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This paper presents the research results whose aim is to understand the mathematics teacher's argumentation during classroom tasks discussion. Aspects of a theoretical stance are used, on the one hand, to show how the teacher's argumentation can be described in the mathematics classroom and, on the other hand, to specify aspects of the discourse in the mathematics classroom. The analysis is illustrated with one episode in a tenth-grade classroom, where the teacher and her students discuss the solution of a task about trigonometric ratios. The results allow recognizing features of the teacher's argumentation features, purposes, and conditions that activate it.

Keywords: Argumentation in mathematics classroom, discourse in mathematics classroom, learning opportunities, professional knowing.

Introduction

The argumentation of the mathematics teacher during teaching requires specifying aspects about the participation and discussion in mathematics class. On the one hand, it can be observed how research on and with mathematics teachers has grown in the last two decades, which in addition to being multifaceted, has a broad scope for teaching and learning mathematics (Chapman, 2016). On the other hand, interest in discussion and participation in mathematics class, particularly in the discussion of tasks, is associated with the way in which learning is conceived, this is from a participatory perspective (Krummheuer, 2011), where learning is conceptualized as participation in classroom discourse. The interest here is focused on the teacher, in what some authors have called the Teacher's mathematical discourse (e.g., Planas et al., 2016), that is, consider the teacher's communication of mathematical content in his interaction with his students. In addition to investigating the learning opportunities that these pragmatic considerations can promote; it is interesting to recognize links between the teacher's argumentation and student participation.

Given this concern and considering that: (1) argumentation can be used to deepen the decisions and practices of teachers (Metaxas et al., 2016); (2) there are elements or qualities in the communication between teacher and students that are important for learning mathematics (Drageset, 2014); (3) to describe the discourse in the classroom, detailed frameworks with categories and concepts are needed to describe individual turns (Drageset, 2015), and (4) argumentation in the educational field can be conceived as a social space and discursive (Ayalon & Hershkowitz, 2018). This paper proposes to answer the question: how is the argumentation of the mathematics teacher during class task discussion? The data analysis is illustrated through one episode of a mathematics class lesson in a tenth-grade teacher (15-16-year-old students) in the city of Medellín (Colombia).

Theoretical background

This section comprises two sections; the first one presents considerations on argumentation and the second one presents concerns on class discourse.

Considerations on argumentation

Like different authors, the definition of argumentation presented by van Eemeren and his colleagues (2014, p. 7) is taken up:

> Argumentation is a communicative and interactional act complex aimed at resolving a difference of opinion with the addressee by putting forward a constellation of propositions the arguer can be held accountable for to make the standpoint at issue acceptable to a rational judge who judges reasonably

This consideration regarding argumentation seeks to attend to complex interactions in mathematics classroom, where the teacher and his students argue during the development of a lesson regarding a certain task. It also implies that the object of this research, the mathematics teacher's argumentation, tries to isolate itself from the classical position, in which argumentation is assumed as a set of premises and conclusions formulated with the help of formal symbols, to assume a position closer to language and communication. Considering the argumentation under this theoretical assumption, consists of seeing the argumentation as a type of activity with purpose or intention, so that the activity is recognized as a process whose representation is the use of language and, therefore, the structure of the constellation. of propositions must be analyzed as speech acts that are part of the resolution of differences of opinion.

The difference of opinion does not necessarily take the form of a disagreement, dispute, or conflict, but there is one party that has a position and another that doubts whether to accept that position (van Eemeren et al., 2014). In mathematics class, it is possible that there are doubts regarding a statement, indication or explanation of the teacher, doubts about an answer or procedure different from the one presented by the teacher, or different answers to a task in the students' work, where the teacher's argumentation is required.

This position regarding argumentation also requires considering the process and the product of argumentation. In this document, the process is analyzed based on three dimensions, the first is the communicative dimension, which refers to what the teacher says and why he says it, that is, statements, questions and purposes; the second is the interactional dimension, which refers to the place where he says it and to whom he says it; and since the argumentation takes place in an educational context, the epistemic dimension is considered, which refers to how he says it and why he says it. The product includes each of the episodes selected for the analysis, which begins with an argumentative intervention, in which the difference of opinion on the part of the teacher or a student is made explicit and ends with the closing. In the episode, the professor seeks to convince his students from his point of view, for which he draws on his knowledge and professional experience.

