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ORIGINAL PAPER

Contributions of Digital Videos in Mathematical Modelling Practices: Semiotic Meanings and Resources

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Abstract

The use of videos has gained prominence in the research about mathematics teaching and learning and stands as a challenge within the mathematical modelling community. However, further investigation is needed to explore the contributions of videos to mathematical modelling practices, including their roles in shaping contexts, resources, problems, and tasks. We investigated how video-responses can contribute to mathematical modelling practices. The research was developed in an online course with 20 in-service teachers who worked in pairs to solve video-problems and produced a new video showing their solution (referred to as video-responses). The research data were collected during video problem-solving and included interviews and the videos produced by the participants. In this paper, we used the construct Humans-with-Media along with Social Semiotics to investigate the mathematical modelling practices of four pairs of teachers. The data analysis, conducted using thematic analysis, reveals that video —with their multimodality and rich semiotic resources—offer meaning-making in modelling practices. This study also identifies simultaneity, contrast, sequencing, replication, complementarity, and juxtaposition as key features that combine semiotic resources in videos, contributing to the meaning-making process in modelling practices. Additionally, meanings of other aspects of modelling were uncovered and are reported in this study.

Keywords Video-responses · Digital videos · Semiotic · Multimodality

1 Introduction

The use of videos has gained prominence in the research in mathematics education (Lobato et al., 2023; Engelbrecht & Borba, 2024; Ng et al., 2015). Studies on the contributions of videos to mathematics learning enrich the body of mathematics education research in general, and mathematical modelling in particular (Cevikbas et al., 2023; Greefrath & Vos, 2021).

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In the last decades, the use of videos has diversified due to the development of the fourth phase of digital technologies (Engelbrecht & Borba, 2024), which is characterized by the widespread accessibility and robustness of the Internet and devices for capturing, storing and editing images and videos. In mathematical modelling, different types and uses of videos are recognized, both for presenting tasks or problems (e.g., video-based modelling tasks/problems, Greefrath & Vos, 2021) and for documenting the processes students engage in when solving a problem (e.g., teacher/student-created videos, Sánchez-Cardona et al., 2021).

Typically, video usage is examined within a wider context of digital tools and technologies for student and teacher learning (Engelbrecht & Borba, 2024; Greefrath & Vos, 2021). Recent studies have argued the need to investigate the contributions of videos to formative assessment of teachers' modelling knowledge, the development of modelling projects (Sánchez-Cardona et al., 2021) and tasks that promote mathematics learning, and the development of mathematical modelling competencies (Aytemiz & Çakmak Gürel, 2023; Canedo Junior & Borba, 2023; Wirth & Greefrath, 2024). In their literature review on mathematical modelling, Cevikbas et al. (2023) found that videos, along with other digital resources, may support students and teachers making predictions about mathematical relationships or technological constructions.

Student-created videos have demonstrated significant contributions in domains beyond Mathematics Education. For instance, these videos have been shown to enhance learning, engagement, participation, and the development of critical thinking skills in Online Education and STEM Education (Atkinson et al., 2024; Stanley & Zhang, 2018). However, their contributions to non-algorithmic areas of Mathematics Education, such as mathematical modelling, remain largely unknown. Therefore, this study aims to explore the contributions of videos created by modelers from a social semiotic perspective. The findings are expected to enhance the understanding of videos' educational potential and expand perspectives on their role in social semiotics and the development of mathematical modelling practices.

2 Review of Literature on Modelling and Digital Videos

This literature review focuses on publications addressing the use of videos in mathematical modelling education for both teachers and students. Despite the limited research in this area, we identified studies regarding two themes. First, some studies examine the design and contributions of video-based mathematical modelling tasks (Aytemiz & Çakmak Gürel, 2023; Canedo Junior & Borba, 2023; Greefrath & Vos, 2021; Wirth & Greefrath, 2024). Second, researchers have designed videos of mathematical modelling lessons for the professional development of mathematics teachers (Ng et al., 2015; Zuo et al., 2024).

Within the first theme, we found different terms for modelling tasks. Greefrath and Vos (2021) considered videos as a typology of ICT-based modelling tasks. Other terms include video-based modelling tasks (Aytemiz & Çakmak Gürel, 2023), instructional video (Wirth & Greefrath, 2024), and video-problems (Canedo Junior & Borba, 2023). Greefrath and Vos (2021) analysed three video-presented modelling tasks, noting that some offer authentic contexts, replace verbal descriptions, and assist students in understanding the task context. Wirth and Greefrath (2024) designed a framework based on heuristic work examples and instructional videos, and they used it to design and implement a video with upper-secondary students. They showed that students perceived advantages and challenges related to certain design features and the processes of self-regulated learning. Aytemiz and Çakmak Gürel (2023) discussed the impact of video-based modelling tasks on eighth-grade student modelling competencies. They compared eighth-grade secondary school students' modelling competencies when tasks were presented on a video versus a worksheet. This study showed that the video-based modelling tasks incorporated students' real-life experiences into the modelling process. Finally, Canedo Junior and Borba (2023) reported how videos shape teachers' mathematical modelling work when the problem is proposed and answered with this medium. In the video-responses, the authors found an underutilization of the audio-visual resources, especially evident in their reproduction of the modes that characterise the blackboard and the chalk.

In the second theme, researchers explored the potential of videos for developing teachers' professional competencies. In this regard, Ng et al. (2015) used video excerpts from a teaching experiment. These videos were used to capture discussions between researchers and teachers during the design phase, and to capture the dynamics of teacher-student and student-student interactions in the teaching experiment phase. The results confirm that videos provide authentic and reliable classroom teaching situations, serving as credible evidence to promote reflection on teachers' own practices. Additionally, videos can be powerful tools for demonstrating good practices within real classroom constraints. Zuo et al. (2024) investigated the contributions of videos to the development of pre-service mathematics teachers' professional competence in a mathematical modelling course. The videos were selected because they were considered a rich source of authentic classroom interactions. The results showed that participants' topical focus shifted towards modelling thinking and pedagogy. Additionally, they exhibited a significant increase in interpretive comments and utilized task-dimensional knowledge to teach mathematical modelling. Finally, Sánchez-Cardona et al. (2021) asked prospective teachers to create videos to develop modelling projects during a mathematical modelling course. They reported that prospective teachers-created videos are a crucial tool for fostering and showcasing teachers' knowledge of mathematical modelling.

Collectively, the studies provide insights into using videos to enhance mathematical modelling competencies among students and teachers. Common threads across this research were 1) video provides authentic (i.e., real-world) content and contexts for mathematical modelling tasks, and 2) video supports classroom interactions. Although the potential of videos in teaching mathematical modelling is widely recognised, research still lacks insight into how they generate new knowledge on modelling and its teaching. Further studies should explore video design, use, and implementation for different audiences, assess content, length, and interaction benefits and drawbacks, and identify key features for designing and evaluating educational videos.

In their review of research on mathematical modelling and technology, Molina-Toro et al. (2019) highlighted the need to investigate not only the role of technology in the different phases of mathematical modelling but also how technology shapes and reorganises modelling processes. In this regard, this study contributes to addressing these gaps by offering a semiotic perspective on how modeller-created videos, as part of technological resources, can support and reorganise mathematical modelling practices.

