledge; it is necessary to use and learn those skills and that knowledge in order to understand and discuss social issues that are relevant to their communities.

Future research could focus on investigating the contribution of statistical modeling to strengthening teachers' learning in practicing teachers, since, as clarified in the methodology; the participants of

this study had not experienced teaching statistics. It may be informative to study how practicing teachers put their statistical knowledge into action in their teaching activity through a socio-critical perspective of statistical modeling.

A limitation of the study is that the researchers only witnessed the prospective teachers' work on the global warming scenario that was done during class periods, not the work done in small teams or the work done by them outside of class. The study relied on the prospective teachers' whole-class discussions and narratives. A more comprehensive data collection plan in which the researchers are able to track small team discussions during all of the modeling activity would produce deeper insights into the prospective teachers' learning process of the modeling cycle. Future studies that consider richer data in their designs will be helpful to provide further insights into effective ways of promoting prospective teachers' learning processes and the development of social awareness.

Notes

1. Authors' translation.
2. Authors' translation.
3. Names in this article are pseudonyms to guarantee the anonymity of participants according to Colombian research law.
4. SIATA – Sistema de Alerta Temprana de Medellín y el Valle de Aburrá. Early Warning System of Medellín and the Aburrá Valley. It is the flagship project for risk management in the region.

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