Given the purpose and object of research of this study, an adjustment is made to the conditions that should occur in the mathematics class for the development of the argumentation proposed by Solar and Deulofeu (2016). In this way, the following conditions are recognized: (i) Communicative and

interactive strategies, (ii) Focus of the lesson, (iii) Task focus, and (iv) Professional knowledge. The indicators for each dimension emerge from the respective analysis.

Considerations on discourse

Given that the argumentation is expressed mainly orally and by a group of participants (Knipping & Reid, 2015), it is pertinent to consider some class discourse elements. Discourse and its terms are frequently used in studies in Mathematics Education (Lim et al., 2020). However, the term discourse is usually related to different approaches and traditions, which implies that no single interpretation is used (Ryve, 2011). However, as other researchers (e.g., Moschkovich, 2003) and given the perspective in which this work is inscribed, we adopted the notion of discourse presented by Gee (2008, p.161):

> A Discourse is a socially accepted association among ways of using language and other symbolic expressions, of thinking, feeling, believing, valuing, and acting, as well as using various tools, technologies, or props that can be used to identify oneself as a member of a socially meaningful group or "social network," to signal (that one is playing) a socially meaningful "role," or to signal that one is filling a social niche in a distinctively recognizable fashion

Discourse is considered something more than speaking or writing (Moschkovich, 2003), considered the language in use since it can be interpreted differently depending on the context. Furthermore, discourse refers to multiple processes through which people communicate (Planas et al., 2018), which implies considering it as a means and objective (Gee, 2008). For example, a discursive means of the teacher in class around the discussion and resolution of tasks and a goal is teaching-learning objects (Planas et al., 2018). This position is consistent with the purpose of the research since it is interesting to know what the teacher says and how he says it, the identity he takes when he says it, and the acts accompanying it.

In this paper, mathematical discourse is understood as the interventions by the whole class or in small groups, where the teacher and his students discuss mathematical tasks that take into consideration the understanding of concepts, operations, procedures, and their interrelations (Walshaw & Anthony, 2008). The mathematical discourse also includes "not only ways of speaking, acting, interacting, thinking, believing, reading, writing, but also mathematical values, beliefs and points of view" (Moschkovich, 2003, p. 326).

Likewise, this study explores the teacher's mathematical discourse, which is considered an essential component of educational mathematics practice. It is understood as the selection, sequencing, explanation, adaptation, and *argumentation* of multiple situations. The teacher communicates with his students during the solution of a task in class to raise generality mathematics [italics added] (Planas et al., 2018).

It is not possible to characterize discourse as a series of individual actions, but rather as a social practice, where each intervention is related to previous interventions (Drageset, 2014). Therefore, the typification of teacher reactions to student intervention proposed by Ruthven and Hofmann (2016) is used, which includes: Approve, Disapprove, Repeat, Restate, Translate, Redirect, Probe, Expand, Revert and Devolve.

Method

This research corresponds to a study with a qualitative interpretive approach, where observation is used as a tool for data collection. It is intended to explore and describe environments and situations in mathematics class and produce in-depth interpretations to analyze the individual and collective actions of the mathematics teacher.

This article reports on Emma's class (pseudonym), a tenth-grade class with 32 students (pseudonyms used) whose ages range from 15 to 17 years (female group). In preparing her lessons, Emma follows the curricular plan designed by the educational institution, which is consistent with the statements of the Colombian National Curriculum Standards. The data correspond to six lessons guided by Emma. The episode presented in this article corresponds to one of these lessons, where Emma and her students discuss the following task: *Finding the value of trigonometric ratios of notable angles*. The task was developed during two lessons. In the first lesson, Emma, together with her students, finds the value of the sine, cosine, and tangent. In the second lesson, there is an autonomous work by the students accompanied by Emma interventions. The episode that is presented take place during the second lesson. Each episode begins with an argumentative intervention and ends with the closure. The argumentative intervention is preceded by turns that contextualize it. Not always, there is an argumentative intervention and a closure. More than one argumentative intervention or more than one closing may occur.