3 Theoretical Background

3.1 Humans-with-Media: Technologies and Mathematical Modelling

The central premise of the human-with-media (H-w-M) construct is that knowledge is produced by collectives comprising human actors interacting with various technologies and media. H-w-M is a metaphorical representation wherein humans and media intertwine so that these collectives become the unit that produces knowledge (Borba et al., 2024). In H-w-M, mathematical modelling can be understood as a dynamic, collaborative practice where H-w-M collectives, embedded in social and cultural contexts, construct, explore and use mathematics to solve, make sense of and interact with the problems, phenomena and contexts. Technology are not just tools, but active participants that reorganise mathematical thinking and modelling processes (Molina-Toro et al., 2019). Thus, in this practice, an H-w-M collective explores, interprets, and communicates worldviews. Within this understanding of modelling, it becomes evident that all knowledge is a partial and situated production, shaped by subjective decisions, contextual influences, and reorganised by the media involved in the process.

Despite not being a rigid or linear practice, there are some general stages of modelling that can be identified: problematising, investigating, mathematising and answering (Canedo Junior & Borba, 2023). These stages not only outline the sequence of actions in problem-solving but also reveal ways in which H-w-M collectives engage in the production, use, and communication of knowledge. These practices are not just about performing mathematical procedures but also involve a process of meaning-making and understanding how knowledge is generated, used and justified.

Our understanding of modelling practices includes actions and aspects to enrich modelling practices (Rendón-Mesa et al., 2023). They facilitate the connection between stages and provide tools for analysing and solving problems in a comprehensive manner. In the *problematisation stage*, the context, topic, and hypothesis guide the formulation of a problem. The *investigating stage* involves aspects such as identifying variables and gathering data to simplify and recognise patterns. *Mathematisation* is based on the construction and use of mathematical language, techniques, procedures, and argumentation. The *answering stage* is reinforced by evaluating responses, comparing results, communicating solutions and ideas, and establishing connections with other stages of the modelling practices. In a social semiotic perspective, recognising the meanings about these aspects produced by H-w-M collectives is crucial because these meanings can reflect how modellers make sense of process, mathematics, and connections to real-world contexts, and how they navigate the problem-solving process.

3.2 Social Semiotics and Digital Videos in Modelling Practices

Social semiotics seeks to understand the social dimensions of meaning, its production, interpretation and circulation, and its implications (Jewitt et al., 2016). From the perspective of social semiotics, multimodality consists of considering communicative processes in the sense of including not only transcriptions of speeches and written productions but also how semiotic resources (images, footage, animations, writing, orality, various sounds etc.) are combined throughout communication and influence the production of meanings (Bezemer & Jewitt, 2010; Jewitt & Kress, 2010). This perspective extends beyond traditional written and spoken language to a multimodal approach, where colours, gestures, sounds, and animations significantly shape meaning.

In the context of modelling practices, social semiotics provides a lens for analysing how students use a range of communicative modes to produce and share mathematical ideas. Representation and communication in modelling practices are inherently multimodal, as modellers draw on diverse resources to express complex mathematical processes. The combination of these modes is not merely additive but transformative, as each mode contributes uniquely to the overall process of meaning-making (Bezemer & Jewitt, 2010). Analysing these combinations enables researchers to understand how the production of mathematical knowledge is shaped by the interplay of semiotic resources (Borba et al., 2022).

H-w-M found an ally in social semiotics, which strengthens the conception of multimodality and the use of different communicative resources and their transformative role in the production of knowledge. Articulating H-w-M with social semiotics led to an analytical lens for understanding how videos transform the mathematics expressed in them (Borba et al., 2024). Digital videos enable the articulation of mathematical processes and ideas in ways that transcend traditional tools such as pen and paper. In this sense, videos are not merely digital replicas of problems and solutions but active participants that transform the mathematics represented within them.

Canedo Junior and Borba (2023) introduced the concepts of video-problems (VPs) and video-responses (VRs) to highlight innovative approaches to structuring tasks and responses in modelling practices. VPs and VRs contribute to the reorganisation of modelling practices by combining semiotic resources to present contexts and scenarios that make problem-solving both meaningful and necessary. The multimodal affordances of videos provide a rich platform for students to express their understanding through the combination of resources, fostering deeper engagement and the articulation of mathematical ideas.

4 Research Question

Although in recent years there has been an increasing interest in the use of videos in the education field, Lobato et al. (2023) identified a research gap in the literature about the mathematical meanings developed by students when watching or creating these educational videos. The reviewed literature indicates that videos have primarily been used to enhance students' competencies and to understand their perceptions. Except for Canedo Junior and Borba (2023), there is a noticeable gap in studies exploring the nature of mathematical knowledge and modelling using digital videos.

Researching this topic is crucial as it provides insights into how students and teachers incorporate technologies in mathematical modelling, interpret modelling activities, and use videos' various semiotic resources. The present paper tackles this issue by providing evidence and interpretations that contribute to understanding the potential of videos in mathematical modelling practices. Drawing on the tools provided by H-w-M and social semiotics, we understand meaning-making and communication as two key aspects of mathematical knowledge production. Therefore, this article seeks to answer the following research question: What are the contributions of videos in mathematical modelling practices?

5 Methodology

5.1 Context and Participants

The setting of this study was an outreach online course named "Trends in Mathematics Education: Mathematical Modelling and Digital Videos". The course was developed at the São Paulo State University in Brazil. Twenty in-service mathematics teachers participated in the course; they were admitted following a selection process based on the following criteria: (i) potential application of learning in their professional practice, (ii) the order of applicants' registration, and (iii) the geographic diversity of the participants' contexts. The second author of this article was the instructor of the course and was supported by other researchers from the Research Group on Informatics, Other Media and Mathematics Education (GPIMEM). The course comprised 60 hours of instruction (30 synchronous and 30 asynchronous). Guest researchers were invited to the course to teach three thematic units, namely (i) theoretical and technical aspects of video production, (ii) considerations about modelling in mathematics education with an emphasis on the integration with the elementary school curriculum, and (iii) production of videos considering the Digital Mathematical Performance and the mathematical surprises as proposals to integrate arts and mathematics.

5.2 Data

The course took place between May and June 2019 in weekly sessions of four hours each. In the second session, the participants watched four VPs that were constructed by the first author with the help of GPIMEM. After watching the videos, participants were instructed to work in pairs. They developed their work from one of the four videos, solved the problem through modelling, and presented the solution in a VR. Table 1 shows the distribution of the eight pairs and the VP they responded to with a VR.

Every pair was required to upload their video response to a YouTube channel and share the video link as a post on a Facebook group created for the course. The data analysed in the study included (i) interviews with each of the pairs (n=8 audio recordings) and (ii) the VR produced by each pair (n=8 VR); and (iii) the logs written individually for each of the pairs (n=8 documents).

5.3 Data Analysis

Data analysis consisted of a cyclical process that included six phases of thematic analysis, namely: familiarisation, coding, theme development, revision, theme definition, and reporting (Braun et al., 2019) and contrasted with six steps for video analysis proposed by Scucuglia (2012): viewing and describing, coding, critical events, transcription, and storyline and composing narrative. In the familiarisation phase, each researcher reviewed all the data individually to ensure the comprehensive documentation and structuration of the data set, as well as its sufficiency to answer the research question.

After the initial review, researchers gathered to discuss their impressions of the data. As a result, the researchers decided to narrow down the data set to four pairs of teachers; they are Pair 1 (Ari and Vera), Pair 2 (Cadu and Leda), Pair 5 (Ana and Silvia) and Pair 6 (Gabi and Rosa). These pairs constructed robust VRs that allowed the examination of their actions, understandings, and procedures in the mathematical modelling practices. Additionally, their videos show the diversity of ways to solve some of the video problems. These pairs were also selected for their disposition to participate in interviews.

The analysis involved two phases. In the first phase, we reviewed each video response independently and in-

Table 1 VP and participants

VP	Description	Participants (pseudonyms)
Water for Responsible Consumption	The VP addresses sustainable water consumption. Firstly, it presents an experiment that seeks to measure the flow of a residential shower. It then questions viewers about the water we consume in our daily activities.	Pair 1: Ari and Vera Pair 2: Cadu and Leda Pair 3: Teco and Dino Pair 4: Manu and Vidal
Time to Charge the Mobile	The VP discusses the cell phone battery charging process. Firstly, it shows how cell phones have become indispensable nowadays, which creates the need to keep the batteries of these devices always charged. It then provides information and questions about the duration of the battery charging process.	Pair 5: Ana and Silvia Pair 6: Gabi and Rosa
We and Our Screens	The VP discusses damage caused to vision by excessive use of devices such as computers, televisions, and cell phones. First, it raises questions, and then it provides information regarding the ideal distance between the user's eyes and the screen of such devices.	Pair 7: Drica and Tina
Studying the Kick	The VP discusses the trajectory described by a soccer ball after the kick. It shows an experiment that seeks to measure the height and horizontal displacement of the ball along the trajectory. It then presents questions about the quantities involved in this process.	Pair 8: Carla and Edu

ductively selected video fragments that were identified as critical events (Scucuglia, 2012). In this phase, there were no pre-established categories, and some provisional themes were identified. After that, we gathered to share their findings and interpretations considering the theoretical framework. Grounded in social semiotics and H-w-M, we investigated how the participants interpreted and developed modelling practices. We first coded the modes of representation, the semiotic resources, and the ways in which these were combined to address the solution to the VP. As outlined in Sect. 3.2, the combination of semiotic resources creates new opportunities for generating and communicating mathematical meanings. Specifically, we analysed the semiotic resources in VRs (language, gestures, representations, cinematography) and how they combined to convey meaning in modelling practices.

Subsequently, we divided each VR in blocks and identified segments by content description and semiotics resources (see Table 1 in the electronic supplementary material). Later, a new system of codification was built and used to code the four videos. As a result of this discussion, we divided each VR into blocks and identified segments, which were developed into contents, descriptions and the semiotic resources used. In the Table 2, an example for VR 1 is shown. In the Table 1 of the electronic supplementary material, information about each VP is presented.

In the second analysis phase, we examined interviews; data excerpts were identified to support the information built in the previous phase. In a collaborative session, thematic reports were reviewed to construct a coherent narrative related to the research question (Braun et al., 2019). The analysis focused on how semiotic resources combined into modes, how these combinations influenced communication, and what and how meanings emerged. To identify the contributions of videos to mathematical modelling practices, we established specific criteria to determine the originality of the meanings conveyed. That is, we assessed how these meanings emerge as a unique product of the combination of semiotic resources within the videos and how they differ from meanings produced through other resources. To this end, we also evaluated contextual relevance, conceptual accuracy, and the narrative on modelling presented in the video. Meanings were organized in tables (see Tables 3-6), and themes were reevaluated to avoid narrowed analysis or conceptual overlap (Braun et al., 2019), ensuring robust evidence, clear definitions, and well-defined conceptual relationships, as illustrated in Fig. 1.

6 Results

In this section, we focus on the findings of our analysis. We began by examining the contributions of the combinations of semiotic resources to modelling practices and subsequently analysed their contributions to the meanings of various aspects of these practices.

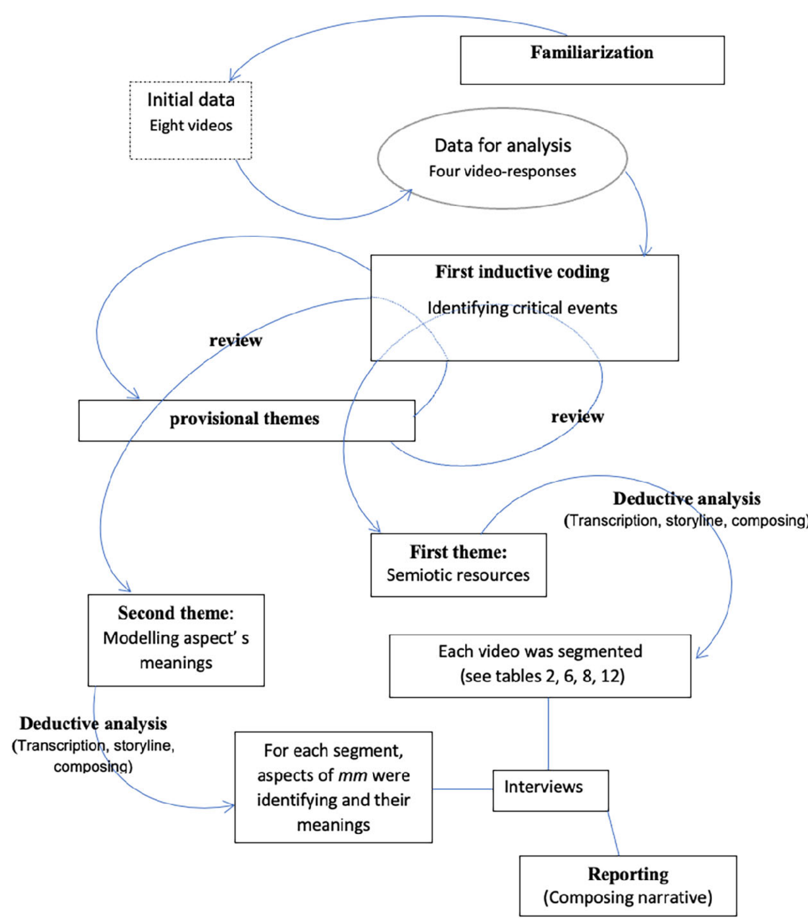
6.1 Contributions of Videos in Mathematical Modelling Practices

Throughout all modelling stages, diverse semiotic resources were used. We structured this section to highlight how semi-

Table 2 Example of table for the analysis of VR 1

Block	Segment	Description/Content	Semiotic Resources
Bl: Introduction. Content: They offer a n envl ronment to present the topic.	Seg 1: Character and setting: Ari is speaking i n front of the camera next to a source of mineral water. Duration: 36 seconds.	Ari greets and mentions the place where she is and the reasons why this place is famous. She states that they recorded the video in this place. The segment ends with transition music and images.	**Oral Language:** *Voice, words* **Body language:** *Deictic, body position, facial expression* **Cinematic language:** *Use of camera, images and text, music, water sound, and scene .*
	Seg 2: Ari addresses the camera while situated against a backdrop of a serene natural envi ronment featuri ng a small river, trees, and a bridge. Duration: 12 seconds.	Ari starts by connecting the previous segment, she says: ”So, the keyword is responsible water consumption”. Next, she asks a question ”How is this related to mathematics?”. She states the intention of responding to the challenges of the video problem and then she gives the word to Vera (another character in the video).	**Oral Language:** *Voice, words* **Body language:** *Deictic, body position, facial expression (facial, eyebrows, head movement, etc.)* **Cinematic language:** *Use of camera, images and text, music, water sound, and scene*.