For data analysis, discourse analysis techniques are used in two actions: fragment and connect (Boukafri & Planas, 2018). Fragment to obtain more manageable units and connect to discuss data and results that have been treated separately. According to Boukafri and Planas (2018), the reiterations of fragmenting and connecting lead to three units of analysis: turn, episode, and lesson. At first, the analysis episodes in each of the lessons are identified, for them a tracking is made in each of the turns, both teacher and students, of the argumentative interventions, and of the respective closings, which indicates the beginning and the end of the argumentation. Then, in the unit turn, the reactions to the students' interventions are identified in response to the teachers turn, using the framework of Ruthven and Hofmann (2016). Responses are taken either when the teacher's turn starts the episode or when a student starts it. The turns are analyzed in the three dimensions: communicative, interactional, and epistemic. The actions of the communicative dimension relate to the framework of Ruthven and Hofmann (2016). Regarding the episode unit, argumentative interventions and closings are retaken, to identify purposes of the teacher's argumentation. And in a third moment, in the lesson unit, the adaptation to the proposal of Solar and Deulofeu (2016) is used to identify the conditions that activated the argumentation in the different episodes within a specific lesson.

Data analysis and findings

The analyzes are exemplified from an episode, where Emma is explaining the procedure that allows us to calculate the value of tan30 °. Together with her students they have reached the expression 1 / √3, the students seem to realize that they must rationalize, to which Emma asks them why they do this, marking the beginning of the episode. Given the interventions of the students, it seems that there is a certain level of understanding of the procedure to follow. Then, however, there is an intervention by Sofia and Mia, which reveals difficulties and requires the teacher's attention.

253	Emma:	[...] At what point do we rationalize? Why do we have to rationalize?
254	Students:	Because there is a root below.
255	Emma:	Because the root can’t be left below and what is that called below?
256	Students:	Denominator.
257	Emma:	Denominator. Well, then it would be a tangent of 30 ° ... here it would be equal to 1 per root of 3, how much does that give me?
258	Students:	Root of 3.
259	Emma:	Root of 3 over the root of 3 by root of 3.
260	Sofia:	3 root of 3.
261	Mia:	Root of 6, right?
262	Emma:	Well, let's look at what did we say, last class? Turn back in the notebook.
263	Alice:	Root 3 square.
264	Emma:	Root of 3 square, root of 3 by root of 3 gives me root of 3 squared and what happens here?
265	Students:	They are canceled.
266	Emma:	What is canceled?
267	Bianca:	The exponent in the root.
268	Emma:	The exponent in the root? ... The exponent with the root. I have a tangent of 30 ° is equal to the root of 3 over 3

In this episode there are two situations that deserve attention within the teacher’s argumentation. The first one, the anticipation of a difference of opinion through Emma’s questions in [253], also considered as the first argumentative intervention. The responses of the students in the interventions [254, 256] allow Emma to identify a certain level of appropriation in said procedure, her intervention in [225, 257] consists of supporting the justification of the students and therefore presenting a partial closing. And the second situation, based on the interventions [260, 261] of Sofia and Mia respectively, show a difference of opinion and therefore correspond to the second argumentative intervention, since in addition to presenting different answers, they warn errors, before which Emma does not explicitly declare the error, but directs the justification for the students to realize it and convince themselves of the expected response, presenting the closure in [268].

Regarding the communicative dimension, it is identified as a question from Emma, with which she seeks to probe the appropriation of what it means and implies rationalizing an expression, marks the beginning of the episode. This question is preceded by interventions from the students, before which Emma raises statements with which she approves, translates, restates, or reverts. Even though in a previous episode the notation of irrational expressions seemed to have become clear, a certain procedure takes place in turn [257] before which the students express in [258] $\sqrt{3}$ as an answer. Emma intervenes in [259] to restate it, to which Sofia raises $3\sqrt{3}$ as an answer, and Mia $\sqrt{6}$ as an answer. In the following interventions, Emma uses questions and assertions, with which she takes up procedures that have already been discussed. Also, in the intervention [255] distinguishes another type of reaction: request, which, given the purpose of this research, makes it necessary to continue expanding the table proposed by Ruthven and Hofmann (2016).