Fig. 1 Data analysis method



otic resources were combined and the contributions they offer to the development of modelling practices. An overview of all stages, including the number of VRs and associated semiotic resources is in electronic supplementary material Table 2.

Problematising: At this stage, semiotic resources serve to contextualise and emphasise the topic's importance and offer an immersive experience. Videos not only present a mathematical problem but also frame it within a meaningful context for the audience.

Participants proposed new questions, hypothesised, or extended their solutions to other problems or contexts. Semiotic resources such as music, verbal language, short videos, and images (both static and moving) were combined to convey the idea that each problem was significant for all human beings (VR1, VR2, and VR4) or mathematics teachers (VR3). For instance, VR1 provided context for the issue of responsible water consumption. In the B1 of VR1, a *simultaneity* of resources such as body language, spoken language, and cinematic techniques can be observed. Here, the camera focuses on Ari in a close-up as he explains the importance of water for society; in the background, a natural setting with trees, small rivers, and the sound of flowing water is shown. While Ari speaks, he also uses body language to gesture and attract the viewers' attention, facial expressions to emphasize key points (eyes and eyebrows), and smiles to convey pleasantness. Cinematic resources were used to complement and reinforce the importance of the topic and to provide an authentic context for the modelling practice (Rendón-Mesa et al., 2023).

Simultaneity and contrast of semiotic resources offer opportunities for the audience to form a close connection with the context and the problem (Immersive experience). With this combination of semiotic resources, the aim was to draw the audience's attention to the issue of water consumption, which is a "priority worldwide, especially its consumption" [Audio-Interview, 12:44]. Similarly, VR4 used music, static image transitions, news, research data, and voice-over to engage the audience in understanding how energy consumption affects the environment and human survival. In addition to an immersive experience, videos create a persuasive effect that encourages viewers to engage with a topic (environmental conservation) and recognise mathematical modelling as a valuable process for studying complex issues.

Investigating: In the videos, we observed evidence of participants narrowing down variables and describing the processes of data collection and organisation. At this stage, the use of mathematical language was also present.

In all videos, this language was combined with cinematic resources (image transitions), oral language (voice-over), and, in some cases, body language. This combination enabled the creators to highlight the selected variables and articulate sequences of data, mathematical graphs, and tables, thereby facilitating the development of strategies to seek solutions. Within this combination of resources, we specifically identified three key characteristics: *sequencing*, *replication*, and *complementarity*.

In VR3, we observed *sequencing*, first when Silvia and Ana stated their hypotheses about the difference in load depending on the variable "type of charger" (original and non-original). Following this, they mention that they asked their students to conduct an experimental process of measurement and data organization. Subsequently, the video includes a sequence of graphs constructed by their students from the data obtained (Fig. 2). In Fig. 2, a series of scenes, images, and graphs are shown, providing a step-by-step idea and a real-life experience. The scene begins with a call to the audience (Fig. 2-a) on how to collect the data, followed by a transition to (Fig. 2-b), introducing the next images that contain all the graphs constructed from the first test (Fig. 2-c) to the last test (Fig. 2-d). Additionally, in this video, this sequence served as a "bridge" or transition between the research phase and the mathematisation phase.

Background music and moving images enhance transitions, making the problematisation more engaging. The harmony between music and transitions reinforces the idea of a process, while icons and images contextualise and enrich the problem presented in the video. In this case, videos not only contribute to the cohesion of the content but also create opportunities to make the process clearer, more engaging, and more comprehensible for the audience.

Additionally, replication was evident in videos that included scenes reinterpreting a segment of the VP in their own way. For instance, in VR2 (Fig. 3), both Cadu and Leda incorporated video clips demonstrating how they collected data on water consumption in their respective homes. These video segments were supplemented with images summarising the collected data, which were subsequently used to develop the mathematical model and address the VP question concerning consumption by the environmental ministry standards of Brazil (Fig. 3-c). This interplay between scenes and images not only supports the mathematisation stage but also situates the problem within the specific context of consumption in participants' homes. In doing so, it reinforces both the authenticity of the context and the practice of problem-solving (Rendón-Mesa et al., 2023).

Through *replication* and *complementarity*, videos contribute to the reproduction of experiences and the personification of the problem. That is, the audience can adapt and reformulate the problem according to their own contexts and interests.

Mathematising: In this stage, patterns and relationships between variables were identified and expressed using mathematical language. In the VRs, images, videos, and mathematical language were used to describe the process of solving the questions. This included video segments showing the situation, measurement methods, and variable control. Mathematical language was employed to represent models, data, calculations, and results, while a voice-over described the process. Thus, the video contributes to a multimodal representation, providing opportunities to express patterns, relationships between variables, and results in an accessible and visually comprehensible manner.

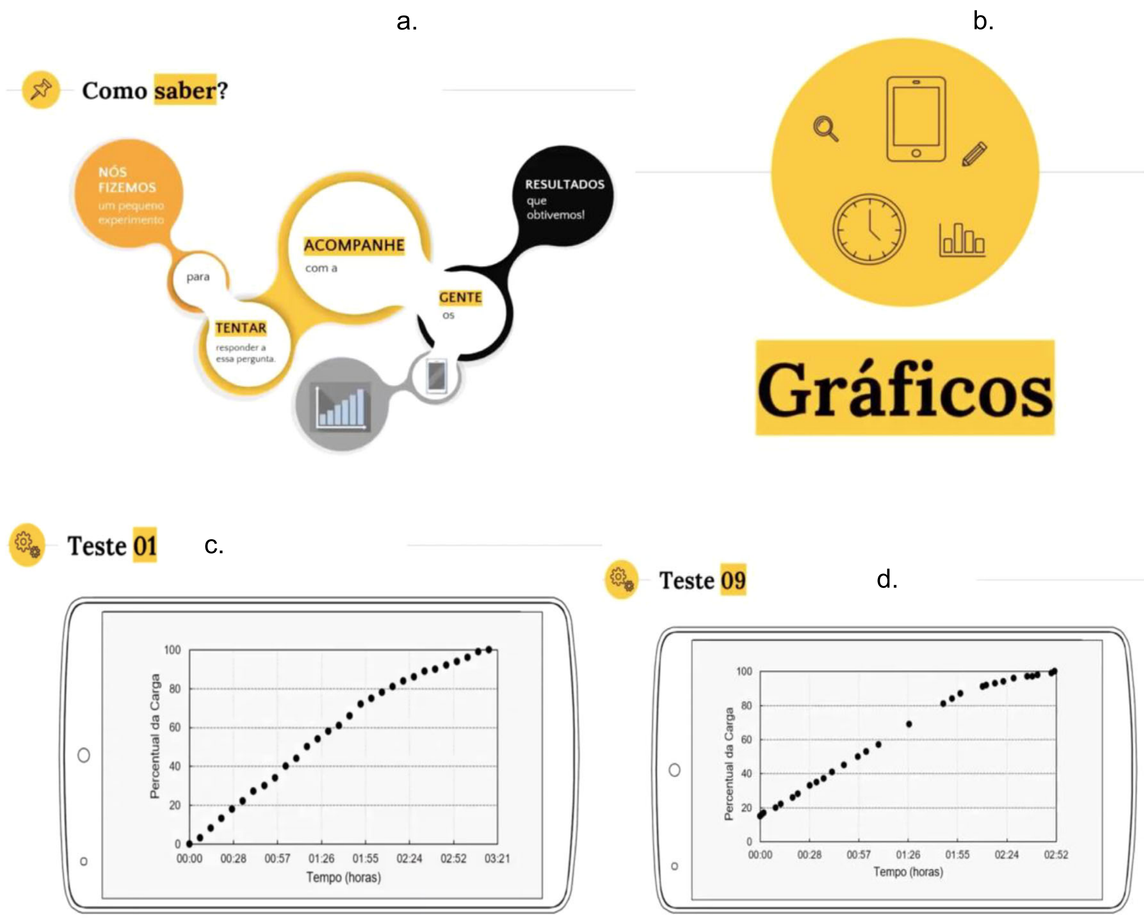


Fig. 2 Sequence of images in the mathematisation stage in VR3

In the VRs, mathematical language was combined with body language, oral and written language, and cinematic resources *to present* mathematical models, conjectures, step-by-step procedures, and *to emphasise* the key elements in the argumentation. For example, in VR3, mathematical-chemical resources, oral and written language, and cinematic resources were combined with body language and deictic gestures to present a mathematical model and explain the battery charging process. In this part of VR3, attention shifted to a video segment in which a chemistry teacher describes the mathematical model that explains the charging phenomenon. This combination of semiotic resources is distinguished by the inclusion of an embedded video, which not only guided the audience but also represented mathematical concepts, such as linearity within a specific interval (see Fig. 4).

The embedded video contributes as both an explanatory and mediating tool; in this case, it not only guides the audience but also describes processes, visualises concepts, and establishes interrelations between mathematical concepts and those from other disciplines.

Answering: This stage recognises the participants' actions in providing concrete answers to the challenges posed in the VP. For example, we observed that the participants in VR4 used background music, images, text, and voice-over to emphasise the moment when they would answer the question that initiated their modelling process. They stated, "Now we can answer our initial question: it is advisable to stop charging the cell phone at 75% battery when starting from any percentage lower than this value." In this part of the video, we highlight the *juxtaposition of resources*; while Gabi used a voice-over to summarize her findings, she juxtaposed a series of images (Fig. 5) related to acceptance, mobile charging, and environmental care.

Through the sequence and juxtaposition of images, she used coherence and contrast to reinforce the meaning of actions that can be considered environmentally friendly and those that cannot. Based on the principles of social semiotics, the videos enabled well-argued, multimodal responses. Rather than giving direct mathematical answers, participants structured their responses using images, music, text, and other semiotic resources. The layering of these re-

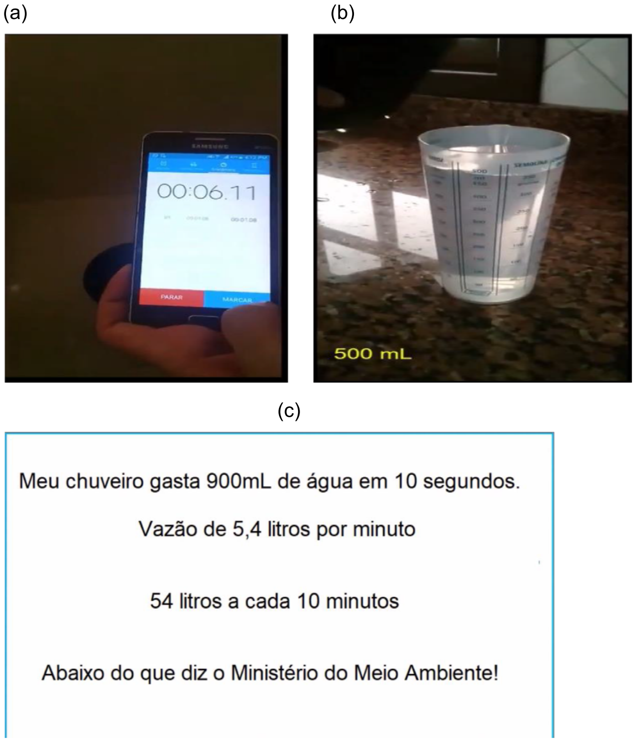


Fig. 3 Replication of the experience of water measuring VR2

sources emphasises the moment of response and reinforces the idea of a conclusion supported by evidence.

Each video concluded with a concrete answer to the questions posed in each video problem. Additionally, some videos presented new questions (VR1), generated recommendations on the topic (VR1, VR3), and offered reflections (VR1, VR3, and VR4). In VR1, the authors stated, “our consumption is below the average established by the WHO.” They then proposed the following problem: “How many days can a swimming pool of 20x10x2 meters guarantee water for personal consumption? (based on the previously calculated daily consumption)” [VR1, 3:49]. In VR3, they concluded,

> “working on this joint project with our students, we identified important knowledge areas to address. Our students noticed that the differences in charging stages were like an affine function but changed after a certain point as the charging process slowed. After discussing with the chemistry teacher, it seems that this part of the charging process behaves like a logarithmic function. Various questions arise, showing that working with a modelling situation can be very productive and even integrate other knowledge areas, providing students with comprehensive understanding. We hope you feel motivated to incorporate modelling in your classes” [VR3, 5:06].

The extension of the problem to other contexts and the formulation of new questions can be seen as an effort to emphasise the significance of studying the topic and its broader implications. In the context of modelling, these elements establish interconnections between the stages of answering and problematising.

According to Social Semiotics, videos contribute to the structuring of a modelling narrative, which involves planning the response in an audiovisual format, organising ideas with clarity and coherence, and emphasising effective communication. This narrative also establishes interconnections between different stages of the modelling process and provide a dynamic perspective on these practices.

The evidence presented in this section collectively highlights key features of the combination of semiotic resources in the creation of VRs. Video production requires participants to actively engage with the material, plan the content, make decisions regarding semiotic resources, and design a structure that effectively communicates a story with clarity and coherence. The participants in this study committed to providing a VR to the challenges presented in the VPs. We identified key features in the combination of semiotic resources, such as simultaneity, contrast, sequencing, replication, complementarity, emphasis, and juxtaposition. These appeared at different modelling stages with varying purposes. Additional details are in electronic supplementary material (Table 1).