In the interactional dimension of this episode, participation, media, and class norms, convincing and discussing, stand out as characteristics. The intervention [262] seems to be interesting, in which Emma refers to previous lessons by inviting the students to review her notebook. In addition to involving them in the answer to a question, she attends to a question of the students in handling with roots, using an indication to be followed by all, because she knows that it is a question that had been discussed.

On the other hand, they are characteristics of the epistemic dimension: treatment of the mathematical object when requesting clarity [255], taking up other lessons to verify the use of a certain procedure [262], error handling [262] and, procedures and answers to verify [253, 255, 257, 264, 266, 268] and validate a given answer [268]. Emma's actions described in this dimension allow us to suggest how her experience in this degree of schooling allows her to justify to the students the use of a certain procedure, insist on when and why it should be done and anticipate possible difficulties with the treatment of the same.

Before describing the purposes of the professor's argumentation in this episode, the particularity of it is highlighted, in it an argumentative intervention is identified [257] during an explanation process, which alludes to the fact that within the professor's mathematical discourse also of explanations there is also argumentation, and it reaffirms the consideration of explanation as a different process from argumentation. Emma seeks to justify the procedure for solving the task, to achieve this, she poses a question [253], considered here as an auxiliary argumentative intervention, which is preceded by interventions by the students, in which answers to the question [255, 257] and therefore a first closure. However, the episode does not end there, it is only until the intervention [268] that the closing of the episode can be recognized, when Emma states the answer to the task. In this way, the purposes in this episode are highlighted: to clarify the properties of the mathematical object, the root, involved in the solution procedure of the task [253, 255, 257, 264, 266], to clarify the solution procedure of the task [255, 257, 262, 264, 266, 268], and dealing with different points of view that do not match the expected response of task [262].

In relation to the conditions that triggered the argumentation, the following are identified: (1) The communicative and interactive strategies, the questions associated with the task solution procedure draw attention, in which Emma seeks to retake procedures, so much so that they were treated in the same lesson or in previous lessons [257, 262, 264], which seems to be related to the statement of the task "Finding the value of the trigonometric ratios". It seems that the students still have difficulties in handling procedures: rationalization, root management, and fractional operations, necessary to respond to the task. (2) The approach to the lesson, the argumentative intervention refers to understanding, that is, Emma observes the work of her students and begins the argument by asking the reason for a procedure [253]. And (3) professional knowledge, Emma seeks to link the work done in previous lessons [262], since she seems to be aware that, to respond to this type of task, the students should be able to handle different concepts and procedures. In addition, it is repetitive the action of inviting students to name mathematical objects in an appropriate way [255, 266].

Conclusions

We can affirm that the argumentation of the mathematics teacher constitutes a complex formed by three articulated dimensions: communicative, interactional, and epistemic, whose objective is to educate students in mathematics. The primary intention of the teacher is for students to understand mathematical objects, and for this she puts into play resources that are in these dimensions. The use of the teacher's reaction typology for the identification of own actions in each characteristic stands out as a success, and the contribution of this research by expanding said typology is worthy of note. It is important to point out how communicative actions allow observing the participatory perspective

of learning, to which the research alludes, where not only the teacher's intervention is recognizable, but also that of the students.

We recognized how Emma links her students in answering questions or situations in a class lesson, how she raises justifications to convince students of a certain answer to a task or question, and how they use students' concerns to open the space for discussion and participation, and how she seems to be interested in the students not only correcting an answer but also being participants and aware of the errors when carrying out a procedure for solving a task. The link between the actions of the epistemic dimension with professional knowledge is also evident, since Emma's actions indicate her experience, which is, it can be corroborated in how she raises justifications for certain procedures, insisting on when, how and why they should be made.

The purposes of the argumentation warn how Emma in addition to presenting the solution of a certain task, she is interested in having her students participate in the class lesson. It was useful to recognize the argumentative interventions and the closings in each episode, which in addition to delimiting said episodes, allowed us to recognize situations in math class lessons where the teacher argues. The conditions that activate the argumentation are recognized both in the interventions of the students and of Emma, they account for specific moments of a class lesson where the teacher should be attentive and prepared to face them.

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