6.2 Exploring the Meanings of Aspects of Mathematical Modelling

From our theoretical perspective, each stage of the modelling process can be characterised by specific aspects that help define and delineate it. In this study, we identify these aspects, and the meanings observed in the videos.

6.2.1 Meanings in the Problematising Stage

In VR1, VR3, and VR4, modelling was understood as a process involved in solving a real situation using mathematics. Although the VP offered sufficient information and data to build a mathematical model, the participants in VR1 decided to use different semiotic resources to justify the importance of the topic and the questions posed in the VP. In contrast, VR3 and VR4 adopted the topic from the VP but formulated their own questions based on their interests. For example, in VR3, participants framed cell phone charging as a classroom issue, focusing on students needing outlets. In VR4, battery charging was linked to its environmental impact, emphasizing global awareness and collective responsibility. Table 3 outlines the meanings identified in the problematising stage and the supporting video evidence.

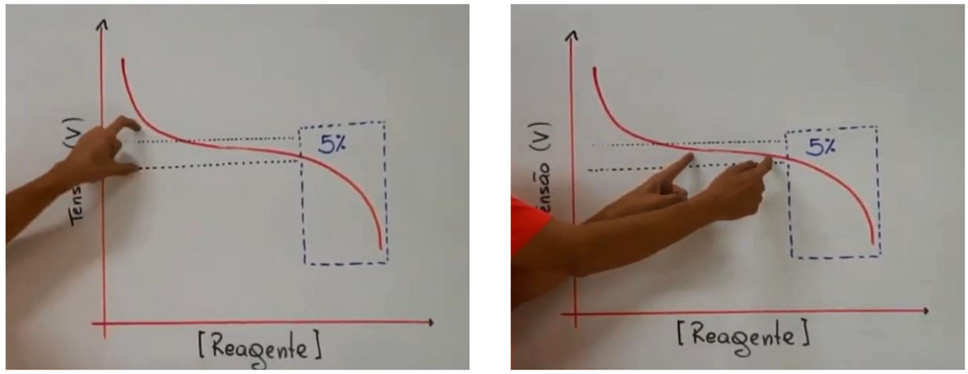


Fig. 4 Deictic gestures to represent mathematical content

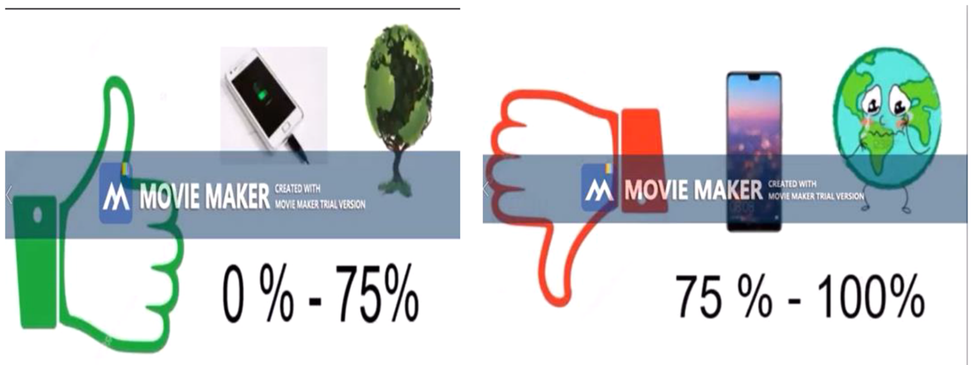


Fig. 5 Juxtaposition of resources

Table 3 Meanings in the problematising stage

VR	Meaning	Description
VR1	The topic (responsible water consumption) is important for people in general. Mathematics is of great importance in studying social issues.	The set design recreates the importance of water. The character (Ari) expresses "We are in the Caldas baths, one of the most important mineral water sources" (Seg 1). He also states, "the keyword is responsible water consumption", and "What does it have to do with mathematics?" (Seg 2). In the transition from Seg 1 to Seg 2, there is a set of images from a natural park with various water sources. The sound of water and music are aligned with the images.
VR3	The topic (charging the battery) is of importance for people in general. Mathematics enables the study of daily life issues.	A set of data about the use of cell phones for daily professional tasks is provided. Data about the number of cell phones in Brazil is presented, and the need for charged batteries is mentioned. The Seg 1 shows some iconography of mobile plugged into a source of energy, warning signals when the cell phone is discharged, and emphasises some key issues.
VR4	The topic (time of battery charge) is of importance to people and the conservation of the planet. Mathematics contributes to the study of environmental and consumption issues.	The set design recreates news about energy consumption, environmental contamination, and their global implications. It presents information about general blackouts and highlights the importance of regulating energy consumption.

6.2.2 Meanings in the Investigating Stage

In this stage, the aspects of modelling were developed mainly with the oral, written, and cinematic resources. The experimental part replicated the way it was exemplified in the VP. In Table 4, we present these meanings, and the evidence found in the video.

6.2.3 Meanings in the Mathematising Stage

The mathematising stage includes exploration of mathematical ideas, concepts and procedures. In the VR2, the mathematising process was characterised by the collection of data and by replicating the strategy provided in the VP.

Additionally, the variety of aspects and meanings around modelling was also minimal. In the VR3, it is possible to

Table 4 Meanings of the aspects of modelling in the investigating stage

Aspect	VR	Meaning	Description
Simplification	VR1	Simplification is a possible "step" in the study of a problem. The "idealised situation can occur" in reality. Opportunity to study the situation using school mathematics content.	Vera says "[...] to simplify the hypothesis of the situation, we will admit the quantity of water as a constant to time" and adds, "keeping the pattern of the opening of the water tap, even to have a constant water temperature".
	VR4	There are explanations from other disciplines that can be coherent with the findings of the mathematical model.	The model, explained from the chemistry, has a section that is "almost linear" and could help understand students' findings.
Selecting the variables	VR3	The knowledge of factors that can influence the situation provides information about variables to consider.	Participants argue that they have observed that the use of original and generic chargers generates different charging patterns.
Data collection and analysing	VR1	The data supplied in the video problem are accommodated to the simplified situation and provide information about the model.	Vera takes the data from the video problem and organises them into two columns. She connects the data from one column with the other through a set of hyphens (—). This approximates the tabular representation without delimiting the variables (symbols) represented.
	VR3	An experimental environment can be generated by inviting students to gather data using tools at their disposal.	They present the data obtained by students using an iconographic language. The data obtained by students with their mobile batteries is organised using Cartesian planes.
	VR4	The data gathered from the experience can be organised and represented to obtain information and results.	They created tables and histograms to represent the data and derive conclusions.
Experimentation	VR1	The model that was built with some data can be used with other data from an experience that contains the same variables and relations. The experience can be recreated to obtain new data.	As part of the challenge in the VP, Vera took data in her home and used the model to obtain results.
	VR2	The experimentation is restricted to the replication of the experience given by others. It is observed to gather data from their context.	Both characters replicated the experience of obtaining water from the shower, measuring with bowls, and using the cell phone as a chronometer.
	VR4	Data can be built from an organised experimental process.	Two experimental processes are presented. The first is a way to collect data, and the second is a form of data validation.

find meaning as a process of studying a topic (affine function) from a particular context and a real need. Nevertheless, this process is not linear and is not over after solving the challenge. On the contrary, it is enriched by experimentation and data collection as well as the understanding of other disciplines about a phenomenon. Different from VR1, VR2, and VR4, this VR3 was not limited to an immediate response or just a replication. The authors of VR3 contextualised the task from the formulation and justification of the question, stating the hypothesis and other variables. In Table 5, we present these meanings, and the evidence found in the video.

Table 5 highlights the emergence of key mathematical semiotic resources (symbolic-numeric, algebraic, graphic, and oral) combined with cinematic elements (image transitions, voice-over, images, videos) and body language (facial expressions, deixis). These resources contribute to support the inductive construction of models, their contextual connections, and argumentation about their validity. Overall, reorganising semiotic resources generated new meanings in model production and the mathematising stage.

6.2.4 Meanings in the Answering Stage

In the VR3, participants also presented some meanings derived from the tendencies in the data (represented in the plot) and the explanations by the chemistry teacher. Table 6 presents these meanings, and the evidence found in VRs.

The meanings presented in Sect. 6.2, taken together, reveal differentiated interpretations, actions and processes of mathematical modelling. These processes emphasised the creation of well-reasoned answers to the questions posed in the video problems (VPs), with a particular focus on ensuring the relevance of these answers to societal concerns. The meanings of modelling aspects were enriched by background music, which not only enlivened the video environments but also encouraged viewer engagement. In some cases, gestures and other deictic registers reinforced and complemented verbal, oral, symbolic, and graphic language.

Table 5 Meanings in the mathematising stage

Aspect	VR	Meaning	Description
Producing the model	VR1	Inductive processes to build models. Mathematical model as a representation of a generalisation. Previous knowledge of a first-degree function as a "model" of a constant variable.	From the data organisation (Fig. 6) there are patterns of data organisation, and she presents a function in the form $f(x) = ax+b$. Relation between $f(x) = ax+b$ in general and $v(t) = 80t$ (Fig. 7)
	VR2	It seems like Construction of representations. Design of tables to organise data and contrast it. Construction of plots to identify patterns and make comparisons.	Some tables were presented with the data gathered from each participant, and participants compared their self-consumptions and answered the challenge question.
	VR4	Construction of representations. The representations help organise the data and establish relationships more directly.	From the tables and histograms, relations of higher, equal, or lower energy consumption can be visualised.
Mathematical model	VR1	Model exemplification. The model can provide information about cases.	The function t =0 in $V(t)=80t$ is evaluated.
	VR3	Mathematical model as tendency. Inductive processes were used as ways to construct models. Mathematics models conceived as a representation of a tendency. Other meaning was mathematical model offers information about the behaviour of a phenomenon.	From the organisation of data in an appropriate scale, there are two observed tendencies. In the first part, a lineal section, and in the other part, a growing concave upward curve. Some interpretations for covariation are presented, for example, one about the constant growth of the charge "getting slower every time" is presented in the last section.
Implications of modelling	VR1	Mathematical models help understand a situation. It is possible to take action to improve the situation based on the findings of the mathematical model. It is possible to generate reflections about the modelled situation.	Vera lists a set of actions that could diminish the flux of water and reduce its consumption. Both Ari and Vera point out that poor people can be more affected by irresponsible water consumption.
Interpretation of results	VR2	The consumption ranges established by some which authorities provided information to "control" their water consumption.	When comparing the results between the characters and guided by the orientations of the Brazilian Ministry of the Environment, they established that something different is happening in Cadu's house, and thus, further inquiry is required.

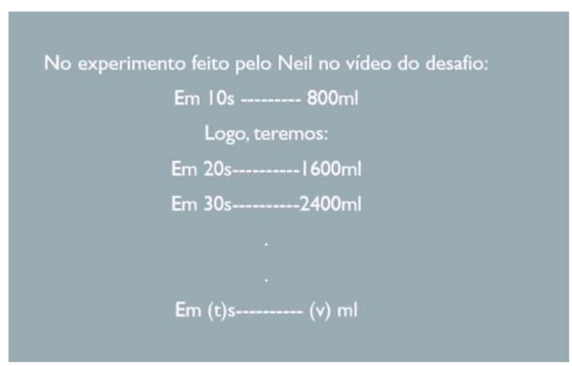


Fig. 6 Organisation of data of VP1 in VR1

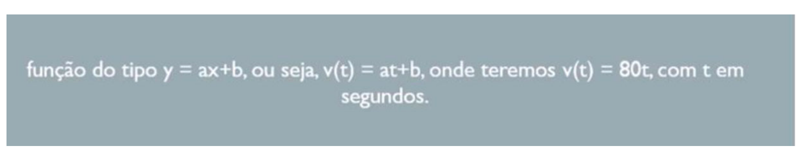


Fig. 7 Contrast between a first-degree function and the function that models the problem

7 Discussion

Videos are widely used in mathematics education in various formats for different educational purposes (Engelbrecht & Borba, 2024; Lobato et al., 2023). However, little is known about students' appropriation of mathematical meanings from instructional videos (Lobato et al., 2023) or their use in non-algorithmic domains such as modelling (Wirth

Table 6 Meanings in the answering stage

Aspect	VR	Meaning	Description
New mathematical task possibilities	VR1	Other ways of calculating water consumption could trigger other reflections about responsible consumption.	A new task that involves calculating of the water in a swimming pool of 20 mx10mx2m and answering the question, "how many days can it provide water for a person?" is stated.
Interdisciplinary option	VR3	The understanding of a phenomenon requires knowledge about other areas.	They invited a teacher to explain the model from a chemical perspective.
Validation	VR3	There are explanations from other disciplines that can be coherent with the findings of the mathematical model.	The model, explained from the chemistry, has a section that is "almost linear" and could help understand students' findings.
Implications of the modelling	VR3	There are explanations from other disciplines that can be coherent with the findings of the mathematical model.	The model, explained from the chemistry, has a section that is "almost linear" and could help understand students' findings.
Problem-solving	VR1	The solution requires the contrast of empirical data with the expected results according to some organisations.	Vera compared the results of the model with data from the VP and her data with the values from the WHO.
	VR3	The trends observed in the empirically collected data can be explained using a known mathematical model.	Some real-world problems can be solved through collaboration with other teachers and interdisciplinary relationships.
	VR4	The solution requires the comparison of the empirical results. The more empirical evidence there is, the more results are trustworthy.	Vera compared the results of the model to data from the VP and her data to the values from the WHO.

& Greefrath, 2024). To understand the contributions that modeller-created videos offer in modelling, we presented a set of VPs to participants of a mathematical modelling teacher education course and analysed the VRs they generated to the proposed challenges.

Regarding the modelling practices, we observed that the four VRs analysed in this paper reflect four different approaches to problem-solving. For example, while VR1 aimed to directly answer the VP's questions by combining various semiotic resources to offer an authentic environment, VR2 focused on replicating the experience with its own data and providing answers to the VP's questions. Both VR3 and VR4 constructed autonomous videos, containing their own questions, processes, solutions, and answers, thus not requiring prior VP review for understanding. VR3 was driven by an interest in using modelling in mathematics class, whereas VR4 aimed to independently demonstrate that the problem of battery charging is related to global warming, emphasising that answering the question helps raise environmental awareness.

As outlined in Sect. 6.1, VRs narrate the development of mathematical practice. Digital videos, as multimodal resources, provide a rich semiotic network for engaging in mathematical modelling practices. Within the stages of modelling, videos can contribute to establish a connection between the audience, the context, and the problem while also offering an immersive experience and a persuasive effect. Additionally, digital videos play a role in reinforcing and emphasising key ideas, as well as in arguing for the relevance of the problem, the process, and its solutions. These contributions demonstrate that videos can integrate different semiotic resources to support the understanding of modelling as an authentic practice—that is, one developed within a context that provides evidence of its real-world existence or global significance, encompassing authenticity in context, practices, and impact (Greefrath & Vos, 2021; Rendón-Mesa et al., 2023).

Across all the VRs, different characteristics and purposes were observed in the combination of semiotic resources. For example, embedded scenes were utilised to highlight arguments and explanations of the model, while the simultaneity of resources was employed to enhance the authenticity and significance of the topic, context, or solutions. We provided evidence that the combination of resources contributed to the emergence of multimodal representations and exemplifications of mathematical objects. The videos also enabled the layering of images, sound, text, and gestures which, according to our findings, help reinforce key meanings and guide the viewer's interpretation. Therefore, the videos contribute to the reorganisation of responses to modelling problems, framing them as situated narratives grounded in the creators' experiences, contexts, and interests.

Previous studies have highlighted the need to provide experiences, evidence of actions, learning, decisions, and reflections produced by students and teachers during their modelling projects, as well as the purposes of using technologies in these works (Sánchez-Cardona et al., 2021; Villarreal et al., 2018). Videos also contribute to production of meanings about modelling practices. As presented in this study, mathematical modelling can be understood as a practice for solving real problems through mathematics. This meaning has been widely recognised in the literature on mathematical modelling at different educational levels (e.g., Ocampo-Arenas & Parra-Zapata, 2022; Sánchez-Cardona et al., 2021). This meaning was common in VR1, VR2, and VR3; however, the use of mathematics and the production of mathematical models were different in each VR, even when responding to the same VP. For instance, in VR4, the data were organised to identify tendencies, discussed with other specialists, and enriched with interdisciplinary knowledge.

In VR1, the models were adjusted to data based on the simplifications of the problem. Other studies (Carreira et al., 2020) have also shown the meanings that participants have attributed to certain aspects of modelling. Carreira et al. (2020) demonstrated that students at different educational levels could reveal meanings to produce models (e.g., a mathematical model means finding rules that allow them to accept or reject a criterion). Like these authors, we agree that models can be produced based on mathematical knowledge; however, we also argue that some of them are produced based on the media involved in the modelling process. For example, for Carreira et al., the creation of simulations was fundamental for validating the models and results produced in the modelling process. In our case, in addition to simulations, we observed that in VR4, the sequences of images, the contrast between scenes, the background music, and the emphasis on voice intonation combined to create an environment of confidence in the simulations performed.

As presented in Sect. 6.2, the VRs reveal meanings for various aspects of modelling. While these meanings may be shared with other media, such as pencil and paper, our study demonstrates that videos contribute to understanding modelling practices as complete and situated stories. These stories are constructed based on the decisions made by their creators and reflect the aspects they consider relevant in the process. They uniquely combine semiotic resources to address multiple purposes across different modelling stages. According to our findings, the combination of visual, auditory, cinematic, body, and mathematical elements allowed for contrasting, arguing, emphasising, extending, and reflecting on key aspects of modelling practices. Jewitt et al. (2016) pointed out that "meanings derive from social action and interaction using semiotic resources as tools" (p. 58). While modeller-created videos can convey modelling meanings alongside other resources and media, this study highlights that the purposes and ways in which these meanings emerge are distinct, conditioned by the creators' interests and objectives.

The analysis of VRs showed that, in some cases, the information provided in the VP can limit the processes of the solution. In the case of Cadu and Leda (VR2), the solution to the challenge was restricted to a "replication" of an experience described in the VP. This solution is far away from a conception of modelling as a process that is rich in actions, paths, processes, and results. A possible explanation for this is that these participants perceived this challenge as simple. This perceived simplicity could have inhibited diverse semiotic resources (modes) and the emergence of new contexts and questions thus constraining the construction of models of different mathematical nature (Borba et al., 2024).

The conditioning of the topic, the action, and the replication of the experience widen the list of challenges found by Wirth and Greefrath (2024) for the use of videos in mathematical education. According to these authors, the two main challenges for the instructional use of videos are the length of the videos and the transferring of what students had studied to the subsequent related modelling problem. The results of this study show a new challenge: It is necessary to ensure that the information offered in the VP is enough to develop the mathematical modelling process that includes a diversity of aspects and semiotic resources but, at the same time, is not restrictive of the activity of the mathematical modeller (student or teacher).

8 Conclusion

This study enriches the scarce body of literature on videos in mathematics education for non-algorithmic domains such as modelling. Two significant findings from this study can be highlighted.

The first is the contribution of videos to mathematical modelling, with evidence demonstrating features such as simultaneity, contrast, sequencing, replication, complementarity, and juxtaposition as combinations of semiotic resources observed in the VRs. This result highlights how combining different semiotic resources shapes the meaning of modelling and its key aspects. The second important finding is the emergence of new challenges in using educational videos, particularly VPs and VRs.

The results suggest that the richest VRs in terms of segments and semiotic resources are also richer in terms of the aspects of modelling and the diversity of meanings. Despite this being a qualitative correspondence, it opens new paths to conduct further research on this type of correspondence both in modelling and other non-algorithmic domains.

In the teaching of modelling, there is still an open debate about task management by teachers, especially about the appropriate ways of guiding the activities of students and providing adequate information that will avoid limiting the development of modelling by students. The findings of this study extend this debate to videos and caution that some VP elements may unintentionally inhibit the students' modelling activity.

Despite the uniqueness of the studied cases, the scope of this study is limited to the conditions under which the course was developed and the methods and frameworks of the research; therefore, further research is needed to explore other forms of modelling, other meanings, and other potentialities that other semiotic resources might contribute.

The type of video produced by the participants responds to a VP of mathematical modelling; other studies incorporating different formats deserve to be developed. This would provide us with evidence to understand more deeply the conditions that other resources generate in mathematical modelling.

Although some implications can be drawn for teacher education, our theoretical framework and methodology did not allow us to focus on the opportunities that these types of productions offer for professional development. Therefore, other analyses might yield different results that could contribute more directly to the design of courses, interactions with teacher educators, and teacher education programs.

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